From laminar to turbulent flow inside a precessing cylinder

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Applications

Flying objects

- Asymmetry → Precession
- Propellant liquid → Fluid motion
- Torque → leads to trajectory deviation

Geophysics

- Precession (23°, 26 000 years)
- Liquid core → Fluid motion
- Influence on Earth magnetic field
Dimensionless numbers

Six parameters:

\[ H, R, \Omega_1, \Omega_2, \alpha, \nu \]

Four dimensionless numbers:

Aspect ratio:

\[ h = \frac{H}{R} \]

Rossby number:

\[ Ro = \frac{\Omega_2 \sin(\alpha)}{\Omega_1 + \Omega_2} \ll 1 \]

Frequency ratio:

\[ \omega = \frac{\Omega_1}{\Omega_1 + \Omega_2} \in [0, 2] \]

Reynolds number:

\[ Re = \frac{(\Omega_1 + \Omega_2)R^2}{\nu} \in [100, 10^5] \]
Experimental setup

$$\Omega_1 \in [0, 0.70 \text{ rad/s}]$$
$$\Omega_2 \in [0, 0.6 \text{ rad/s}]$$
$$\alpha \in [1^\circ, 10^\circ]$$

Experimental methods used:
- Laser sheet and camera for PIV measurements
- Kalliroscope particles
Linear theory (in the cylinder frame of reference)

Dimensionless Navier-Stokes equations and inviscid boundary conditions:

\[
\begin{align*}
\frac{\partial \mathbf{v}_b}{\partial t} + 2 \mathbf{z} \times \mathbf{v}_b + \nabla p &= -2 \cdot Ro \cdot \omega r \cos(\omega t + \theta) \mathbf{Z} \\
\nabla \cdot \mathbf{v}_b &= 0 \\
\mathbf{v}_b \cdot \mathbf{n} &= 0
\end{align*}
\]

Solution:

\[
\mathbf{v}_b = 2Ro \cdot \mathbf{i} \cdot r \cdot e^{i(\omega t + \theta)} \cdot \mathbf{Z} + e^{i(\omega t + \theta)} \sum_{j=\infty}^\infty A_j \mathbf{u}_j e^{ik_jz} + C.C
\]

- Small forcing term: frequency \( \omega \), \( m = 1 \)
- Particular solution (shear along the \( z \) direction)
- Solution of the homogeneous problem (Kelvin modes)
Visualization of modes (vorticity, $\alpha = 1^\circ \, Re = 10000$)

Mode 1 ($\omega = 1.75$)

Mode 2 ($\omega = 0.5$)

Mode 3 ($\omega = 0.4$)

Mode 5 ($\omega = 0.2$)

Amplitude of the 1st, 2nd, 3rd, 4th and 5th modes

(Meunier et al., *J. Fluid Mech.* (2008), vol. 559, pp. 405-440)
Experimental result

$h=1.62, \ Ro \approx 0.0031, \ \omega \approx 1.18, \ Re \approx 6000$

**Kalliroscope Visualization (real time)**

- High Re → instability
  - nonstationary
  - disorganized
- Manasseh (94, 96)

**PIV measurement (accelerated 10 times)**

- Structure with a high azimuthal number
  - \( m = 5 \)
Experimental vorticity

\[ h = 1.62, \text{1st res. mode 1, } Re \approx 6000, Ro = 0.0031 \]

\[ z = \frac{h}{4} \rightarrow m_1 = 5 \]

\[ z = 0 \rightarrow m_2 = 6 \]

\[ m_2 - m_1 = 1, k_2 - k_1 = k, \text{ and } \omega_2 - \omega_1 = \omega \]

\[ \rightarrow \text{Instability of precession = triadic resonance (Lagrange et al., PF 2008)} \]
Linear stability analysis

Resonant condition:

\[ \begin{align*}
A e^{i(\omega t + \theta + kz)}
\end{align*} \]

Amplitude equations:

\[ \begin{align*}
\partial_t A_1 &= A^* N_1 A_2 - i \Delta k_1 Q_1 A_1 - \frac{1}{\sqrt{Re}} S_1 A_1 - \frac{1}{Re} V_1 A_1 \\
\partial_t A_2 &= A N_2 A_1 - i \Delta k_2 Q_2 A_2 - \frac{1}{\sqrt{Re}} S_2 A_2 - \frac{1}{Re} V_1 A_2
\end{align*} \]
Stability diagram (1st res. of mode 1, \( h = 1.62 \))

The threshold stable/unstable is well predicted by the theory.
Growth rate measurement

\( h = 1.62, \text{ 1}\textsuperscript{st} \text{ res mode 1} \)

\[ \frac{\sigma}{|A|} \]

\[ |A| \propto Re^{1/2} \]

\( Re \approx 6000, \; Ro = 0.0031 \)

Inviscid

\( Re \to \infty \)

\( Re = 6000 \)

\( Re = 1000 \)

Open symbols: \( Re < 6000 \)

Closed symbols: \( Re > 6000 \)

Good agreement between experiments (symbols) and theory (lines)
Variation of the precession frequency $\omega$

$(h = 1.62, \text{Re} = 6500)$

Outside of the resonance: $k_2 - k_1 \neq k \rightarrow$ detuning effects

$\rightarrow$ weaker instability
Non-linear effects

The total flow is:

\[ \mathbf{v} = A \mathbf{v}(r) e^{i(\omega t + \theta + k_z r)} + A_1 \mathbf{v}_1(r) e^{i(\omega_1 t + \theta + k_z r)} + A_2 \mathbf{v}_2(r) e^{i(\omega_2 t + m_1 \theta + k_z r)} + A_0 \mathbf{v}_0(r) \mathbf{u}_0 \]

Base flow  First and second free Kelvin mode  Geostrophic mode

Amplitude equations:

\[
\begin{align*}
\partial_t A &= 2ifRo - \frac{1}{\sqrt{Re}} SA - i\xi A_0 A \\
\partial_t A_1 &= A^* N_1 A_2 - \frac{1}{\sqrt{Re}} S_1 A_1 - i\xi_1 A_0 A_1 \\
\partial_t A_2 &= AN_2 A_1 - \frac{1}{\sqrt{Re}} S_2 A_2 - i\xi_2 A_0 A_2 \\
\partial_t A_0 &= \frac{1}{\sqrt{Re}} \left( \frac{-2}{h} A_0 + \chi_1 |A_1|^2 + \chi_2 |A_2|^2 \right)
\end{align*}
\]

Good prediction of the threshold for the intermittency

(h = 1.62, 1st res. mode 1, Ro = 0.0031)
Fixed point and time average values

\[ (h = 1.62, \text{1st res. mode 1}) \]

- **Stable**
- **Unstable and stationary**
- **Unst. and intermittent**

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Instability → Sub-critical bifurcation
Intermittency → Super-critical bifurcation

Even if the flow is turbulent !!!

\[ \langle |A_1| \rangle \sim A_{1f} \]
\[ \langle |A| \rangle \sim A_f \]
Conclusion

- PIV measurements / Kalliroscope visualisations inside a precessing cylinder
- Instability of precession = Mechanism of triadic resonance
- Linear stability analysis (with detuning and viscous effects)
- Good agreement theory/experiment (stability diagram / growth rate)
- Non-linear effects (intermittency / mean flow)

Perspectives

- Precession of an ellipsoid
- Influence of a magnetic field
- Destabilisation of a flying object