

*Tsinghua-Princeton-CI Summer School*  
*July 19-25, 2016*

**Lectures on**  
**Dynamics of Combustion Waves**  
**in Premixed Gases**

Professor Paul Clavin  
Aix-Marseille Université  
ECM & CNRS (IRPHE)

*Theoretical analyses (analytical studies of simplified models)*  
*+ laboratory experiments*

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Combustion is a fascinating phenomenon coupling complex chemistry to transport mechanisms and nonlinear fluid dynamics. This book provides an up-to-date and comprehensive presentation of the nonlinear dynamics of combustion waves and other non-equilibrium energetic systems. The major advances in this field have resulted from analytical studies of simplified models performed in close relation with carefully controlled laboratory experiments. The key to understanding the complex phenomena is a systematic reduction of the complexity of the basic equations.

Focusing on this fundamental approach, the book is split into three parts. Part I provides physical insights for physics-oriented readers, part II presents detailed technical analysis using perturbation methods for theoreticians, and part III recalls the necessary background knowledge in physics, chemistry and fluid dynamics. This structure makes the content accessible to newcomers to the physics of unstable fronts in flows, whilst also offering advanced material for scientists who wish to improve their knowledge.

**Paul Clavin** is Professor Emeritus at Aix-Marseille Université and is an honorary member of the Institut Universitaire de France (Chair of Mécanique Physique 1993-2004). In 1995 he founded a renowned research institute, the Institut de Recherche sur les Phénomènes Hors Équilibre (IRPHE) and has received major awards from the Société Française de Physique (Plumey 1988), French Academy of Sciences (Grand Prix 1995) and from the Combustion Institute (Zeldovich Gold medal, San Francisco, August 2014).

**Geoff Searby** is retired Director of Research at the Institut de Recherche sur les Phénomènes Hors Équilibre (IRPHE). He is a renowned specialist of the physics of thermo-acoustic instabilities in combustion chambers and rocket motors, and his experiments have made major contributions to the understanding of the dynamics of flame fronts. In 2004 he obtained a major award from the French Academy of Sciences.

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Clavin and Searby

Combustion Waves and Fronts in Flows

# Combustion Waves and Fronts in Flows

Flames, Shocks, Detonations, Ablation Fronts and Explosion of Stars

Paul Clavin and Geoff Searby



730 pages In press; to appear in September 2016

## Lecture 1: **Orders of magnitude**

1-1: Overall combustion chemistry

1-2: Combustion waves in gaseous mixtures

1-3: Arrhenius law

1-4: Hydrocarbon/air flames

1-5: Instabilities of flames

# I – 1) Overall combustion chemistry

reactants → products + heat release

Lavoisier 1777



binding energy of small molecules ≈ a few eV

$$1\text{eV/molecule} \approx 23 \text{ kcal/mole} \quad \Rightarrow \quad T_b - T_u \approx 2000 \text{ K}$$

$$(C_p \approx 10 \text{ cal/mole/K})$$

reaction time  $\tau_r(T)$  extremely sensitive to temperature:

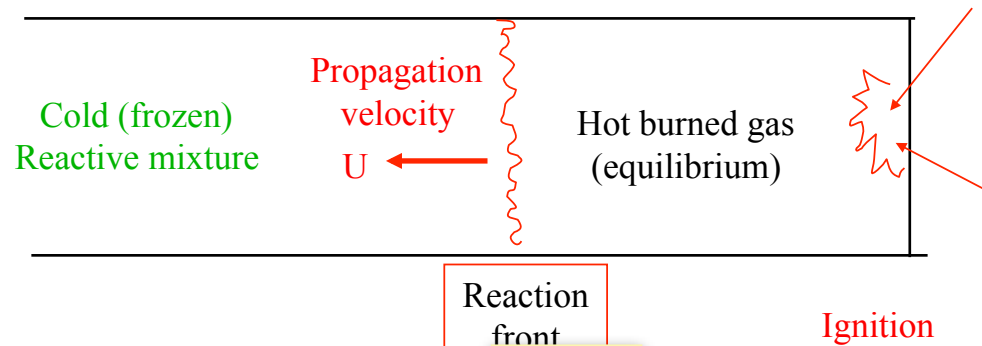
$$T < 500\text{K} : \tau_r(T) \approx \infty \text{ (frozen mixture of reactants)}$$

$$T \approx 2500\text{K} : \tau_r(T) \approx 10^{-6} \text{ s.}$$

Euler 1738



thermal feedback ⇒ combustion waves



Davy 1830



# I – 2) Combustion waves in gaseous mixtures

Flames : 10 cm/s – 10 <sup>acetylen/oxygen</sup> m/s,  $\Delta p/p \approx -10^{-5}$

Laminar propagation

Fast deflagrations :  $\approx 100$  m/s,  $\Delta p/p \approx -10^{-1}$

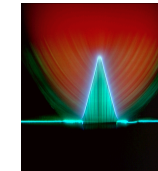
Turbulent propagation

Detonations :  $\approx 2000$  m/s,  $\Delta p/p \approx +30$

Cellular structure



*E. Mallard*  
*H. Le Chatelier*  
Mallard, Le Chatelier 1883



Laser Tomography  
L.Boyer 1980  
Bec Bunsen  
J. Quinard 2000



John H.S. Lee 1990



Berthelot, Vielle 1884



Shchelkin 1960

## Dimensional parameters

chemical energy/unit mass  $q_m \Rightarrow T_b/T_u = 5 - 10$

sound speed,  $a_b/a_u = \sqrt{T_b/T_u}$

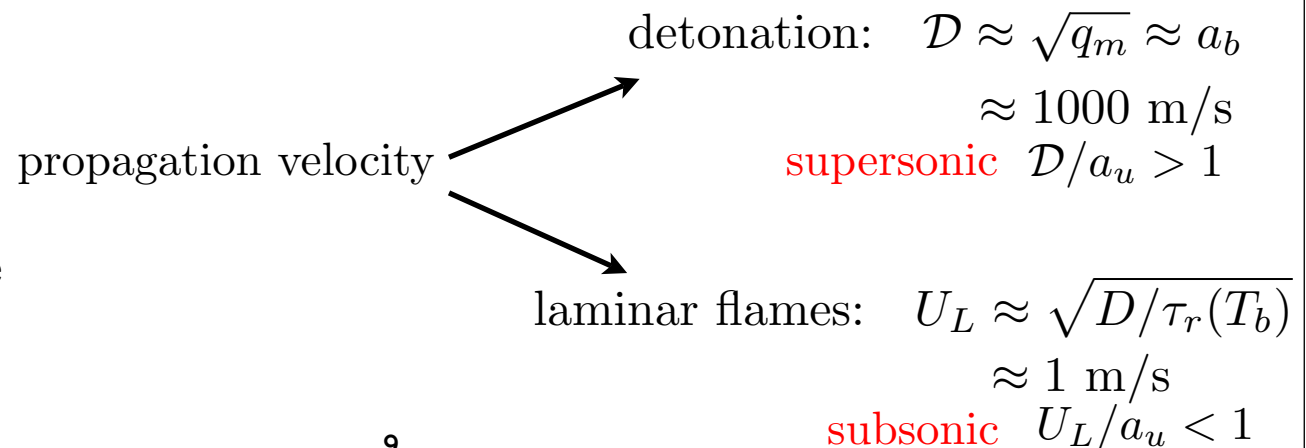
reaction rate  $1/\tau_r(T_b)$

molecular and thermal diffusion coefficients  $D \approx D_T$

## Dimensional analysis

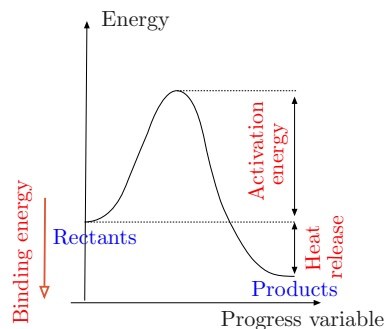
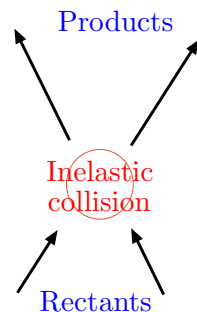
$$[q_m] = (\text{velocity})^2$$

$$[D] = (\text{velocity})^2 \times \text{time}$$



# I – 3) Overall reaction rate. Arrhenius law

Collision in gases



$$\frac{E}{k_B T_b} \approx 8$$

$$e^{-E/k_B T_b} \approx 3 \times 10^{-4}$$

Kinetic theory of gases  $\Rightarrow$  Arrhenius law

MB distribution  $\propto e^{-\frac{1}{2} \frac{mv^2}{k_B T}}$

$$\Rightarrow \frac{1}{\tau_r(T)} = \frac{1}{\tau_{coll}} e^{-E/k_B T}$$

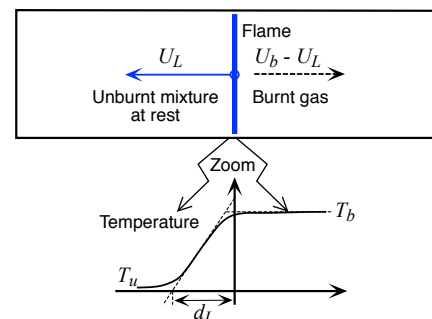
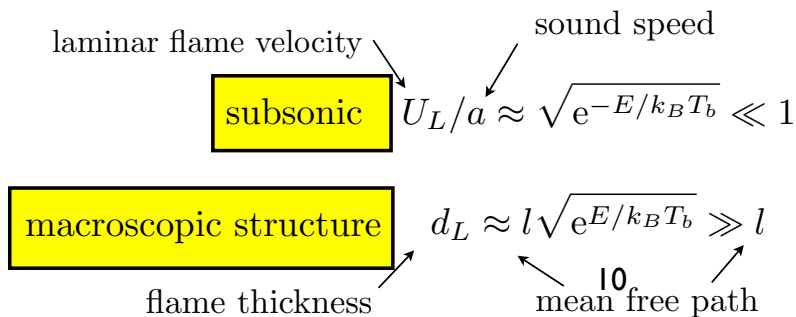
elastic collision rate  $1/\tau_{coll} \approx 10^9 \text{ s}^{-1}$

$$\Rightarrow 1/\tau_r(T_b) \approx 3 \times 10^5 \text{ s}^{-1}$$

$$T_b/T_u = 8 \Rightarrow \tau_r(T_u) \approx 10^{10} \text{ years !!}$$

Kinetic theory of gases  $\Rightarrow$  Flame properties

$$D \approx a^2 \tau_{coll} \approx l^2 / \tau_{coll}$$



# Back to the kinetic theory of gases

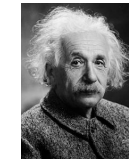
## Molecular diffusion $\equiv$ Random Walk



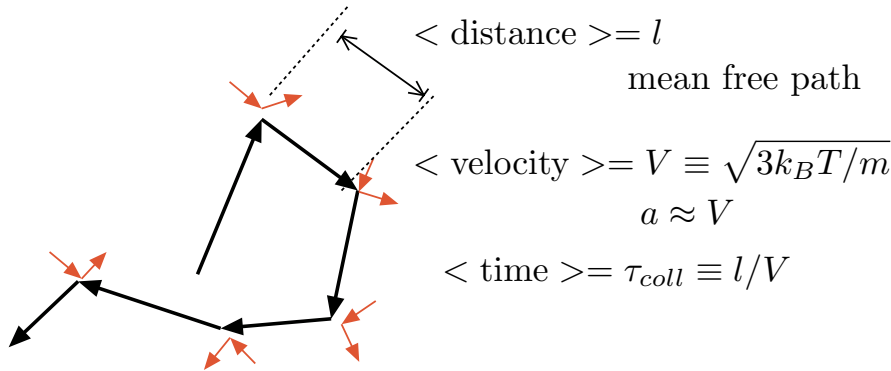
Maxwell 1867



Boltzmann 1877



Einstein 1905



spreading :  $\frac{1}{(4\pi Dt)^{3/2}} e^{-r^2/4Dt}$

$D = lV = l^2/\tau_{coll} \approx a^2\tau_{coll}$

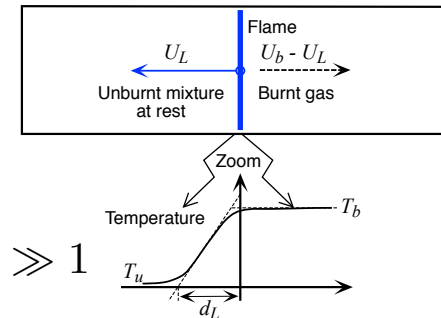
## Flame structure

$U_L \approx \sqrt{D/\tau_r(T_b)}$

$\frac{1}{\tau_r(T_b)} = \frac{1}{\tau_{coll}} e^{-E/k_B T_b}$

$U_L/a \approx \sqrt{e^{-E/k_B T_b}} \ll 1$

$d_L/l \approx U_L \tau_r/l \approx \sqrt{D\tau_r}/l \approx \sqrt{e^{E/k_B T_b}} \gg 1$



## Limitations of the dimensional analysis

$e^{-E/k_B T_b} \approx 3 \times 10^{-4}$

$a \approx 500 \text{ m/s}$

$l \approx 10^{-7} \text{ m}$



$\Rightarrow$

$U_L \approx 8.6 \text{ m/s}$

too large

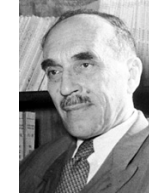
10 – 50 cm/s

$d_L \approx 0.6 \times 10^{-5} \text{ m}$

too small

hydrocarbon/air

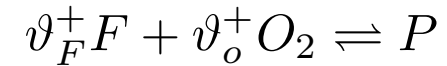
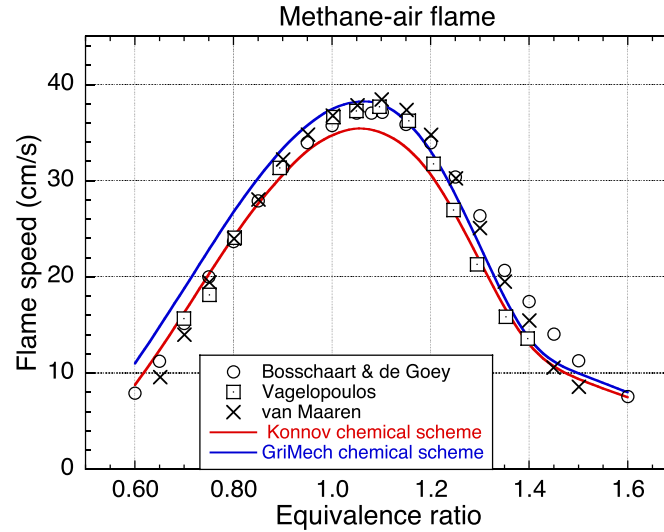
1 – 10<sup>-1</sup> mm



Semenov 1934

# I – 4) Hydrocarbon/air flames

Methane-air flame



Equivalence ratio

$$\phi = \frac{N_F/N_{O_2}}{\nu_F^+/\nu_{O_2}^+}$$

$\phi = 1$  : stoichiometry

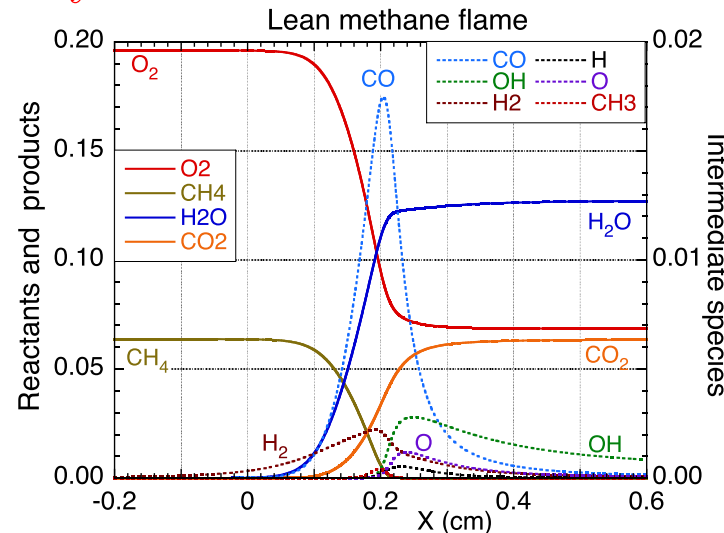
$\phi > 1$  : fuel rich

$\phi < 1$  : fuel lean

near to the flammability limit

$$\phi = 0.65$$

”thicker flame”



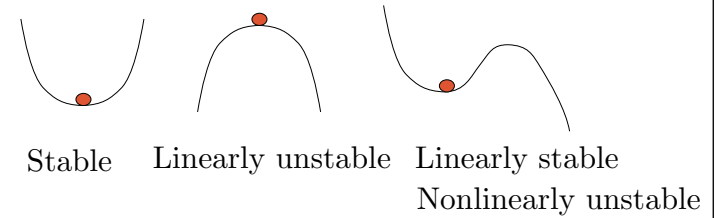
$$\begin{aligned} d_L &\approx U_L \tau_r(T_b) \\ &\approx \sqrt{D_T \tau_r(T_b)} \\ &\approx D_T \sqrt{\tau_r(T_b)} / D_T \\ &\approx D_T / U_L \end{aligned}$$



# I – 5) Instabilities of flames

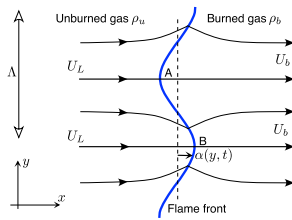
## Intrinsic instabilities

Planar flames are linearly unstable:



- hydrodynamic instability of the flame front

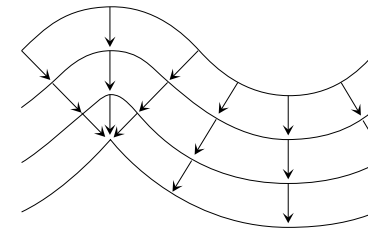
$$\rho_u > \rho_b$$



induced flow



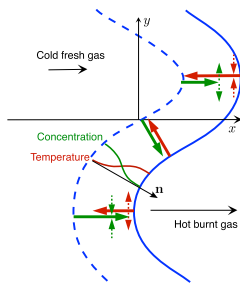
Propane lean flame



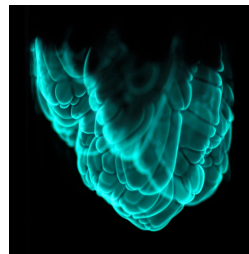
Cusp formation  
Huygens construction

- diffusional-thermal instability of the inner flame structure

$$D_T < D$$



Unstable inner structure



Propane rich flame

## System instability (combustion in a cavity)

The coupling of flames with acoustics can be unstable

Thermo-acoustic instabilities (Rayleigh criterion)



Lord Rayleigh 1878

Combustion chambers

Rocket engine

Gas turbines

## Vibratory instability of flames in tubes

Lean methane-air flame

$$\phi = 0.73 \quad U_L = 23 \text{ cm/s}$$

$$\phi = 0.8 \quad U_L = 30 \text{ cm/s}$$

**Acoustic instability  
in  
Premixed Flames**

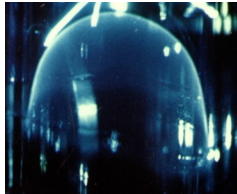
© IRPHE  
G. Searby

G. Searby IRPHE 2006

Tomography cut: L. Boyer 1980

## Effect of acceleration

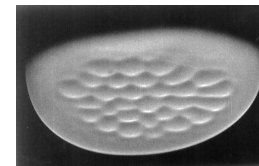
### Gravity



Propane flame propagating upward

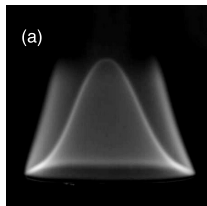


Slowly downward propagating flame

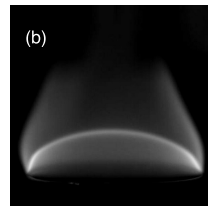


slightly faster

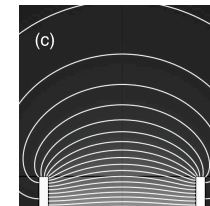
### Effect of an acoustic field on a Bunsen flame



Methane rich Bunsen flame  
 $\phi = 1.5$



in the presence of  
an axial acoustic field



equipotential surface  
in the absence of flame