Memory-Induced Low Frequency Oscillations in Closed Convection Boxes

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The origin of the low frequency oscillation of the large-scale, recirculating flow specific to the high Rayleigh number regime in closed convection cells is investigated. It is shown how the oscillations result from the delayed coupling of the boundary layer instabilities by the slow convective motion of the recirculation. The model developed explains (i) the form of the dependence of the oscillation frequency with the Rayleigh number $f \sim \mathrm{Ra}^\gamma$, including the prefactor, the "anomalous" value of the exponent $\gamma \approx 0.49$, and (ii) the opposition of phase of the oscillations between the top and bottom boundary layers.

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A series of experiments [1] have reported on the existence of a turbulent regime specific to high Rayleigh numbers (i.e., for $\mathrm{Ra} = \alpha g \Delta L^3/\kappa\nu$ above 4×10^7) in closed convection boxes (see [2] for a review). In this regime, called "hard turbulence," the heat flux through the box (Nusselt number) has been shown to be proportional to $\mathrm{Ra}^{2/7}$ rather than to $\mathrm{Ra}^{1/3}$, the latter scaling relation being characteristic to moderated Rayleigh numbers ("soft turbulence") [1]. Flow visualizations [3,4] and numerical simulations [5] have also revealed in this high Rayleigh number regime the existence of a quasi-two-dimensional large-scale flow inducing a recirculation motion which occupies all of the available space in the confinement of the box (Fig. 1). This large-scale flow plays a key role for the determination of the Nusselt-Rayleigh dependence. The exponent $\frac{1}{3}$ relates to situations where the thickness λ_{th} of the thermal layers at the wall of the box is such that the Rayleigh number based on this scale λ_{th} is critical; the $\frac{2}{7}$ dependence comes from the fact that λ_{th} now depends on the linear size of the box L through the characteristic recirculation velocity $u(L)$, which itself depends on L [1].

A concomitant observation in the high Rayleigh number regime is the low frequency, regular pulsation of the large-scale flow, as revealed by direct visualizations [3] and by the peak detected on the temporal temperature spectra in the boundary layers close to the walls [1,6]. The features of this phenomenon are as follows: (i) the oscillation frequency increases like Ra^γ with $\gamma = 0.490 \pm 0.005$ and (ii) the boundary layers on top and bottom of the cell oscillate in opposition of phase [1,6].

The purpose of this Letter is to show how this oscillation can be explained by the memory effect associated with the recirculation flow in the confined space of the box.

Self-sustained oscillations are a common feature of a large variety of marginally stable recirculating flows. The underlying mechanism relies on the interplay between the linear growth of the primary instability disturbances (most of the time a shear instability of temporal growth rate r) and the delay of the nonlinear saturation due to the slow convective motion in the recirculation loop [7,8]. If τ is the recirculation time, the envelope equation for the disturbances $A(t)$ reads [7]

$$\frac{d}{dt}A(t) = rA(t) - \mu A(t-\tau)^2 A(t). \qquad (1)$$

This evolution equation, which has some similarities with the delayed logistic equation and reduces to the Landau model for $r\tau < \pi/4$, displays nonlinear self-sustained oscillations whose period can be computed from the dynamical parameters r and τ; the parameter μ sets the amplitude of the oscillation only [8].

The top and bottom boundary layers in the convection box are permanently sheared by the recirculation motion [9]. The horizontal shear velocity corresponds, by mass conservation, to the buoyancy driven ascending and descending velocity which scales like the free fall velocity $u(L) \sim (\alpha \Delta g L)^{1/2}$ where α, Δ, and g are the thermal expansion of the fluid, the temperature difference between the top and bottom of the cell, and the acceleration due to gravity, respectively, L being the height of the cell (assumed here to present an aspect ratio of unity). The Reynolds number of the boundary layers sheared by the velocity $u(L)$ parallel to the walls $\mathrm{Re}_L = u(L)L/\nu = \mathrm{Ra}^{1/2}\mathrm{Pr}^{-1/2}$ where $\mathrm{Pr} = \nu/\kappa$ is, considering the value of the Rayleigh

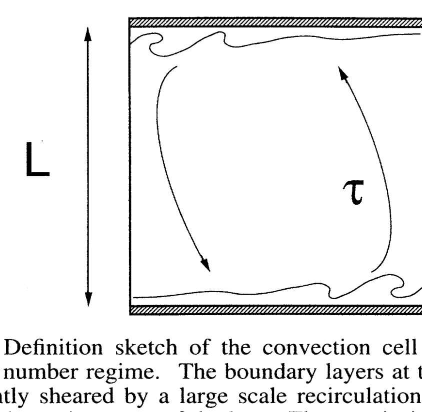

FIG. 1. Definition sketch of the convection cell in the high Rayleigh number regime. The boundary layers at the walls are permanently sheared by a large scale recirculation flow which occupies the entire space of the box. The transit time from one side of the cell to the other is τ.

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number in the hard turbulence regime ($\mathrm{Ra} > 10^8$) and for $\mathrm{Pr} \approx 1$, at most transitional. Indeed, the critical Reynolds number for the onset of boundary layer instability over a flat plate with vanishingly small incoming perturbations is about 3×10^5 [10]. Owning to the permanent turbulent excitations present in the center of the cell and to the perturbations coming from the, say, top boundary layer, the bottom boundary layer is thus liable to produce disturbances via a primary instability of growth rate r (estimated below), disturbances which are further convected by the large scale motion to reach, after a time interval τ, the top boundary layer and excite there the growth of the new perturbations, the overall process being self-sustained. This qualitative scenario can be translated in the spirit of formulation (1) in a system of two coupled nonlinear delayed equations, each of them describing the amplitude of the instabilities of each (top and bottom) boundary layer

$$\frac{d}{dt}A(t) = rA(t) - \mu[A(t-\tau)^2 + cB(t-\tau)^2]A(t), \quad (2a)$$

$$\frac{d}{dt}B(t) = rB(t) - \mu[B(t-\tau)^2 + cA(t-\tau)^2]B(t), \quad (2b)$$

where r, τ, and μ have the same meaning as in (1) and c is a positive coupling parameter lower than unity. For $0.2 < c < 0.8$, the solutions of this model are regular, periodic oscillations, the amplitudes $A(t)$ and $B(t)$ oscillating in phase opposition (Fig. 2). The second maximum of the amplitudes could in principle be visible in the temporal traces of the bolometers and should show up as a distinct peak at half frequency. The natural random fluctuations of τ, however, as explained in [7], broaden the resonance peak and hide this period doubling when the base flow is turbulent. The period T of the sum $A(t) + B(t)$ can be computed following the same lines of Ref. [8] and reads

$$T \approx \tau[2 + a(1+c)e^{br\tau}/2] + \frac{\ln c}{2br}, \quad (3a)$$

$$a = \frac{(1-\beta)^2}{1-\beta^2}, \quad b = 2(1-\beta^2), \text{ with } \beta \approx 0.45. \quad (3b)$$

The frequency of one oscillator is $f = 1/2T$. The quantities r and τ are, in the present case, estimated as follows. The time lag τ is the transit time from the bottom to the top of the convection cell (and vice versa) $L/u(L)$. The velocity $u(L)$ has been measured by Wu [6] (see also [9]) and is equal to $0.31(\kappa/L)\,\mathrm{Ra}^{1/2}$ for an aspect ratio 1.0 cell, therefore

$$\tau = \frac{L}{u(L)} = \frac{L^2}{0.31\kappa\,\mathrm{Ra}^{1/2}}. \quad (4)$$

The growth rate r is the temporal growth rate of the boundary layer shear instabilities. The thickness of a turbulent boundary layer $\delta(x)$ at a distance x from the beginning of its development is $\delta(x) = 0.37x[u(x)x/\nu]^{-1/5}$ [10]. The Reynolds number $\mathrm{Re}_{x=L} = u(L)L/\nu$ being

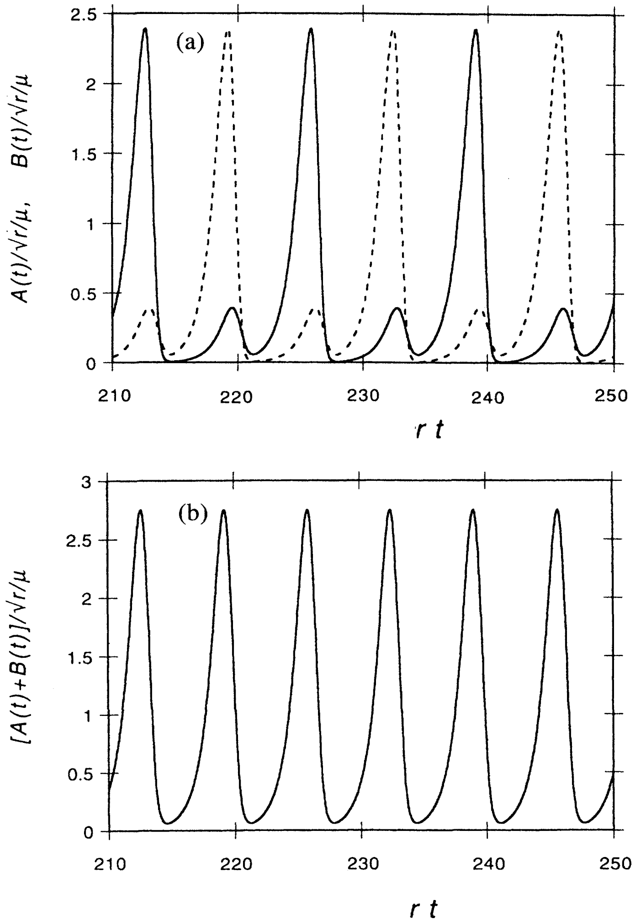

FIG. 2. (a) Temporal evolution of the amplitudes $A(t)$ and $B(t)$ of the system of coupled oscillators [Eqs. (2a) and (2b)], normalized by $\sqrt{r/\mu}$. Note the phase opposition between the oscillators. (b) Temporal evolution of the sum $A(t) + B(t)$, normalized by $\sqrt{r/\mu}$. $r = 4$, $\tau = 0.35$, $\mu = 4$, and $c = 0.5$.

close to transitional at $x = L$, the turbulent contribution $\delta(L)$ to the total thickness δ of the layer is $\delta(L) = 0.37L\,\mathrm{Re}_c^{-1/5}$ with $\mathrm{Re}_c = 3 \times 10^5$. To this quantity $\delta(L)$ adds the thickness of the viscous sublayer $\delta_v = \nu/u^*$ where u^* is the friction velocity related to the large-scale velocity $u(L)$ by $u^* \approx 0.05u(L)$ and ν the viscosity of the fluid [10]. The total thickness δ over which acts the total velocity difference $u(L)$ is thus

$$\delta = \frac{\nu}{u^*} + 0.37L\,\mathrm{Re}_c^{-1/5}. \quad (5)$$

The temporal growth rate r corresponds to the inverse of the time necessary to accelerate an element of fluid of size δ to the velocity u^*:

$$r = \frac{u^*}{\delta} = \frac{0.135\,\mathrm{Re}_c^{1/5}}{2.29\,\mathrm{Re}_c^{1/5}/\mathrm{Ra}^{1/2} + 1}\,\frac{u(L)}{L} \quad (6)$$

assuming $\mathrm{Pr} \approx 1$. The above description of the structure of the boundary layer is consistent with the measurements of Belmonte *et al.* [9] who show that the production of the smallest scales in the layer occurs at a distance from the plate decreasing like $\mathrm{Ra}^{-1/2}$ for $\mathrm{Ra} > 10^9$. This is a clear indiction of the turbulent nature of the boundary layer where the thickness of the viscous sublayer, which sets the size of the smallest structures in the layer scales as $\delta_v = \nu/u^*$ with $u^* \sim u(L) \sim \mathrm{Ra}^{1/2}$.

The product $r\tau$ is always larger than the critical value $\pi/4$ for the existence of sustained nonlinear oscillations in the range of Rayleigh numbers considered [$\mathrm{Ra} > 4 \times 10^7$, see Fig. 3(b)]. The oscillation frequency of one oscillator (one boundary layer) is then given by (3)

$$\frac{\kappa}{fL^2} = \frac{x}{\mathrm{Ra}^{1/2}} + y \tag{7a}$$

with

$$x = \frac{1}{0.16}[2 + a(1 + c)e^{br\tau}/2] \tag{7b}$$

and

$$y = \frac{\kappa \ln c}{bL^2 r} \tag{7c}$$

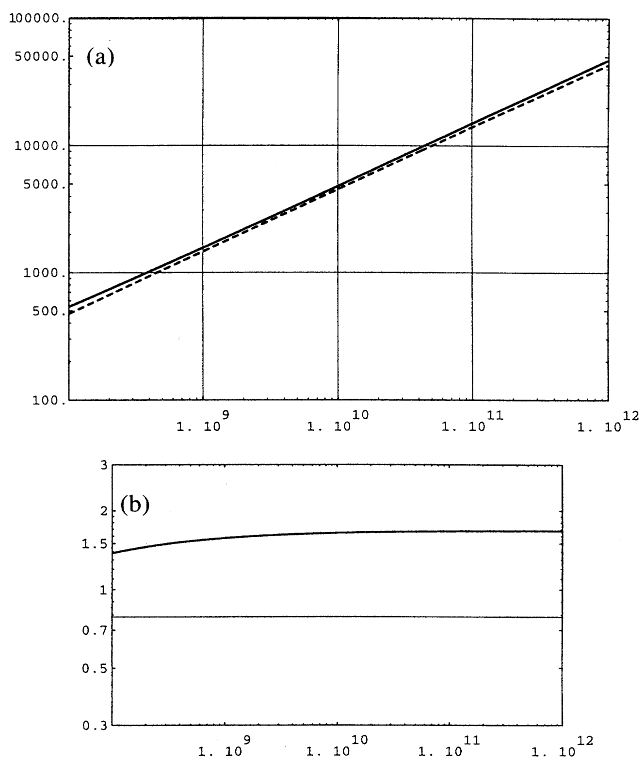

FIG. 3. (a) fL^2/κ given by the present model [Eqs. (7) with $c = 0.5$, solid line] compared with the experimental law $0.057\mathrm{Ra}^{0.49}$ of Refs. [1,6] (dashed line) as a function of the Rayleigh number Ra. (b) The product $r\tau$ on the range of Rayleigh numbers considered. $r\tau$ is always larger than $\pi/4$ (horizontal line), the minimum value for the amplitudes $A(t)$ and $B(t)$ to display sustained oscillations in phase opposition.

In the form of Eqs. (7), together with (4) and (6), the nondimensional frequency fL^2/κ is a function of the Rayleigh number Ra only. It is compared to the experimental law $fL^2/\kappa = 0.057\,\mathrm{Ra}^{0.49}$ in Fig. 3(a). Since not only the trend but also the absolute values of the frequencies are predicted with a good precision, the fit is, considering the simplicity of this model, excellent. The oscillation frequency f is essentially proportional to the inverse of the transit time τ, which decreases with Rayleigh number like $\mathrm{Ra}^{-1/2}$ [Eq. (4)]: The period T represents a little bit more than five transit times $L/u(L)$. However, the expression of f incorporates correcting factors involving the growth rate r. The growth rate increases slightly faster than $u(L)/L$ with increasing Ra via its sensitivity to the thickness of the viscous sublayer $\delta_v \sim \mathrm{Ra}^{-1/2}$, whose contribution to δ is mostly sensitive at moderate Rayleigh numbers [up to $\mathrm{Ra} \approx 10^9$, see Fig. 3(b)]. As a consequence, through the exponential factor in the expression of the period T, the dependence of fL^2/κ on Ra bends slightly towards a dependence less severe than $\mathrm{Ra}^{1/2}$ at moderate Ra and thus imposes a slight deviation to the exponent $\frac{1}{2}$ on the fit by a unique power law on the whole range of Rayleigh numbers ($\gamma \approx 0.49$).

The interpretation of the onset of low frequency oscillations, in phase opposition between the top and bottom boundary layers of high Rayleigh number convection cells provided in this paper, is substantially different from the ones proposed up to now [1,11,12]. It amounts to a caricature of the convection box by a set of two oscillators (the boundary layers), coupled by a slow, large-scale recirculation flow. This convective flow plays a crucial role in the origin of the oscillatory pattern by the time delay that it imposes on the propagation of the coupling between the oscillators [7]. By its generality, this model is a useful tool to understand the origin of antisymmetric oscillations as a result of the coupling of shear instabilities by a symmetric recirculation zone. This is, in particular, the case for the problem of the Bénard-Karman vortices alternatively (in phase opposition) shed from the recirculation zone in the wake of a cylinder [13,14]. The application of formula (3) with the appropriate parameters r and τ indeed leads to a Strouhal number $S_t = fD/u$, where D is the diameter of the cylinder and u the incoming velocity, close to 0.2 [15]. In that case, the secondary maximum of the velocity fluctuations in the shear layers bordering the recirculation zone immediately downstream from the obstacle should be detectable at moderate Reynolds number.

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[1] B. Castaing, G. Gunaratne, F. Heslot, L. Kadanoff, A. Libchaber, S. Thomae, X. Z. Wu, S. Zaleski, and G. Zanetti, J. Fluid Mech. **204**, 1 (1989).

[2] E. D. Siggia, Annu. Rev. Fluid Mech. **26**, 137 (1994).
[3] G. Zocchi, E. Moses, and A. Libchaber, Phys. Rev. E **47**, R2253 (1993).
[4] A. Tilgner, A. Belmonte, and A. Libchaber, Phys. Rev. E **47**, R2253 (1993).
[5] J. Werne, Phys. Rev. E **48**, 1020 (1993).
[6] X. Z. Wu, Ph.D. thesis, University of Chicago 1991 (unpublished).
[7] E. Villermaux, Nature (London) **371**, 24 (1994).
[8] E. Villermaux and E. J. Hopfinger, Physica (Amsterdam) **72D**, 230 (1994).
[9] A. Belmonte, A. Tilgner, and A. Libchaber, Phys. Rev. Letters **70**, 4067 (1993).
[10] H. Schlichting, *Boundary Layer Theory* (McGraw-Hill, Inc., New York, 1987).
[11] S. Zaleski, C. R. Acad. Sci. (Paris) Série 2 **t. 313**, 1099 (1991).
[12] J. Werne, Phys. Rev. E **49**, 4072 (1994).
[13] H. Bénard, C. R. Acad. Sci. (Paris) **t. 147**, 839 (1908).
[14] Y. Pomeau, Chaos, Solitons and Fractals **5**, 1755 (1995).
[15] E. Villermaux (to be published).