Internal wave beams: transport and attractors

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**Supervision**

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Waves and Instabilities in Geophysical and Astrophysical Flows,
Porquerolles 09
Internal wave beams: transport and attractors

Internal wave beam 2D

A viscous internal wave beam is described by

$$\psi(\chi, \eta, t) = \int_0^\infty e^{-\nu k^3 \chi} \cos(k\eta - \omega t) dk.$$  \hspace{1cm} (1)

1st order approximation, in tilted coordinate frame. Integration of kinematic equations $u(\chi, \eta) = -\psi_\eta$, $w(\chi, \eta) = \psi_\chi$, gives Poincaré plot.

Colour: energy. Black dots: position of particles per period starting from red position, 50 periods.
Internal wave beams in the lab

- To test the theory we need
  - Clear beam(s), steady state
  - Many periods for Poincaré plot
- In finite tank, i.e. physical setting, reflections play a role

- Internal wave attractor provides controlled reflection
Internal wave beams: transport and attractors
Focus on attractors

Reflection from sloping boundaries

In almost all domains focussing dominates *Maas & Lam. JFM '95*
Internal wave attractors

- Clearly defined direction of energy propagation
- Strong beams, linear waves (non-breaking)
- Steady state

Hazewinkel et al. JFM 598
Internal wave beams: transport and attractors
Focus on attractors

Growth and equilibrium

Forced stage of experiment; growth of the attractor

\[ A = \frac{\rho_z'}{\bar{\rho}_z}, \text{ i.e. the vertical perturbation density gradient relative to the unperturbed background stratification.} \]
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Focus on attractors

Decay

The unforced aftermath
Variations in frequency
Internal wave beams: transport and attractors
Focus on attractors

Variations in frequency
Attractors are found in other domains than trapezoidal
Towards transport by internal wave beams

- To test the theory we need
  - Clear beam(s), steady state
  - Many periods for Poincaré plot

The internal wave attractor provides us with

- Clearly defined direction of energy propagation
- Strong beams, linear waves (non-breaking)
- Steady state
Internal wave beams: transport and attractors
Transport by internal wave beams
Experiments on transport

Particle drift? PIV particles, once per period.
Internal wave beams: transport and attractors
Transport by internal wave beams
Experiments on transport

Integrated Eulerian velocity fields

Obtain Eulerian fields from PIV, for many periods 32 obs/T

$$\dot{x} = u(x, z, t) \rightarrow x(t), \text{ 3T full resolution followed by } 100T \text{ plotted once per period}$$
Internal wave beams: transport and attractors
Transport by internal wave beams
Experiments on transport

Integrated Eulerian velocity fields

yield particle displacements

Conclusion: Net drift along beam, against energy propagation direction.

But: the integration does not bring buoyancy into account, so is this what the particles do?
Internal wave beams: transport and attractors
Transport by internal wave beams
Experiments on transport

PIV II: net Lagrangian velocity

particle tracking over one period (i.e. disregarding oscillation as in movie)

Weaker net vertical motion due to stratification (neglected in theoretical model; 1st order approximation)
Is our naive model realistic for particles in a stratified environment?

On a particle the forces are $\mathbf{F} = \mathbf{D} \text{rag} + \mathbf{B} \text{uoyancy}$

- $\mathbf{D} = 6\pi \nu \rho_* R_p \mathbf{V}_{\text{rel}} = 6\pi \nu \rho(z) R_p (\mathbf{v}(x,z) - \dot{x})$

- $\mathbf{B} = - \frac{4\pi R_p^3 g}{3} \Delta \rho \approx - \frac{4\pi R_p^3 g}{3} (\rho_* + z \frac{\partial \rho}{\partial z} - \rho_*) = - \frac{4\pi R_p^3 g}{3} (z \frac{\partial \rho}{\partial z})$

Particles small $\rightarrow$ non-inertial; $\mathbf{F} = 0$

Neglecting $\mathbf{B}$ gives $\mathbf{D} = 0 \rightarrow \mathbf{v}(x,z) = \dot{x}$ Our naive model.
If buoyancy is accounted for

\[ \dot{x} = u(x, z, t) \]  \hspace{1cm} (2)

\[ \dot{z} + \tau z = w(x, z, t) \]  \hspace{1cm} (3)
Internal wave beams: transport and attractors
Transport by internal wave beams
Particle motion in stratification

But still vertical transport exists

- Suppresses net vertical motion.
- Does this also describes the fluid?
  - Horizontal convergence into beam
  - Growth of density perturbation; steady state steady?
  - Driving currents
In NS buoyancy was included so how come we have to bring it in again by hand?

\[(\rho_0(z) + \rho)_t + \mathbf{u} \cdot \nabla (\rho_0(z) + \rho) = 0 \quad (4)\]

\[\rightarrow\]

\[b_t + \mathbf{u} \cdot \nabla b + w = 0 \quad (5)\]

but what if \(\rho_0(z, \epsilon t)\), since we regard the slow time for the transport
Conclusion

- Transport has opposing components along inclined beam
  - Observed integrated Eulerian velocity
  - Theoretical drift from zero mean streamfunction
- Buoyant particles are transported horizontally into the beams (convergence) and subsequently in the vertical

Questions

- Does the transport depend on the tracer considered? (Diffusivity, how fast is the exchange?)
- Do internal wave beams drive currents?

references: Hazewinkel et al.08 JFM, *Transport by IW beams* submitted to Geophysical Research Letters
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Transport by internal wave beams
Particle motion in stratification

Questions?

Film: Blown by the attractor