

*Tsinghua-Princeton-CI Summer School*  
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**Lectures on**  
**Dynamics of Combustion Waves**  
**in Premixed Gases**

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**Contents**

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Combustion is a fascinating phenomenon coupling complex chemistry to transport mechanisms and nonlinear fluid dynamics. This book provides an up-to-date and comprehensive presentation of the nonlinear dynamics of combustion waves and other non-equilibrium energetic systems. The major advances in this field have resulted from analytical studies of simplified models performed in close relation with carefully controlled laboratory experiments. The key to understanding the complex phenomena is a systematic reduction of the complexity of the basic equations.

Focusing on this fundamental approach, the book is split into three parts. Part I provides physical insights for physics-oriented readers, part II presents detailed technical analysis using perturbation methods for theoreticians, and part III recalls the necessary background knowledge in physics, chemistry and fluid dynamics. This structure makes the content accessible to newcomers to the physics of unstable fronts in flows, whilst also offering advanced material for scientists who wish to improve their knowledge.

**Paul Clavin** is Professor Emeritus at Aix-Marseille Université and is an honorary member of the Institut Universitaire de France (Chair of Mécanique Physique 1993-2004). In 1995 he founded a renowned research institute, the Institut de Recherche sur les Phénomènes Hors Équilibre (IRPHE) and has received major awards from the Société Française de Physique (Plumey 1988), French Academy of Sciences (Grand Prix 1995) and from the Combustion Institute (Zeldovich Gold medal, San Francisco, August 2014).

**Geoff Searby** is retired Director of Research at the Institut de Recherche sur les Phénomènes Hors Équilibre (IRPHE). He is a renowned specialist of the physics of thermo-acoustic instabilities in combustion chambers and rocket motors, and his experiments have made major contributions to the understanding of the dynamics of flame fronts. In 2004 he obtained a major award from the French Academy of Sciences.

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Clavin and Searby

Combustion Waves and Fronts in Flows

# Combustion Waves and Fronts in Flows

Flames, Shocks, Detonations, Ablation Fronts  
and Explosion of Stars

Paul Clavin and Geoff Searby



730 pages In press; to appear in September 2016

## Lecture 1: **Orders of magnitude**

1-1: Overall combustion chemistry

1-2: Combustion waves in gaseous mixtures

1-3: Arrhenius law

1-4: Hydrocarbon/air flames

1-5: Instabilities of flames

## Lecture 2: **Governing equations**

2-1. Conserved extensive quantities

2-2. Continuity

2-3. Fick's law. Diffusion equation

2-4. Conservation of momentum

2-5. Conservation of total energy

*Thermal equation*

*Inviscid flows in reactive gases*

*Conservative forms*

*One-dimensional inviscid and compressible flow*

2.6. Entropy production

## Lecture 3: **Thermal propagation**

3-1. Quasi-isobaric approximation (Low Mach number)

3-2. One-step irreversible reaction

3-3. Unity Lewis number and large activation energy

3-4. Zeldovich & Frank-Kamenetskii asymptotic analysis

*Preheated zone*

*Inner reaction layer*

*Matched asymptotic solution*

3-5. Reaction diffusion waves

*Phase space*

*Selected solution in an unstable medium*

## Lecture 4 : **Hydrodynamic instability of flames**

- 4-1. Jump across an hydrodynamic discontinuity
- 4-2. Linearized Euler equations of an incompressible fluid
- 4-3. Conditions at the front
- 4-4. Dynamics of passive interfaces
- 4-5. Darrieus-Landau instability
- 4-6. Curvature effect: a simplified approach

## Lecture 5: **Thermo-diffusive phenomena**

### 5-1. Flame stretch and Markstein numbers

*Passive interfaces*

*One-step flame model*

*The second Markstein number*

### 5-2. Thermo-diffusive instabilities

*Planar flames for  $Le \neq 1$*

*Jump conditions across the reaction layer*

*Linear equations and linear analysis*

*Cellular instability ( $Le < 1$ )*

*Oscillatory instability ( $Le > 1$ )*

## Lecture 6: **Thermal quenching and flammability limits**

6-1. Extinction through thermal loss

6-2. Basic concepts in chemical kinetics

*Combustion of hydrogen*

*Two-step model. Crossover temperature*

*One-step model with temperature cutoff*

6-3. Flame speed near flammability limits

## Lecture 7: **Flame kernels and quasi-isobaric ignition**

7-1. Introduction

7-2. Zeldovich critical radius

7-3. Critical radius near the flammability limits

7-4. Dynamics of slowly expanding flames

7-5. Quasi-steady dynamic of thin flames

*Semi-phenomenological model*

*Opened-tip Bunsen flames*

## Lecture 8: Thermo-acoustic instabilities

### Lecture 8-1. Rayleigh criterion

*Acoustic waves in a reactive medium*

*Sound emission by a localized heat source*

*Linear growth rate*

### Lecture 8-2. Admittance & transfer function

*Flame propagating in a tube*

*Pressure coupling*

*Velocity and acceleration coupling*

### Lecture 8-3. Vibratory instability of flames

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*Flame propagating downward (sensitivity to the Markstein number)*

*Bunsen flame in an acoustic field*

## Lecture 9 : **Turbulent flames**

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### 9-2. Turbulent diffusion

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*Rough model of turbulent transport*

*Well-stirred flame regime*

### 9-3. Strongly corrugated flamelets regime

*Kolmogorov's cascade*

*Gibson's scale*

*Elements of fractal geometry*

*Self similarity of strongly corrugated flames*

*Co-variant laws*

### 9-4. Turbulent combustion noise

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*Sound generated by a turbulent flame*

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*Model of hyperbolic equations for the formation of discontinuity*

*Riemann invariants*

*Rankine-Hugoniot conditions for shock waves*

*Mikhelson (Chapman-Jouguet) conditions for detonations*

### 10-2. Inner structure of a weak shock wave

*Formulation*

*Dimensional analysis*

*Analysis*

### 10-3. ZND structure of detonations

### 10-4. Selection mechanism of the CJ wave

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*Flow of burnt gas in spherical CJ detonations*

*Point blast explosions*

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*Critical energy*

### 11-2. Spontaneous initiation and quenching

*Initiation at high temperature*

*Spontaneous quenching*

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*Experiments*

*Runaway phenomenon*

*Mechanisms of DDT*

## Lecture 12 : Galloping detonations

### 12-1. Physical mechanisms

*Instability mechanism*

*Two limiting cases*

### 12-2. General formulation

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*Strong shock in the Newtonian approximation*

### 12-3. Strongly overdriven regimes in the limit $(\gamma - 1) \ll 1$

*Distinguished limit*

*Integral-differential equation for the dynamics*

*Oscillatory instability*

### 12-4. CJ detonations for small heat release

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*Near CJ regimes for small heat release. Transonic reacting flows*

*Slow time scale*

*Asymptotic model for CJ or near CJ regimes*

*Results for simplified chemical kinetics*

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*Classification of normal modes*

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*Stability of shocks in ideal gases*

*Stability of reacting shocks*

# Lecture 14: **Nonlinear dynamics of shock waves** **Mach stem formation**

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*What is a Mach stem ?*

*Mach stems and cellular detonations*

*Spontaneous formation of Mach stems*

## 14-2. Multidimensional dynamics of shock fronts

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## 14-3. Shock-vortex interaction

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*Composite solution*

*Model equation*

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## Lecture 15 : Cellular detonations

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*Order of magnitude. Scaling*

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*Inner structure*

*Matching*

*Linear growth rate*

*Weakly nonlinear analysis*

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*Formulation*

*Scaling*

*Model for CJ or near CJ regimes*

*Multidimensional stability analysis*