On the log-Poisson statistics of the energy dissipation field and related problems of developed turbulence

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An energy cascading model of intermittency involving rare localized regions of both large and/or weak energy dissipation (dynamical intermittency) is considered and compared to the case of intermittency arising from a large number of regions with nearly equal dissipation rates (space intermittency). The latter leads to the log-normal statistics of the dissipation rate while the first scenario leads to shifted log-Poisson distributions either for a large or for weak energy dissipation. The only difference between these two cases is that small values of dissipation (with respect to the maximum of PDF) are more probable for intermittency of the regions with weak dissipation than for intermittency of the regions with large values of dissipation. Some consequences are derived which show that Novikov's inequalities are valid for intermittency with rare regions of a weak dissipation only. Different experimental data of probability distributions of dissipation are presented and compared to theoretical predictions. Some experimental evidences of quasi-two-dimensional vortical structures with weak dissipation are discussed. They suggest that the scenario involving dynamical intermittency with holes of dissipation could apply to a real world turbulence. © *1996 American Institute of Physics.* [S1070-6631(96)00912-9]

I. INTRODUCTION

One of the main and, still, incompletely understood features of fully developed turbulent flow is the intermittency of the energy cascading process. Physically speaking, this means that active (in the sense of energy dissipation) and inactive regions in the flow do not spread uniformly over the entire turbulent region but concentrate into some volumes. Several models of intermittency have been proposed. The first one is the well known log-normal model[1,2] of the energy dissipation field $\epsilon(\mathbf{x},t) = \nu/2\Sigma_{i,j}(\partial u_i/\partial x_j + \partial u_j/\partial x_i)^2$. This model can be interpreted as one of the class of multifractal models,[3] which are based on the assumption of multifractality of the energy dissipation measure. The variety of intermittency models and their assumptions about the energy dissipation distributions are reviewed and compared in Refs. 4, 5. Recently, She and Lévêque[6] have proposed a model of turbulence which leads to a log-Poisson statistics for the energy dissipation field $\epsilon(\mathbf{x},t)$, as pointed out by Dubrulle[7] (see also Ref. 8). By contrast to previous pictures, intermittency in this model results from the existence of localized, rare non-dissipative regions in the flow.

In this paper, we study the small scale intermittency statistics on the basis of a dynamical cascade process in the spirit of the Novikov–Stewart[9] model of which the dynamical version is the well known β-model proposed by Frisch *et al.*[10] The model presented here has some common points with several previous models such as p-model of Meneveau and Sreenivasan[11,4] (Sec. I), models of distribution of the multipliers[12–15] and also may be interpreted from a fractal point of view.[16] This model is a cascade model which includes dissipatively active and passive localized regions in the flow. The cascade breakdown of these regions of dissipation contains several free parameters. By construction, the energy dissipation field is self-similar if the parameters of the model do not depend on the scale of the dissipative structures. In the case of a large number of breakdowns we obtain as the limiting cases a log-Poisson and a log-normal distribution for the energy dissipation. The log-Poisson distribution is valid either for strong dissipatively active or passive structures with the difference that the former violates Novikov's inequalities; the scaling of dissipation is a power law. So we arrive at the conclusion that small scale intermittency results from the existence of the holes of dissipation. This is the main result of the paper. In the last part we give various experimental evidences supporting the existence of the holes of dissipation in high Reynolds number turbulence. This concept of intermittency originating from holes of dissipation developed during last years by different authors is derived here from a different point of view.

Our paper is in direct connection with the results of She and Lévêque,[6] Dubrulle[7] and She and Waymire.[8] In the paper of Novikov[35] these results were estimated from the point of view of a ''gap problem'' of PDF. The cascade picture of energy dissipation breakdown introduced in Ref. 8 is in the spirit of the orthodox mechanism presented below. It was shown in Ref. 8 that this picture corresponds to a random multiplicative process (using the terminology of probability theory) which, with the help of a general decomposition theorem of infinitely divisible processes, leads to a log-Poisson statistics. In our paper we avoid the references to such general theorems, preferring a physical argumentation. This allows us to derive, in the limit case of log-Poisson distribution, concrete PDF's which can be fitted to experimental data. Also we show that the log-Poisson statistics is one of limiting PDF which can be obtained from the dynamical cascade described in the paper. Another limiting case is

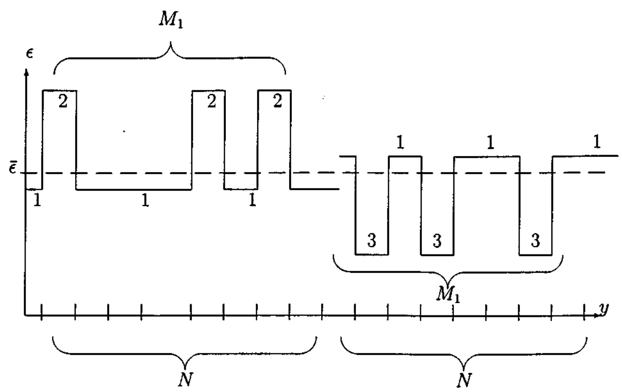

FIG. 1. One-dimensional illustration for energy dissipation: 1—dissipation in non-intermittent volumes; 2 and 3—active dissipation passive (quasi-zero) dissipation volumes.

the log-normal distribution which, in spite of theoretical difficulties that we discuss, is widespread in the literature on turbulence.

The paper is organized as follows. In Sections II and III the model of cascade and the associated distributions of the dissipation rate are described. In Sections IV, V we discuss some consequences on the moments of $\epsilon(\mathbf{x},t)$ and on the variance of the logarithm. The main point is that this model contains both the intermittency originating from very ''active'' dissipative localized regions, and the intermittency coming from filamentary vortex-like structures which are regions of weak dissipation (referred to hereafter as ''passive'' dissipative zones or ''holes'' of dissipation). We show how the log-Poisson statistics of the dissipation field results from a mixture of both regions in the flow. Finally, in Section VI, we comment on some experimental data which support the ideas presented before.

II. DYNAMICAL INTERMITTENCY CASCADE

Let us consider a volume V of turbulence with mean energy dissipation rate per unit mass $\overline{\epsilon}$. The total energy dissipation in the volume is equal to $\overline{\epsilon V}$. Let us assume that in the volume V M_1 active (or passive) regions of larger (or smaller) than $\overline{\epsilon}$ rate of dissipation appear, as shown in figure 1. (The non-intermittent zones are labeled by 1, the intermittent zones with large dissipation by 2, and the intermittent zones with small dissipation by 3.) The volume of each region is equal to V/N (V is divided in N small volumes among which M_1 are more (or less) active than the others $N-M_1$). The mean energy dissipation rate of each of the M_1 active (or passive) volume is assumed to be

$$\epsilon_1^{(1)}=\overline{\epsilon}\left(\frac{N}{M_1}\right)^{\alpha_1}, \tag{1}$$

where the exponent α_1 is such that for $\alpha_1>0$ the corresponding volume is dissipatively active, with increased dissipation rate (regions 2 in figure 1), and for $\alpha_1<0$ the volume is dissipatively passive meaning that its dissipation is less than mean dissipation rate $\overline{\epsilon}$ (regions 3 in figure 1). We also choose $M_1<N/2$ because in other case a given active (or passive) region becomes non-intermittent (non-active) and non-intermittent region becomes passive (or active).

The mean energy dissipation rate in the rest $N-M_1$ regions of volume V is

$$\epsilon_2^{(1)}=\overline{\epsilon}\,\frac{1-(M_1/N)^{1-\alpha_1}}{1-M_1/N}. \tag{2}$$

In (1), (2) the lower index 1 refers to a dissipation in active ($\alpha_1>0$) or passive ($\alpha_1<0$) volumes and the index 2 to a non-intermittent volume. The expression (2) is consistent with the conservation of the total dissipation $\overline{\epsilon V}$ in volume V.

Note that the Novikov–Stewart[9] model is restored if $\alpha_1=1$ in (1), (2), corresponding to $\epsilon_2^{(1)}=0$. In this limit, all dissipation is concentrated in the M_1 active volumes only.

Instead of (1) and (2), we will now use logarithms of the relative dissipation rates, defined as

$$\ln\left[\frac{\epsilon_1^{(1)}}{\overline{\epsilon}}\right]=\gamma_1,\quad \ln\left[\frac{\epsilon_2^{(1)}}{\overline{\epsilon}}\right]=-\kappa_1 r_1\gamma_1,$$
$$\gamma_i\equiv\gamma_i(\alpha_i,r_i)=-\alpha_i\ln r_i,\quad r_i=\frac{M_i}{N}, \tag{3}$$
$$\kappa_i\equiv\kappa(\alpha_i,r_i)=\ln\left[\frac{1-r_i}{1-r_i^{1-\alpha_i}}\right][r_i\gamma_i]^{-1}.$$

Now, in the spirit of a dynamical cascade process (in space, see Ref. 10), we consider that each of the M_1 active (passive) regions consists of N small regions, among which only M_2 are active (passive) compared to the dissipation rate of (1). In these M_2 volumes the energy dissipation rate is equal to

$$\epsilon_1^{(2)}=\overline{\epsilon}\left(\frac{N}{M_1}\right)^{\alpha_1}\left(\frac{N}{M_2}\right)^{\alpha_2}. \tag{4}$$

For the rest $N-M_2$ non-active volumes in each of M_1 volumes of the first level, the energy dissipation rate is

$$\epsilon_2^{(2)}=\overline{\epsilon}\left(\frac{N}{M_1}\right)^{\alpha_1}\frac{1-(M_2/N)^{1-\alpha_2}}{1-M_2/N}, \tag{5}$$

The same process holds for the $(N-M_1)$ non-active volumes and goes on for the subsequent steps of cascade, $i=2,3,\ldots$ etc. In (1)–(5) the upper index corresponds to the step of the cascade, so $\epsilon_1^{(i)}$ and $\epsilon_2^{(i)}$ are the energy dissipation rates in active (passive) and non-active volumes after i steps of cascade.

Now we make the two following assumptions:—(I) **Independency:** The cascade of the dissipation rate in any volume at the step i does not depend on the processes in other regions at any other step j ($j\neq i$);—(II) **Developed turbulence:** The cascade process in any intermittent active (or passive) region is completed when the scale of smallest active (passive) volume l_η reaches the Kolmogorov scale η. The number of cascade levels is large.

This model of cascade contains the following parameters.

(a) The number N defines the breakdown coefficient Q between the characteristic scales of successive steps i and $i+1$ such as

$$\frac{l_{i+1}}{l_i}=N^{-1/3}=Q^{-1}, \quad Q=N^{1/3}. \tag{6}$$

We consider that the parameter Q can take any values $Q>1$ but is constant for all i. Note that for a cascade with a large number of levels, $Q\rightarrow 1$.

(b) The ratio r_i is such that at a given level i,

$$r_i=\frac{M_i}{N}<\frac{1}{2}, \tag{7}$$

defines the number M_i of active (or passive) regions at a given scale l_i which are contained in any of the M_{i-1} corresponding volumes of the previous level. This condition is linked with our definition of active (or passive) regions. Parameters r_i mark the space intermittency of cascade process.

(c) The exponent α_i is indicative of the strength of the relative energy dissipation rate in active (or passive) volumes at a given scale l_i. The corresponding parameters γ_i characterize the dynamical intermittency of cascade.

(d) Parameters κ_i (which depend on r_i, α_i) indicate the level of dissipation in non-intermittent regions.

III. PROBABILITY FUNCTIONS OF THE ENERGY DISSIPATION RATE

We consider a test volume $v_q=VN^{-q}$, $q\geqslant 1$, at the scale defined by (6),

$$l_q=\frac{L}{Q^q}, \quad L=V^{1/3}, \tag{8}$$

where L corresponds to the largest scale in the flow.

This volume is one of the M_1 active (passive) regions of first level with probability M_1/N. With the probability $(1-M_1/N)$, the test volume is covered by non-active regions. So, with probability M_1/N the logarithm of the relative dissipation rate, $\ln(\epsilon/\overline{\epsilon})$, is equal [according to (3)] to $\gamma_1=-\alpha_1\ln(M_1/N)$, and with probability $(1-M_1/N)$ to $-\kappa_1 r_1\gamma_1$.

If the test volume is found to be in one of the M_1 active (passive) volumes of first level, it may be covered with probability M_2/N by one of the M_2 active (passive) regions of the second step. The value of the logarithm $\ln(\epsilon/\overline{\epsilon})$ is now increased by $\gamma_2=-\alpha_2\ln(M_2/N)$ and becomes equal to $\gamma_1+\gamma_2$. Symmetrically, $\ln(\epsilon/\overline{\epsilon})$ is increased by $\gamma_1-\kappa_2 r_2\gamma_2$ with probability $(1-M_2/N)$.

If the test volume is in the other $N-M_1$ regions of first level, $\ln(\epsilon/\overline{\epsilon})$ takes the value $-\kappa_1 r_1\gamma_1+\gamma_2$ with probability M_2/N and the value $-\kappa_1 r_1\gamma_1-\kappa_2 r_2\gamma_2$ with probability $(1-M_2/N)$.

We can now write the generating function of the random value $x=\ln(\epsilon_q/\overline{\epsilon})$ with q steps of the cascade when the smallest scale of energy dissipation volumes becomes equal to l_q (8),

$$\varphi_q(z)=\int P_q(x)z^x dx=\prod_{i=1}^{q}\varphi_{\Delta_i}(z), \tag{9}$$

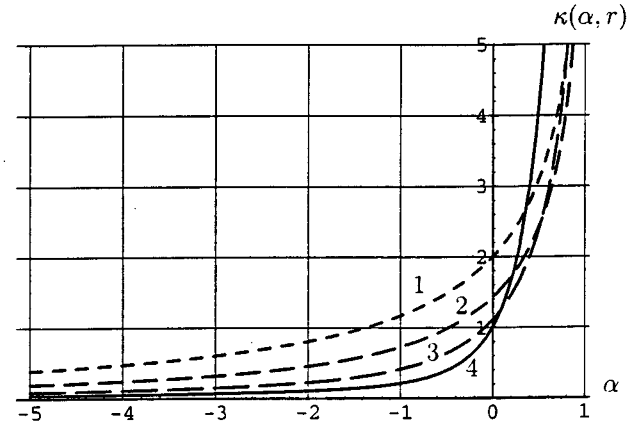

FIG. 2. Function $\kappa(\alpha,r)$ (3) for (1) $r=0.5$; (2) $r=0.3$; (3) $r=0.1$; (4) $r=0.01$.

where $\varphi_{\Delta_i}(z)$ is the generating functions of step i. In (9) the conditions of independency (I) and of developed turbulence (II) have been used, and $P_q(x)$ is the probability of x after q steps of cascade.

We have

$$\varphi_{\Delta_i}(z)=\int P_{\Delta_i}(x)z^x dx,$$

with $P_{\Delta_i}(x)$ the probability of x at step i. For this value and according to the procedure described above, we write

$$P_{\Delta_i}(x)=r_i\delta(x-\gamma_i)+(1-r_i)\delta(x+\kappa_i r_i\gamma_i), \tag{10}$$

which states that the quantity x is equal to γ_i with probability r_i and to $-\kappa_i r_i\gamma_i$ with probability $(1-r_i)$.

We thus obtain from (10) and (9)

$$\begin{aligned}\varphi_q(z)&=\prod_{i=1}^{q}[r_i z^{\gamma_i}+(1-r_i)z^{-\kappa_i r_i\gamma_i}]\\&=\prod_{i=1}^{q}\left[r_i z^{\gamma_i}+(1-r_i)\left(\frac{1-r_i^{1-\alpha_i}}{1-r}\right)^{\ln z}\right]\\&=z^{-\sum_{i=1}^{q}\kappa_i r_i\gamma_i}\prod_{i=1}^{q}[1+r_i(z^{\gamma_i(1+\kappa_i r_i)}-1)].\end{aligned} \tag{11}$$

We shall consider several particular limit cases.

A. Dynamical intermittency

Let us consider the case of a small number of intermittent active (or passive) volumes with finite and equal increments (decrements) γ_i:

$$r_i=\frac{M_i}{N}\ll 1, \quad \kappa_i r_i\ll 1, \quad \gamma_i=\gamma=\text{const.} \tag{12}$$

Figure 2 shows the function $\kappa(\alpha,r)$ for several values of r. So, the condition $\kappa_i r_i\ll 1$ in (12) is valid if α_i are not too close to 1 which is the case of Novikov–Stewart model. In this case the values $\kappa(\alpha,r)$ become very large, and the condition $\kappa_i r_i\ll 1$ is violated.

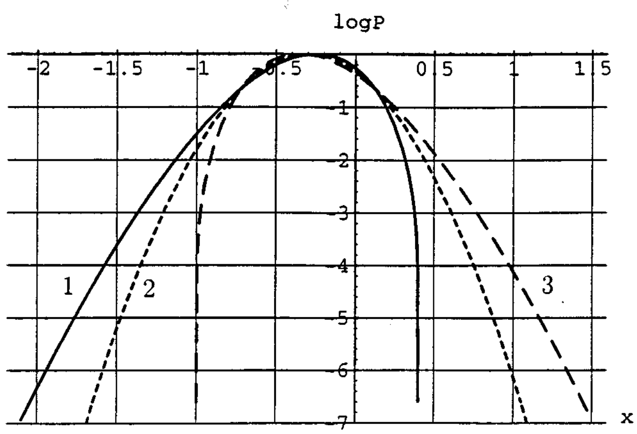

FIG. 3. Normalized probability functions of $x=\ln(\epsilon_q/\overline{\epsilon})$ for (1) distribution (16) with $|\gamma|=0.1$, $n_q=6$, $m_q=3$; (2) normal distribution for x with mean value $\langle x\rangle=-0.3$ and variance $D=0.06$; (3) distribution (15) with $|\gamma|=0.1$, $n_q=6$, $m_q=9$.

For small r_i we also assume that for large enough q, the sums $\Sigma_{i=1}^q \kappa_i r_i$ and $\Sigma_{i=1}^q r_i$ are finite,

$$\sum_{i=1}^{q} r_i = n_q, \quad \sum_{i=1}^{q} \kappa_i r_i = m_q. \tag{13}$$

With (12), (13) the generating function may be approximately represented by

$$\varphi_q(z) \approx z^{-\gamma m_q} \prod_{i=1}^{q} \exp[r_i(z^{\gamma(1+\kappa_i r_i)}-1)]$$

$$\approx z^{-\gamma m_q} \exp[n_q(z^\gamma - 1)]. \tag{14}$$

Introducing instead of z a new variable $\zeta = z^{|\gamma|}$ we obtain from (14) the probability function of the normalized logarithm of the dissipation rate, $k = x/|\gamma|$.

(a) For active dissipative volumes, value $\gamma > 0$ in (12) ($\alpha_i > 0$),

$$P_k^+(m_q, n_q) = e^{-n_q} \frac{n_q^{k+m_q}}{(k+m_q)!}, \quad k \geq -m_q. \tag{15}$$

(b) For passive dissipative volumes, $\gamma < 0$ ($\alpha_i < 0$),

$$P_k^-(m_q, n_q) = e^{-n_q} \frac{n_q^{m_q-k}}{(m_q-k)!}, \quad k \leq m_q. \tag{16}$$

In (15), (16) we assume that $k+m_q$, m_q-k are integers. Only discrete values of k are allowed in the model. This model is essentially a quantized approximation of the real continuous process which can be obtained in the limit case of $Q \to 1$, $q \to \infty$, $r_i \to 0$. Accurate statements can be obtained, but are out of the scope of this paper which focuses on physical arguments.

Equation (15) defines a shifted (by the amount $-m_q$) log-Poisson distribution of the random value $k = x/|\gamma|$ of mean value and variance,

$$\langle k\rangle = n_q - m_q, \quad \sigma_k^2 = \langle (k - \langle k\rangle)^2\rangle = n_q. \tag{17}$$

In (16), the sign of the shift changes; $\langle k\rangle = m_q - n_q$.

In figure 3 the normalized probability functions of $\ln(\epsilon_q/\overline{\epsilon})$ corresponding to the both shifted log-Poisson distributions (15), (16) are plotted for parameters $|\gamma|=0.1$, $n_q=6$, $m_q=9$ [for (15) see curve 3 in fig. 3], $n_q=6$, $m_q=3$ [for (16) see, curve 1 in figure 3]. Curve 1 in figure 3 shows that for cascade with the passive dissipation volumes the small values of $\epsilon_q/\overline{\epsilon}$ (with respect to the maximum of the PDF) become more probable than the larger values of $\epsilon_q/\overline{\epsilon}$. Evidence of this behaviour was indicated by Vincent and Meneguzzi[17] who showed that the dependence of the logarithm of the dissipation PDF is close to linear for large negative values of $\ln(\epsilon_q/\overline{\epsilon})$. This is the case of distribution (16) for large negative k. But it should be noted that for large positive $\ln(\epsilon_q/\overline{\epsilon})$ the probability (16), being equal zero, underestimates the PDF's which can be obtained in experiments or numerical simulations.

B. Space intermittency

Another limit case can be considered when instead of (12) we have

$$\gamma_i \to 0, \tag{18}$$

with any r_i ($\leq 1/2$) and relaxing the condition of constant logarithmic increments (decrements) $\gamma_i = \gamma$. The condition (18) necessary also means $\alpha_i \to 0$. Proceeding to characteristic function, $z = e^{i\xi}$, now we have for the right hand side of (11),

$$\varphi_q(\xi) \approx e^{-i\xi \Sigma_{i=1}^q \gamma_i \kappa_i r_i} \prod_{i=1}^{q} \{1 + i\xi\gamma_i r_i(1+\kappa_i r_i)$$

$$- \xi^2 \tfrac{1}{2} r_i [\gamma_i(1+\kappa_i r_i)]^2\}$$

$$\approx e^{-i\xi x_0 - \xi^2 (D/2)},$$

$$x_0(q) = \sum_{i=1}^{q} \gamma_i r_i [\kappa_i(1-r_i) - 1],$$

$$D(q) = \sum_{i=1}^{q} \gamma_i^2 r_i (1-r_i)(1+\kappa_i r_i)^2. \tag{19}$$

We use in (19) the second order expansion on γ_i as is usually done to obtain central limit theorem (for turbulence, see, for example, Ref. 12), considering that with condition (18) the values $x_0(q) \sim D(q) \sim \Sigma_{i=1}^q \gamma_i^2$ are finite but $\Sigma_{i=1}^q \gamma_i^m \to 0$, $m \geq 3$.

From (3), (18) for κ_i, $x_0(q)$, $D(q)$ we obtain

$$\kappa_i \approx \frac{1}{1-r_i}\left[1 + \frac{\gamma_i}{2(1-r_i)}\right], \quad 1 + \kappa_i r_i = \frac{1}{1-r_i},$$

$$x_0(q) = \sum_{i=1}^{q} r_i \gamma_i^2 \frac{1}{2(1-r_i)}, \quad D(q) = \sum_{i=1}^{q} r_i \gamma_i^2 \frac{1}{1-r_i}. \tag{20}$$

So, if now instead of conditions (13) we assume that sums $x_0(q)$ and $D(q)$ in (20) are finite for $\gamma_i \to 0$, we obtain for (19) a Normal distribution for x, the logarithm of the relative energy dissipation rate $x = \ln(\epsilon/\overline{\epsilon})$, with mean value $\langle x\rangle = -x_0(q)$ and variance $\langle (x - \langle x\rangle)^2\rangle = D(q)$. Remind that this holds for small relative amplitudes of the energy dissipation rate [in active or passive volumes, because $x_0(q)$ and $D(q)$ do not depend on the sign of α_i, $|\alpha_i|\ll 1$]. Of course, log-normal distribution also follows from (14) for $r\ll\gamma\to 0$ with $x_0(q)=\gamma^2 n_q/2$, $D(q)=\gamma^2 n_q$.

In figure 3 we show the log-normal distribution (curve 2 in figure 3) with the mean value $\langle x\rangle=-0.3$ and variance $D=0.06$ in comparison with the log-Poisson distributions (15), (16).

IV. THE HYPOTHESIS OF THE VARIANCE OF THE LOGARITHM OF DISSIPATION

To link the characteristics n_q, m_q (13) of the shifted log-Poisson distributions (15), (16) with the scale l_q (8) of the volume in which we define the energy dissipation rate, it is necessary to specify the distributions of $r_i=M_i/N$.

A. Kolmogorov–Obukhov's model

The simplest choice is a constant value r for all levels i:

$$r_i=\frac{M_i}{N}=r=\text{const}\ll 1. \tag{21}$$

With the condition $\gamma_i=\gamma=$const (12), this choice corresponds to $\alpha_i=\alpha=$const, hence, the values κ_i do not depend on i and $\kappa_i=\kappa$ in (3). We thus have

$$\kappa=\frac{1}{r\gamma}\ln\left[\frac{1-r}{1-re^{\gamma}}\right],\quad \gamma=\alpha\ \ln\left(\frac{1}{r}\right). \tag{22}$$

For n_q, m_q we now obtain from (13)

$$n_q=qr=\mu_r\ \ln\frac{L}{l_q},\quad \mu_r=\frac{r}{\ln Q},\quad m_q=\kappa n_q, \tag{23}$$

where the index q corresponding to the scale l_q is found from (8) as

$$q=\frac{1}{\ln Q}\ln\frac{L}{l_q}. \tag{24}$$

The value n_q gives the variance of the normalized logarithm of dissipation (17), $k=\ln(\epsilon/\overline{\epsilon})/|\gamma|$. So, equation (23) for n_q leads to the Kolmogorov–Obukhov[1,2] assumption about this variance, i.e. a logarithmic dependence on the size of the system:

$$\sigma^2_{\ln\epsilon_q}=\mu\ \ln\frac{L}{l_q},\quad \mu=\mu_r\gamma^2. \tag{25}$$

Note that this result is valid for any distribution of (15), (16) and also for a log-normal distribution (19). The hypothesis of constant ratios $r_i=M_i/N=r$ for all steps of the cascade does not take into account the presence of a viscous cutoff l_η.

B. Power-law model

With increasing step i (reduction of the scales) it seems more reasonable that the values r_i increase (increase of spatial intermittency). Assuming (as usual for cascade or shell models) that for the neighbouring steps the ratio $r_{i+1}/r_i=e^{\delta}$ is constant, expressing the invariance by expansion of the system, we have the relation

$$r_i=re^{\delta i},\quad \delta>0,\quad r\ll 1, \tag{26}$$

which reduces to (21) if $\delta=0$.

But together with relation (26) we have to take into account some ‘‘boundary’’ condition in connection with our condition (7) about the notion of intermittent active (or passive) region, that is $r_i<1/2$. So, for some scale l_λ above the viscous scale l_η the corresponding value r_i, $i\sim i_\lambda$ is of order 1/2. The number i_λ is defined by (8), and we have the following estimation for δ:

$$i_\lambda=\frac{1}{\ln Q}\ln\frac{L}{l_\lambda},\quad re^{\delta i_\lambda}\sim\frac{1}{2},\quad \frac{\delta}{\ln Q}\sim\frac{\ln(1/2r)}{\ln(L/l_\lambda)}.$$

For n_q we obtain

$$\begin{aligned} n_q&=r\sum_{i=1}^{q}e^{\delta i}=\frac{re^{\delta}}{e^{\delta}-1}(e^{q\delta}-1)\approx b_0\left(\frac{L}{l_q}\right)^{\beta_\lambda},\\ \beta_\lambda&=\frac{\ln(1/2r)}{\ln(L/l_\lambda)},\quad b_0=\frac{r}{e^{\delta}-1}.\end{aligned} \tag{27}$$

Equation (27) for the variance of the normalized logarithm of dissipation corresponds to the power-law scaling model with the relation $\beta_\lambda\sim 1/\ln Re_\lambda$ suggested by Castaing *et al.*[18,19] where the standard estimation for the Reynolds number, $L/l_\lambda\sim Re_\lambda^{3/4}$ has been used in (27).

The assumption (21) of constant scale ratios r, leading to the Kolmogorov–Obukhov law (25) for $\ln(\epsilon/\overline{\epsilon})$ appears much adapted to large scales of the inertial range, in which the viscosity plays no role. On the other hand, the scale repartition (26) with its power-law (27) suits to viscous-dominated scales, also called the intermediate dissipation range (Frisch and Vergassola[20]). This point was already pointed out in Ref. 21.

V. MOMENTS OF THE ENERGY DISSIPATION RATE

Let us consider the moments of the energy dissipation rate ϵ_q at scale l_q, which, with the relation $u_q\sim(\epsilon_q l_q)^{1/3}$ give the moments of velocity increments u_q at this scale. Using $x=\ln(\epsilon_q/\overline{\epsilon})$, $k=x/|\gamma|$ and the sifted log-Poisson probability function for x with $\gamma>0$ (15) we have

$$\begin{aligned}\langle\epsilon_q^p\rangle&=\overline{\epsilon^p}\langle e^{pk|\gamma|}\rangle=\overline{\epsilon^p}\sum_{k=-m_q}^{\infty}e^{pk|\gamma|}e^{-n_q}\frac{n_q^{k+m_q}}{(k+m_q)!}\\ &=\overline{\epsilon^p}e^{-n_q}P^{-m_q}e^{Pn_q},\quad P=e^{p|\gamma|}.\end{aligned} \tag{28}$$

For the distribution function (16) with $\gamma<0$, we have the same expression as (28), with $P=e^{-p|\gamma|}$. Hence, for both cases (15) and (16) we obtain

$$\begin{aligned}\langle\epsilon_q^p\rangle&=\overline{\epsilon^p}e^{-n_q}P^{-m_q}e^{Pn_q}=\overline{\epsilon^p}e^{n_q(P-1-p\kappa\gamma)},\\ P&=e^{p\gamma},\quad \gamma=\alpha_i\ \ln 1/r_i=\gamma_i.\end{aligned} \tag{29}$$

A. Intermittency from dissipatively active regions

In the limit $r_i=$const (Kolmogorov–Obukhov assumption) where (23) is valid, we have

$$\langle \epsilon_q^p \rangle = \overline{\epsilon^p} \left(\frac{l_q}{L} \right)^{\tau_p}, \quad \langle u_q^p \rangle \sim \left(\frac{l_q}{L} \right)^{\zeta_p},$$

$$\tau_p = \mu_r (1 + \kappa \gamma p - e^{p\gamma}), \quad \zeta_p = \frac{p}{3} + \tau_{p/3}. \tag{30}$$

The second relation in (30) is the consequence of the relation assumed correct between the statistics of the velocity increments u_q and local energy transfer (or dissipation) rate ϵ_q, $u_q^3 \sim l_q \epsilon_q$ (see, for example, Refs. 22, 34).

As already noted in the Introduction (see Ref. 22), it follows from (30) that the cascade with active dissipation regions, $\gamma > 0$, violates (because of the exponential term $e^{p\gamma}$), for large order $p > 0$, the Novikov inequalities for τ_p, such as $\tau_p > 2 - p + \tau_2$, or the corresponding inequalities for $\zeta_p, \zeta_{2p+2} > \zeta_{2p}$. The present case is even more drastic than for the log-normal model, because in (30), τ_p has an exponential dependence on $p\gamma$. The log-normal model can also be obtained from (30) with $\gamma \ll 1$ [see (18)]:

$$\tau_p = \mu_r [p\gamma(\kappa - 1) - p^2 \gamma^2 / 2].$$

B. Intermittency from dissipatively passive regions

In the case of a cascade with intermittent dissipatively passive regions, $\gamma < 0$, the behaviour of τ_p is quite opposite. We obtain instead of (30)

$$\tau_p = \mu_r (1 - p\kappa |\gamma| - e^{-p|\gamma|}). \tag{31}$$

From (22) with $\gamma = -|\gamma|$ it follows that for $r \to 0$,

$$\kappa |\gamma| = \frac{1}{r} \ln \frac{1 - r e^{-|\gamma|}}{1 - r} \to 1 - e^{-|\gamma|}. \tag{32}$$

This gives from (31) and (30)

$$\tau_1 = 0, \quad \langle \epsilon_q \rangle = \overline{\epsilon.} \tag{33}$$

The expression (30) for τ_p can be written in the form proposed by She and Lévêque,[6]

$$\tau_p = \mu_r (1 - s^p) - p \mu_r (1 - s), \quad s = e^{-|\gamma|} < 1. \tag{34}$$

These formulas were obtained by She and Lévêque[6] with

$$\mu_r = 2, \quad s = \frac{2}{3}, \tag{35}$$

on the basis of the assumption that passive regions of dissipation correspond to filamentary vortical structures. As a generalization of the result of She and Lévêque the similar formula with two arbitrary constants (34) was proposed in the paper[36] where the case with $\mu_r(1-s) = 1$ is considered in connection with the ‘‘gap problem’’ raised by Novikov.[35] Remind that in our case of the shifted log-Poisson distribution (16) (which is the limiting case as the log-normal one) the PDF for large positive $\ln(\epsilon_q / \overline{\epsilon})$ is zero.

The parameters (35) give a numerical estimate for the constant μ in the Kolmogorov–Obukhov law (25) [because of relation (34) for $|\gamma|$]:

$$\mu = \mu_r \ln^2 \frac{1}{s} = \mu = 2 \ln^2 \frac{3}{2} \approx 0.3288, \tag{36}$$

which is closed to the generally accepted value.[23]

Another measure of intermittency is associated with the exponent of the six-order structure function, τ_2, $\zeta_6 = 2 + \tau_2$. From (34), (36) we have

$$\tau_2 = -\mu_r (1-s)^2 = -\mu \frac{(1-s)^2}{\ln^2 s} = -\frac{2}{9}, \tag{37}$$

for the parameters of (35). It is important to note that in Ref. 6 the quantity $-\tau_2$ was identified with μ. Actually, it is verified for log-normal model only which corresponds to the limit $\gamma \to 0$, $s \to 1$ in our case.

For completeness, let us also mention that since

$$\frac{\langle (\epsilon_q - \langle \epsilon_q \rangle)^2 \rangle}{\langle \epsilon^2 \rangle} = \frac{\langle \epsilon_q^2 \rangle - \langle \epsilon_q \rangle^2}{\langle \epsilon^2 \rangle} = 1 - \left(\frac{l_q}{L} \right)^{\mu_r (1-s)^2}, \tag{38}$$

for small scales ($l_q \to 0$) or large Reynolds numbers ($L \to \infty$), the influence of the term $\langle \epsilon_q \rangle^2$ in (38) decreases.

At large p, the equations (30) and (34) give

$$\langle \epsilon_q^p \rangle = \overline{\epsilon^p} \left(\frac{L}{l_q} \right)^{p \mu_r (1-s)}, \quad p \to \infty. \tag{39}$$

We can obtain this result by noticing that for large p, the contribution of the passive regions of dissipation is small, $(\epsilon_1^{(q)} / \overline{\epsilon})^p = s^{qp} \to 0$ [see (1)]. The non-active volumes for the corresponding dissipation after q steps of cascade give, according to (2), with constant $r_i = r$, $\alpha_i = -|\alpha|$, $s = r^{|\alpha|}$, $r \to 0$,

$$\frac{\epsilon_2^{(q)}}{\overline{\epsilon}} = \left(\frac{1 - rs}{1 - r} \right)^q \approx (1 + r(1-s))^q \approx e^{qr(1-s)}$$

$$= \left(\frac{L}{l_q} \right)^{\mu_r (1-s)}, \tag{40}$$

where equations (23), (24) have been used. So as $\epsilon_2^{(q)} \approx \epsilon_q$, equation (39) is recovered from (40). Taking $\mu_r(1-s) = 2/3$ we obtain[24] $\langle \epsilon_q^p \rangle \sim l_q^{-2/3p}$. This gives one of She–Lévêque's[6] assumptions:

$$\xi_q^\infty = \lim_{p \to \infty} (\langle \epsilon_q^{p+1} \rangle / \langle \epsilon_q^p \rangle) \sim l_q^{-2/3}.$$

Another assumption in Ref. 6 is

$$\frac{\langle \epsilon_q^{p+1} \rangle}{\xi_q^\infty \langle \epsilon_q^p \rangle} = A_p \left(\frac{\langle \epsilon_q^p \rangle}{\xi_q^\infty \langle \epsilon_q^{p-1} \rangle} \right)^\beta, \quad 0 < \beta < 1,$$

which corresponds to the hypothesis of a log-Poisson statistics.[7]

We can estimate the constant μ_r on the basis of our model from the following considerations. Instead of the original constants Q, r, and $\alpha = -|\alpha|$ (or γ) we can use $\mu_r = r / \ln Q$ and $s = e^{-|\gamma|} = r^{|\alpha|}$. Let us consider the case $r \to 0$ and $|\alpha| \to \infty$. This particular case corresponds to a flow with rare structures with zero dissipation, $\epsilon_1^{(1)} / \overline{\epsilon} = s = 0$. The amplitude of the velocity field near one of these structures can be represented in the form $u(x) = u_0 e^{-a/x} + x e^{-bx^2}$, where x is a distance from the centre of structure. This function describes the rigid-body rotation near $x = 0$ with zero dissipation where the influence of the external (surrounding) flow is small. When distance x is large the velocity becomes equal to the velocity of the surrounding flow u_0 with weak dependence on x. The dissipation of this velocity field is proportional to $[(a/x^2)e^{-a/x}]^2$, with a maximum at the distance $x_0 \sim 2a$ from the center. Thus $\epsilon_{x_0} \sim x_0^{-2}$ and hence, $\epsilon_{x_0}^p \sim x_0^{-2p}$. Modelling non-active regions as such filamentary structures, one finds $\langle \epsilon_q^p \rangle \sim l_q^{-p\mu_r}(q \gg 1)$ with $\mu_r = 2$, which corresponds precisely to She–Lévêque's result.

C. "*P*"-model

Equation (11) gives the generating function of logarithm of the dissipation rate, $x = \ln(\epsilon_q/\overline{\epsilon})$, which depends on parameters r, Q (or μ_r) and α (or $s = r^{-\alpha}$). Another particular case can be considered with $r_i = r = 1/2$. This case corresponds to the p-model of Meneveau and Sreenivasan.[11] Indeed the moments of x are evaluated from the second equality in (11) by the equation

$$\langle x^n \rangle = \frac{\partial^n}{\partial(i\xi)^n}\varphi_q(e^{i\xi}) = \frac{\partial^n}{\partial(i\xi)^n}\left(\frac{1}{2}\right)^q [s^{i\xi} + (2-s)^{i\xi}]^q, \quad (41)$$

where we used the relations

$$z^\gamma = e^{\gamma i\xi} = s^{i\xi}, \quad s = r^{-\alpha}, \quad 1 - r^{1-\alpha} = 1 - \frac{1}{2}s, \quad r = \frac{1}{2}.$$

For the moments of ϵ_q, we obtain from (41):

$$\begin{aligned}\langle \epsilon_q^p \rangle &= \overline{\epsilon^p}\langle e^{px}\rangle = \overline{\epsilon^p}\sum_{n=0}^{\infty}\frac{1}{n!}p^n\langle x^n\rangle \\ &= \overline{\epsilon^p}\sum_{n=0}^{\infty}\frac{1}{n!}p^n\frac{\partial^n}{\partial(i\xi)^n}\left(\frac{1}{2}\right)^q [s^{i\xi}+(2-s)^{i\xi}]^q \\ &= \overline{\epsilon^p}e^{-ip\partial/\partial\xi}\left(\frac{1}{2}\right)^q [s^{i\xi}+(2-s)^{i\xi}]^q \\ &= \overline{\epsilon^p}\left(\frac{1}{2}\right)^q [s^p+(2-s)^p]^q. \end{aligned} \quad (42)$$

Choosing the breakdown coefficient $Q = 2$ (as in Ref. 11), we have $q = \ln(L/l_q)/\ln 2$, and (42) gives

$$\langle \epsilon_q^p \rangle = \overline{\epsilon^p}\left(\frac{l_q}{L}\right)^{1-p-\log_2[(s/2)^p + (1-s/2)^p]}. \quad (43)$$

This corresponds to a p-model with parameter

$$\tilde{p} = \frac{s}{2}. \quad (44)$$

The values $\tilde{p} \approx 0.3$ proposed by Meneveau and Sreenivasan and $s = 2/3$ of She and Lévêque are in agreement with (44). The parameter α which corresponds to the model of Meneveau and Sreenivasan is equal to $\alpha = \ln(2\tilde{p})/\ln 2 < 0$. This scenario, as the one of She and Lévêque's, thus corresponds, in our classification, to a cascade involving passive regions ($\alpha < 0$).

VI. COMPARISON WITH EXPERIMENTAL DATA

Here, we present a selection of various experimental data supporting some statements made above. In particular, we start with a possible evidence for weakly dissipative objects in the flow.

A. Localized holes of dissipation

We have explained how the (shifted) log-Poisson statistics of the dissipation or transfer rate ϵ could be indicative of a scenario involving rare, localized holes of dissipation in the flow materialized by quasi-one-dimensional vortical filamentary structure having a solid-body rotating core. Some evidences of such objects firstly were revealed in numerical experiments (see, for example, Refs. 25, 26, 27). First experimental observations of such structures was done by Douady, Couder, and Brachet[28] (see also the book by Frisch[29] and references therein).

Villermaux *et al.*[30] have given more detailed experimental evidence for the existence, in a three-dimensional, homogeneous, isotropic turbulence, of such objects. Their statistical importance was estimated to be of one event per 100 large-scale turnover time for Re about 10^3. These authors have also shown how a Kelvin–Helmholtz instability of large-scale sheet-like structures could explain the formation of these intense roll-up vortices. Their arguments also provide an estimation of the statistical frequency of such events, which writes $p \sim \exp(-0.7Re^{1/6})/8$ quantitatively consistent with their observations and numerical simulations.[17] The probability of realization of the favorable conditions for the formation of these intense vortices is thus a decreasing function of the Reynolds number.

Let the regions with reduced dissipation rate (passive, $\alpha_i = \alpha < 0$) appear above some critical level i_c with constant ratio r. At the level $i_\lambda, i_\lambda > i_c$, the value of dissipation rate in the passive volume becomes proportional to $r^{|\alpha|(i_\lambda - i_c)}$. The probability of appearance of such volumes is [using (24)] proportional to $r^{i_\lambda - i_c}$,

$$r^{i_\lambda - i_c} \sim \left(\frac{l_\lambda}{l_c}\right)^{\ln(1/r)/\ln Q}.$$

The diameter of the structures, typically given by l_λ,[30] decreases with the Reynolds number since $l_\lambda \sim LRe^{-1/2}$ or $l_\lambda \sim LR_\lambda^{-3/4}$. Thus the probability decreases accordingly.

B. Distributions of ϵ

In figure 3 we have presented the general form of the shifted log-Poisson PDF for a cascade with holes of dissipation (curve 1). This form of the energy dissipation PDF was experimentally found by Naert[31] and the same form for small $\epsilon_q/\overline{\epsilon}$ was obtained in our experiments. The measurements were made on the axis of a round jet ($R_\lambda = 835$) and on the axis of the wind tunnel of the ONERA in Modane

TABLE I. Characteristics of the turbulent flows: Axisymmetric jet ($R_\lambda = 835$) and wind tunnel of ONERA ($R_\lambda = 2485$ and 3374), f_s and f_k are, respectively, the sampling and the Kolmogorov frequencies, U_{mean} the mean velocity, u' the r.m.s. of the velocity signal, and λ and η, respectively, the transversal Taylor and Kolmogorov length scales.

R_λ	U_{mean} (m/s)	u' (m/s)	λ (cm)	η (mm)	f_s (kHz)	f_k (kHz)
835	6.48	1.65	0.78	0.14	25	7.53
2485	20.45	1.58	3.19	0.326	25	10.00
3374	15.77	2.38	2.80	0.236	25	10.63

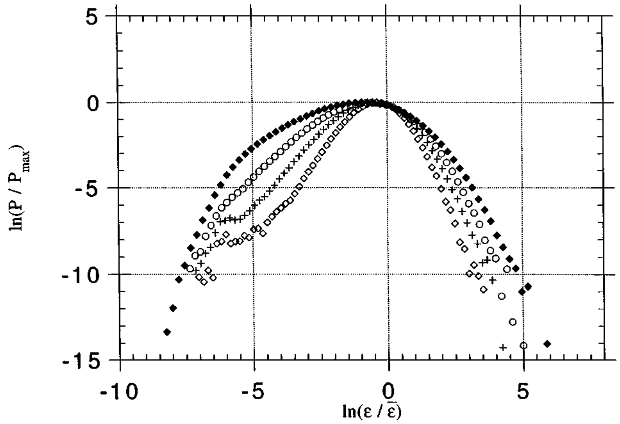

FIG. 4. Normalized probability distributions of $\ln(\epsilon_l/\overline{\epsilon})$ on the axis of a round jet: $\diamond$, $l=278\eta$; $+$, $l=110\eta$; $\bigcirc$, $l=55\eta$; $\blacklozenge$, $l=28\eta$.

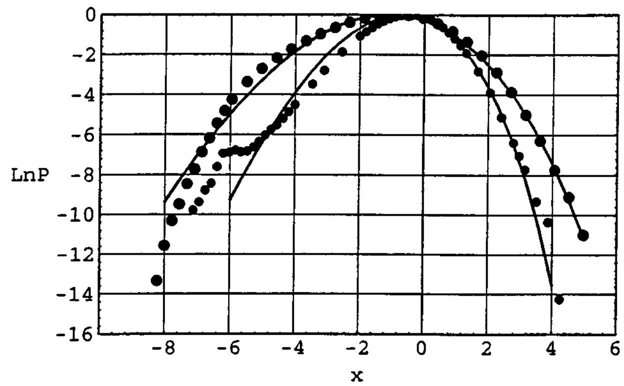

FIG. 6. Approximations of the experimental PDF's of $x=\ln(\epsilon_l/\overline{\epsilon})$ for a round jet ($R_\lambda=835$) by the shifted log-Poisson distribution (16) (solid lines): large circles, the data for $l=28\eta$, the corresponding solid line is the distribution (16) with $n_q=53$, $m_q=47$, $|\gamma|=0.2$; small circles, the data for $l=110\eta$, the solid line corresponds to the distribution (16) with $n_q=30$, $m_q=26$, $|\gamma|=0.2$.

($R_\lambda=2485$). Both these flows have large enough Reynolds numbers and a well defined inertial range (two decades of scaling). So we can expect the existence of the universality of small scale statistics for both flows considered.

We also made experiments in the boundary layer of the ONERA wind tunnel ($R_\lambda=3374$). The main characteristic parameters of these flows are indicated in Table I. Details are given in Refs. 32, 33. In particular, the spatial resolution of velocity gradients is about 4η in the jet and 2.5η in the ONERA wind tunnel, where η is the Kolmogorov scale. The energy dissipation rate ϵ_l averaged over a length of size l was estimated as in Ref. 34.

Figures 4 and 5 show normalized PDF's of ϵ_l for different length size l, respectively, for the round jet and on the axis of the wind tunnel ($R_\lambda=2485$). They were experimentally obtained with sample of about 3×10^7 data leading to a reliable statistical convergence down to a probability level of 5×10^{-5}. Note in figures 4, 5 the increasing probability of small ϵ_l when the scale l is reduced.

In figure 6 the experimental distributions in the case of the jet are plotted for $l=110\eta$ and $l=28\eta$. The fit of the theoretical distribution (16) leads to the values of parameters: $n_{q1}=30$, $m_{q1}=26$, $|\gamma|=0.2$ for $l=110\eta$ and $n_{q2}=53$, $m_{q2}=47$, $|\gamma|=0.2$ for $l=28\eta$. Therefore, parameters n_q, m_q increase when the size l is reduced, in accordance with the equations (14), (23), which give

$$n_q=qr, \quad m_q=\kappa n_q, \tag{45}$$

corresponding to an increase of the number q of cascade step. In the case of the jet, we observe that both for $l=110\eta$ and for $l=28\eta$, the value of $\kappa=m_q/n_q$ is rather the same, namely $\kappa\approx47/53\approx26/30\approx0.88$. This corresponds to (45).

The same approximation was made for the PDF of ϵ_l with $l=10\eta$ in the wind tunnel (figure 7). In this case $|\gamma|=0.5$, $n_q=12.5$, $m_q=9$. These values of m_q, n_q, as equations (45) show, indicate that for the wind tunnel the ratio r is less than in the case of jet.

Quite opposite is the shape of PDF's, shown in figure 8, for data in the boundary layer (near the wall of tunnel at a distance corresponding to the logarithmic zone of the mean velocity profile; see Table I, $R_\lambda=3374$). In this case, the structures with small energy dissipation at each level of the

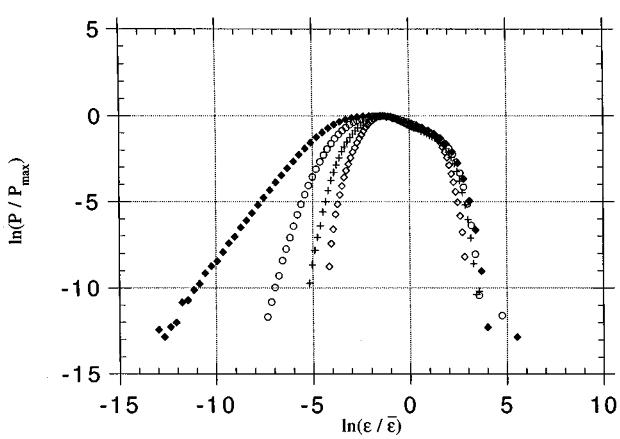

FIG. 5. Normalized probability distributions of $\ln(\epsilon_l/\overline{\epsilon})$ on the axis of a wind tunnel: $\diamond$, $l=376\eta$; $+$, $l=100\eta$; $\bigcirc$, $l=28\eta$; $\blacklozenge$, $l=10\eta$.

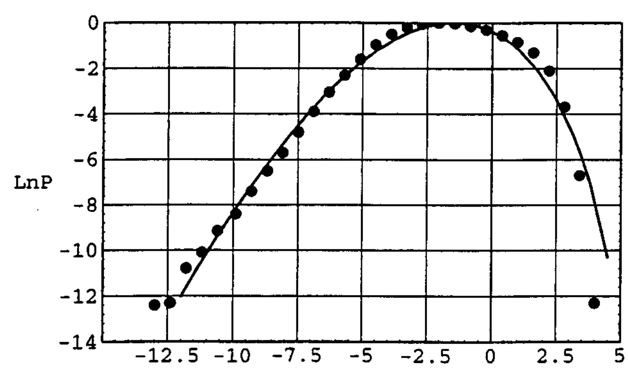

FIG. 7. Approximations of the experimental PDF's for axis of wind tunnel ($R_\lambda=2485$) by the shifted log-Poisson distribution (16) (solid line): circles, the data for $l=10\eta$, the corresponding solid line is the distribution (16) with $n_q=12.5$, $m_q=9$, $|\gamma|=0.5$.

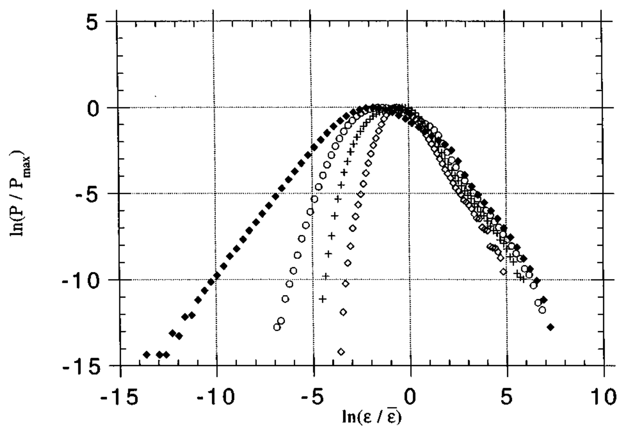

FIG. 8. Normalized probability distributions of $\ln(\epsilon_l/\overline{\epsilon})$ in a shear layer of the wind tunnel: $\diamond$, $l=400\eta$; $+$, $l=100\eta$; $\bigcirc$, $l=27\eta$; $\blacklozenge$, $l=11\eta$.

cascade seem to be suppressed by the large scale shear flow. Moreover, the structures with active dissipation are more probable, and the PDF's of dissipation agrees better with the theoretical distribution (15) with the shape like curve 3 in figure 3.

The PDF's of dissipation rate ϵ_l in the jet case for large length size l ($l=400\eta$, $l=100\eta$) exhibit an intriguing feature at small ϵ_l (figures 4, 6). Tails of the PDF have a plateau which disappears when the scale l is decreasing to the dissipation range. Such a plateau was also observed in Ref. 31.

The probability level about of this plateau is 3×10^{-4} (ten times larger than the level of statistical convergence) when the scale l over which the dissipation is averaged is large enough. The plateau is all the more obvious that the averaging scale l is large. Specifically, the plateau becomes apparent when l is of the order of λ or more (see the curve corresponding to $l=55\eta\approx\lambda$ in figure 4 and figure 10).

The plateau can be explained by an increased amount of localized filamentary vortices with given (small) values of dissipation ϵ_l. Generally speaking, the form of PDF with increased probability for small ϵ_l may be represented as superposition of PDF's with some ''plateau'' ascribed to structures of different scales with reduced dissipation, intrinsic to the present dynamical cascade model. But a plateau is a manifestation of structures with small dissipation which are external to a described cascade. It corresponds, as seen in figure 9 which shows a time series of the dissipation signal, to rare, localized events of low dissipation. Also included in figures 9(b), (c) is the band of ϵ's corresponding to the width of the plateau: since the depth of these holes of dissipation is well below the standard variation of ϵ, and since these events are very sharply defined, their probability density is constant.

The probability of occurrence of these events is therefore

$$p\approx\int_{-\infty}^{\epsilon^*}P(\epsilon)d\epsilon+P(\epsilon^*)(\epsilon_0-\epsilon^*), \tag{46}$$

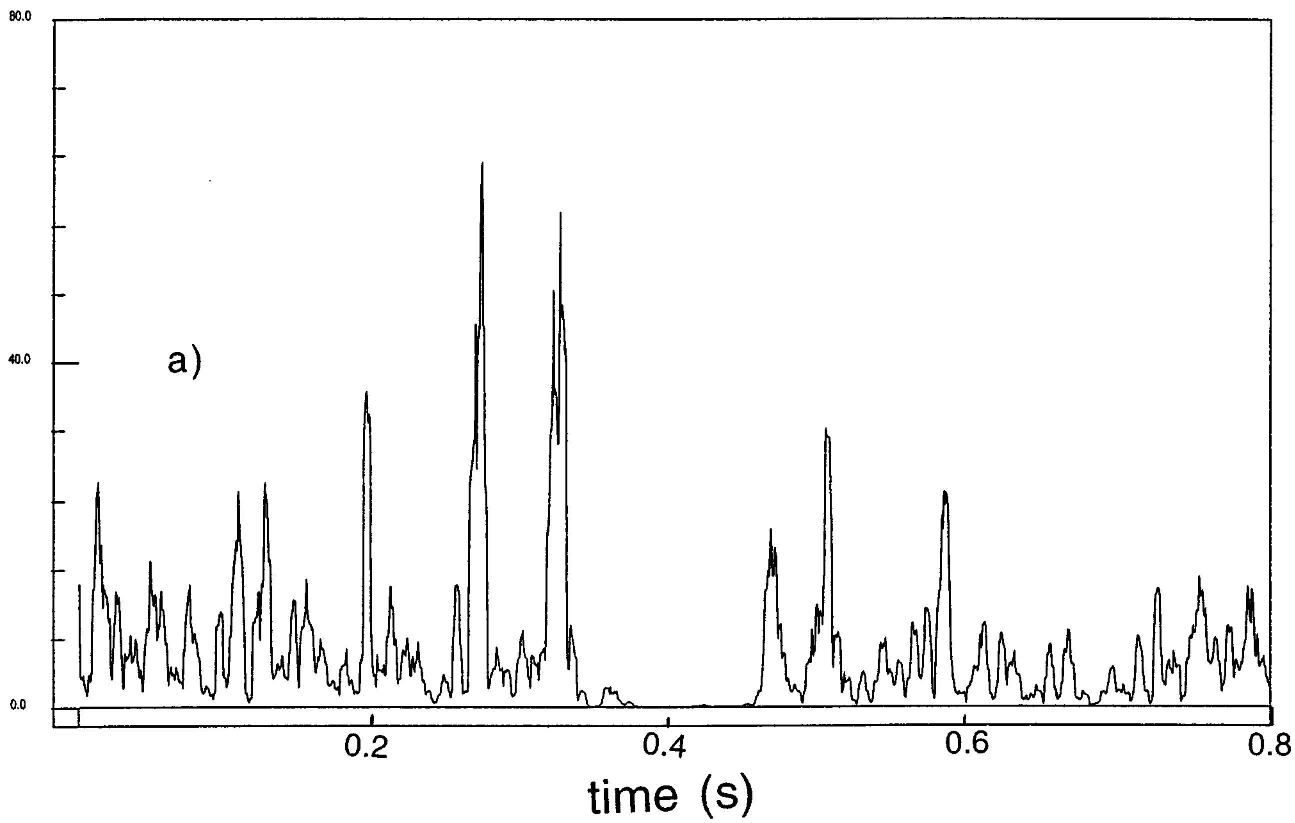

FIG. 9. (a) A time series of the averaged dissipation ϵl, with $l=55\,\eta$ in linear–linear coordinates. (b), (c) The same as previously on a log-linear scale: note the presence of strong, sharply localized holes of dissipation.

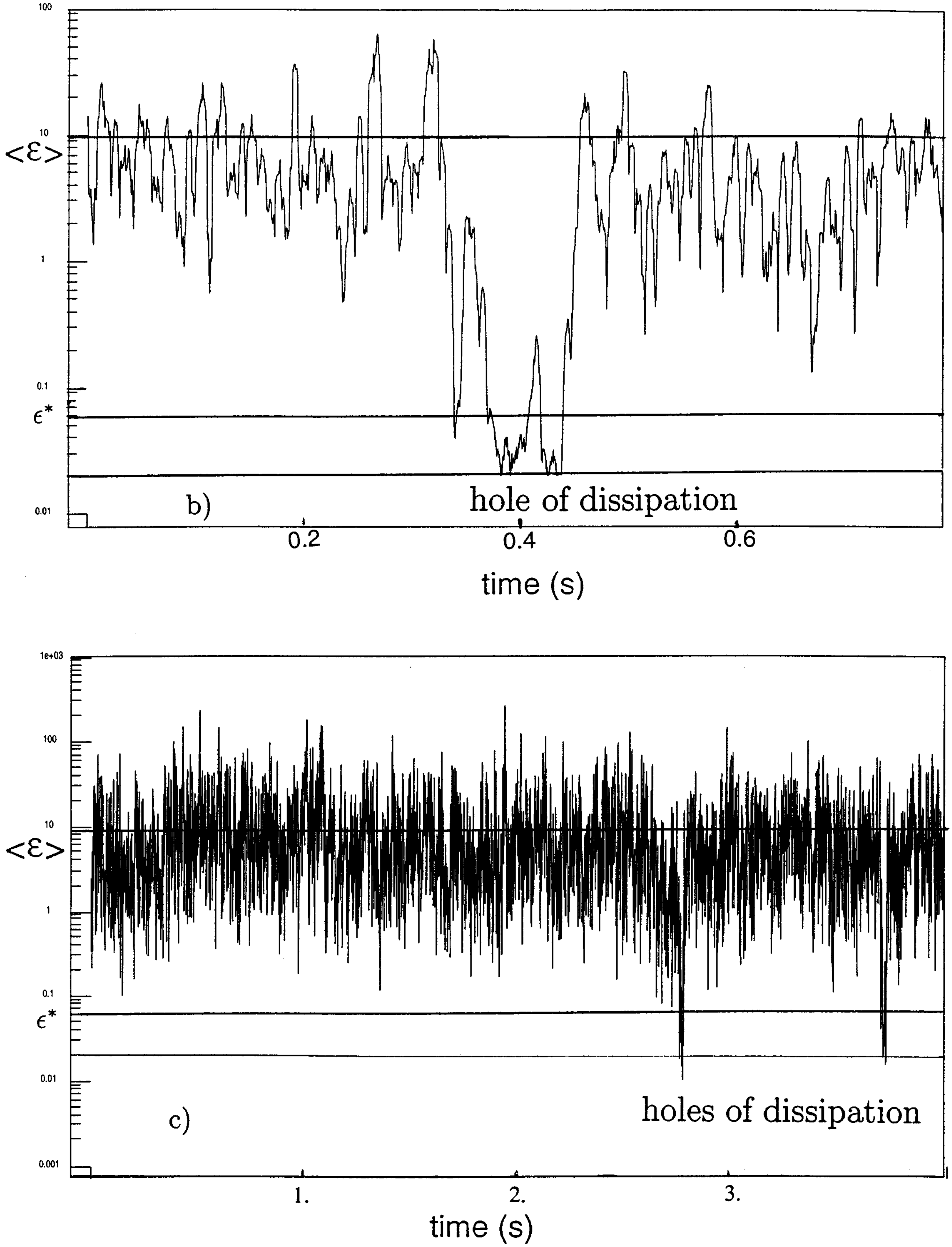

FIG. 9. (*Continued*).

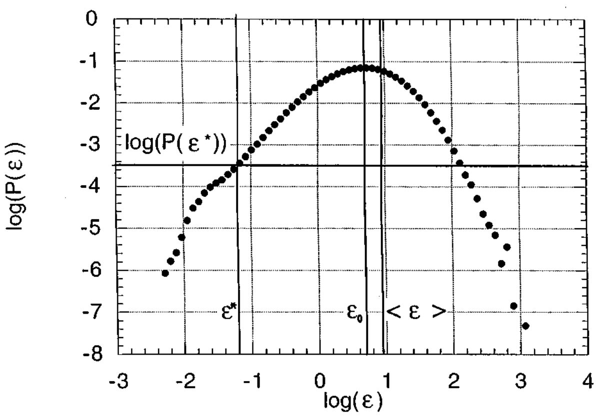

FIG. 10. PDF of ϵ defined by (45) in the jet experiment ($R_\lambda=835$) for $l \approx \lambda = 55\eta$, ϵ^* is the edge of the (birthing at that scale l) plateau, and ϵ_0 is the most probable value of ϵ.

where ϵ_0 is the most probable value of ϵ (see figure 10) and ϵ^* is the limit of the plateau. The integration of the PDF according to (46) provides: $p \approx 10^{-3}$.

This phenomenon is of course reminiscent of the vortex filaments bearing a solid-body non-dissipative rotating core whose relevance for the present discussion has been pointed out in section VI A. Their density in turbulent flows has been estimated to be[30]

$$p \sim \exp(-0.7\, Re^{1/6})/8. \tag{47}$$

According to the Reynolds number of the jet experiment ($Re = R_\lambda^2/15 \approx 46500$), expression (47) leads to $p \approx 2.5 \times 10^{-3}$, in qualitative agreement with the probability estimated from the PDF of ϵ (46).

A strong indication supporting the identification of these holes of dissipation to vortex filaments is the fact that the plateau disappears when the averaging scale l is smaller than the Taylor microscale λ. Indeed, the diameter of the solid-body rotation core of these intense vortical structure is of the order of λ (see Ref. 30). They thus become transparent in the PDF's of ϵ, when ϵ is averaged on a scale smaller than λ.

However, and despite these encouraging coincidences, the fact that these presumable intense, weakly dissipative filamentary structures are revealed through a plateau in the PDF of ϵ in a jet experiment compels us to prudence: the geometry of the jet is not free from external intermittency and these holes of dissipation could also be associated to the passage of the boundary of the jet through the probe volume.

VII. CONCLUDING REMARKS

We have presented a rather general model of energy cascading intermittency with rare localized regions of dissipation (very large or very weak) which gives the shifted log-Poisson statistics of dissipation. In this scheme, including both dynamical and spatial intermittency, the log-normal distribution appears as a nearly equal energy dissipation rate, corresponding to a strong spatial intermittency. In the case of strong ''dynamical'' intermittency, related to both the weak or strong dissipation zone, only the first case satisfies the Novikov's inequalities. This result suggest that only dynamical intermittency based on an upper bound value of energy dissipation rate is possible. Experimental data are in agreement with this view involving localized filamentary vortices.

Concerning the experimental PDF's of dissipation it is possible to explain the absence of an ideal cutoff on the right tails and the deviations of the left tails using the convolution of a log-Poisson and large-scale log-dissipation statistics. This idea to describe the small scale statistics of velocity differences was proposed in references 18, 19. In this case the large-scale statistics corresponds to a Gaussian distribution and small-scale velocity differences fluctuations are approximated by a log-normal distribution. In our paper we used the simplest (and to some extend more straightforward) interpretation of the experiments. But even this simple fit by the distribution (16) explains some features of the experimental distributions in figures 4, 5 like the increased probabilities for small values $\epsilon_q/\overline{\epsilon}.$

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