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Lectures on Dynamics of Combustion Waves in Premixed Gases

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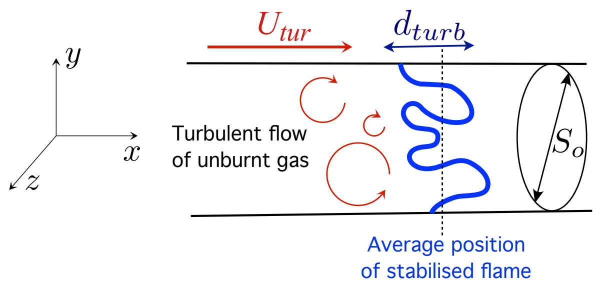
Lecture IX
Turbulent flames

Lecture 9 : Turbulent flames

IX-1) Introduction

The problem of premixed flames in a turbulent flow is still widely open



U_{turb} ?

d_{turb} ?

Experiments are difficult. Experimental data are very scattered

The simplest model has no known solution (Nonlinear stochastic equation)

Reaction-diffusion wave in a turbulent flow (no gas expansion)

$$\partial\theta/\partial t + \mathbf{v}(\mathbf{r},t).\nabla\theta - D_T\Delta\theta = \omega'(\theta)/\tau_{rb}.$$

prescribed turbulent flow (stochastic field)

Same model in the wrinkled flame regime $(l_{tur} \gg d_L,\ \ \tau_{tur} \gg \tau_L \Rightarrow\ U_n = U_L)$

eq. flame surface $\quad G(\mathbf{r},t) = G_0 \quad \partial G/\partial t + (\mathrm{d}\mathbf{r}/\mathrm{d}t).\nabla G = 0 \quad \mathrm{d}\mathbf{r}/\mathrm{d}t = \mathbf{v}(\mathbf{r},t) - U_n\mathbf{n} \quad \mathbf{n} = \nabla G/|\nabla G|$

stochastic eikonal eq.

$$\partial G/\partial t + \mathbf{v}(\mathbf{r},t).\nabla G = U_n|\nabla G|$$

$\mathbf{v} = (u, w_y, w_z)$

$x = \alpha(y,z,t)$

$G - G_0 = x - \alpha(y,z,t)$

$$\partial\alpha/\partial t - u(\mathbf{r}_f,t) + \mathbf{w}(\mathbf{r}_f,t).\nabla_{yz}\alpha = U_{tur} - U_n\sqrt{1+|\nabla_{yz}\alpha|^2}$$

$$\langle S\rangle = \iint \mathrm{d}x\mathrm{d}y\left\langle\sqrt{1+|\nabla_{yz}\alpha|^2}\right\rangle \qquad U_{tur}/U_L = \left\langle\sqrt{1+|\nabla_{yz}\alpha|^2}\right\rangle \qquad U_{tur}S_o = U_L\langle S\rangle$$

The very existence of $\langle S\rangle$ and d_{tur} is questionable

$|\mathbf{v}| \ll U_L: \quad U_{tur}/U_L \approx 1 + (|\mathbf{v}|/U_L)^2$ (Shchelkin 1943, Clavin Williams 1979)

$|\mathbf{v}| \gg U_L: \quad U_{tur} \approx |\mathbf{v}|$ (Damköler 1940)

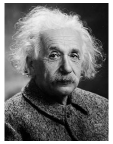
Bending effect

modification to the laminar flame struture

IX-2) Turbulent diffusion

Taylor's diffusion coefficient (analogy with Einstein random walk for molecular diffusion)

G.I Taylor 1922



Einstein 1905

1-D for simplicity: $\mathrm{d}x/\mathrm{d}t = v(t), \quad x(t) = \int_0^t v(t')\mathrm{d}t'$ ← stochastic Lagrangian velocity

ensemble average

$$\langle x^2(t)\rangle = \int_0^t \mathrm{d}t' \int_0^t \mathrm{d}t'' \langle v(t')v(t'')\rangle \qquad \langle x^2(t)\rangle = 2\int_0^t \mathrm{d}t' \int_0^{t'} \mathrm{d}\tau \langle v(t')v(t'-\tau)\rangle$$

turbulence: homogeneous in time $\langle v(t)v(t-\tau)\rangle = \langle v^2\rangle g(\tau) \quad g(0)=1, \quad \lim_{\tau\to\infty} g = 0$

$$\tau_I \equiv \int_0^\infty g(\tau)\mathrm{d}\tau$$

integral time scale

integration by parts $\langle x^2(t)\rangle = 2\langle v^2\rangle \int_0^t (t-\tau)g(\tau)\mathrm{d}\tau$ where $\int_0^\infty \tau g(\tau)\mathrm{d}\tau = O(\tau_I^2) \quad t \gg \tau_I: \; g = 0$

$t \gg \tau_I$, 1-D : $\langle x^2(t)\rangle = 2D_{tur}t$, 3-D : $\langle x^2(t)\rangle = 6D_{tur}t$, where

$$D_{tur} \equiv \langle v^2\rangle \tau_I$$
$$\langle v^2\rangle = (\text{turbulence intensity})^2$$

Rough model for the turbulent transport (analogy with molecular diffusion)

$$\langle \mathbf{v}\theta\rangle \approx -D_{tur}\nabla\langle\theta\rangle, \qquad \langle \nabla.(\mathbf{v}\theta)\rangle \approx -D_{tur}\Delta\langle\theta\rangle$$

limited to scalar mixing with small displacement / size (blobs, sheets ..) $l_I \ll L \quad \left(v_I \approx \langle v^2\rangle^{1/2}, \; l_I \equiv v_I\tau_I\right)$

turbulence intensity — integral length scale

$$D_{tur} = l_I v_I$$

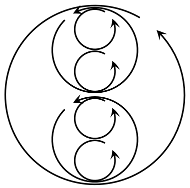
Well-stirred flame regime of Damköhler (1940) $l_I \ll d_L$ and $D_{tur} \gg D_T$

little practical importance

$$U_{tur} \approx \sqrt{D_{tur}/\tau_b} \qquad \frac{U_{tur}}{U_L} \approx \sqrt{\left(\frac{l_I}{d_L}\right)\left(\frac{v_I}{U_L}\right)} \gg 1, \qquad d_{tur} \approx D_{tur}/U_{tur} \gg d_L$$

IX-3) Strongly corrugated flammelets regime

Kolmogorov's cascade (homogeneous, isotropic and fully-developed turbulence)



schematic representation
$l_i = l_I/2^i$

Decomposition into a sum of vorties of different length and time scales

$l_i, \quad \tau_i, \quad v_i \equiv l_i/\tau_i$ — turn-over velocity

$\mathrm{Re}_i \equiv l_i v_i/\nu$ — local Reynolds nb

$\nu \equiv \mu/\rho$ — viscous diffusion coeff

Kolmogorov 1941

Kolmogorov scale $\quad l_K, \tau_K, v_K \quad \mathrm{Re}_K = 1 \quad l_i > l_K \quad v_i > v_K \quad \forall i$

Integral scale $\quad l_I, \tau_I, v_I \quad \mathrm{Re}_I \gg 1 \quad l_I > l_i \quad v_I > v_i \quad \forall i$

Scaling laws (dimensional analysis) $\; l_K \ll l_i \ll l_I$

energy transfer in NS eqs : $\rho(\mathbf{v}.\nabla)v^2/2 \qquad {v_i}^3/l_i \equiv \epsilon \approx \text{cst} \Rightarrow v_i \approx \epsilon^{1/3} l_i^{1/3}, \quad v_i^2 \approx \epsilon^{2/3} l_i^{2/3}, \quad \tau_i \approx \epsilon^{-1/3} l_i^{2/3},$

dissipation rate of energy : $\nu \mathbf{v}.\Delta\mathbf{v} \quad \Rightarrow \quad \epsilon = \nu\, v_K^2/l_K^2 \qquad \epsilon = v_I^3/l_I$

$$\mathrm{Re}_K \equiv v_K l_K/\nu = 1 \quad \Rightarrow \quad \boxed{l_I/l_K \approx \mathrm{Re}_I^{3/4}, \quad v_I/v_K \approx \mathrm{Re}_I^{1/4}, \quad \tau_I/\tau_K \approx \mathrm{Re}_I^{1/2}} \qquad \mathrm{Re}_I \gg 1$$

energy spectrum : $\left\langle v^2 \right\rangle/2 = \int_0^\infty \mathrm{d}k\, E(k) \qquad E(k) \approx \epsilon^{2/3} k^{-5/3}$

definition of strongly corrugated flames

$$\boxed{v_K \ll U_L \ll v_I} \Rightarrow d_L \ll l_\mathrm{K}, \quad \tau_L \ll \tau_\mathrm{K}$$

no modification to the laminar flame structure

$v_K \approx \nu/l_K\,, \;\; U_L \approx D_T/d_L,, \;\; D_T \approx \nu$

Gibson scale l_G (Peters 1986)

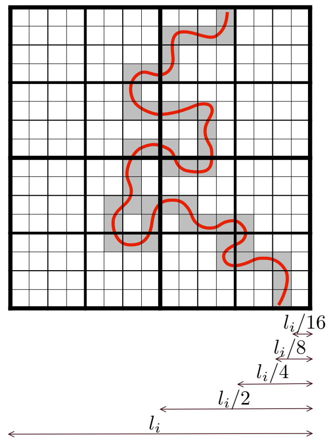
definition of the Gibson scale: smallest size of the wrinkles on the flame front

turn-over time = transit time across the vortex $\quad \tau_i \approx l_i/U_L \;\Rightarrow\; v_i \approx U_L$

$$\boxed{l_G \equiv U_L^3/\epsilon \;\Rightarrow\; l_K \ll l_G \ll l_I}$$

many scales of wrinkles $l_G \ll l_I \Rightarrow$ fractal geometry of the flame front

Elements of fractal geometry



Total surface area in a cube of size l_i, $\quad l_G < l_i < l_I \qquad S_i \approx N_{i,G}\, l_G^2.$

nb of cubes of size l_G that intersect the surface within the volume l_i^3 ↗

Weaker resolution $\quad l_j, \quad l_G < l_j < l_i \qquad S_{i,j} \approx N_{i,j}\, l_j^2$

nb of cubes of size l_j that intersect the surface within the volume l_i^3 ↗

Details of small scales are lost as the size of the box l_j increases

$$S_{i,j+k} < S_{i,j} < S_i \qquad N_{i,j+k} < N_{i,j} < N_i$$

Fractal dimension $\mathrm{D}_f > 2:$ $N_{i,j} \approx (l_i/l_j)^{D_f}$, $\quad S_{i,j}/l_i^2 \approx (l_i/l_j)^{D_f - 2}$

Regular surface: $D_f = 2 \Rightarrow$ total area S_i in a box of size $l_i \qquad S_i/l_i^2 = \lim_{l_j \to 0} S_{i,j}/l_i^2 =$ finite cst.

For a flame of thickness d_L its area is well defined for wrinkles whose scale is larger than d_L, $\; l_j > d_L$

The fractal dimension $D_f > 2$ can concern only scales greater than the smallest wrinkles

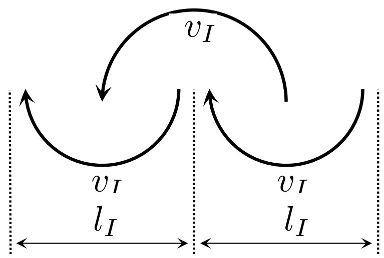
Fractal dimension of a turbulent flame can be meaningful only for $\; d_L < l_G < l_j < l_i < l_I$

Self similarity of strongly corrugated flames

$v_K \ll U_L \ll v_I \quad d_L \ll l_K \ll l_G \ll l_I$

Assumption: the Kolmogorov cascade is not modified by gas expansion   ok for $l_i \gg l_G$

Contamination time vs combustion time



Kolmogorov cascade $\tau_i \approx \epsilon^{-1/3} l_i^{2/3} \searrow l_i \searrow$ $v_i \approx \epsilon^{1/3} l_i^{1/3} \nearrow l_i \nearrow$

Fastest contamination: integral scale $v_I \gg v_i$. $U_{tur} = v_I$?

ok if the combustion time of the vortex is not longer than the turnover time

Self similar law

An effective front of thickness l_i is defined at each scale

A flame velocity U_i can be defined at each scale if $U_i = (S_{i,j}/l_i^2)U_j \qquad U_i/U_j = \langle S_{i,j} \rangle / l_i^2$

At the Gibson scale the combustion time of the vortex = turnover time $U_L = v_G$

Self similarity: same law at all scales $\Rightarrow$ combustion time of the vortex = the turnover time $\forall\, l_i$

$$l_i/U_i = \tau_i \quad \Rightarrow \quad U_i = v_i$$

Kolmogorov cascade $\Rightarrow$ small vortices burn faster than larger ones

$$U_{tur} = v_I, \quad l_{tur} = l_I$$

Fractal dimension of the flame surface:

$$U_i/U_j = \langle S_{i,j} \rangle / l_i^2 \;\Rightarrow\; u_i/u_j = \langle S_{i,j} \rangle / l_i^2 \;\Rightarrow\; \langle S_{i,j} \rangle / l_i^2 = (l_i/l_j)^{1/3}$$

$$\langle S_{i,j} \rangle / l_i^2 = (l_i/l_j)^{D_f - 2} \quad \Rightarrow \quad D_f = 7/3$$

The result is the same for all mixtures...??

Co-variant laws

Pocheau 1994

More general law independent of the turbulent scaling and satisfying additivity

Turbulent energy contained in the range $[l_i, l_j]$: $v_{i,j}^2 \equiv \sum_{k=i}^{j-1} v_k^2$ $\quad v_k^2$: energy in $[k, k+1]$

Co-variant law = same for each couple of length scales $l_i,\ l_j$ $\quad l_i > l_j$

The only co-variant law for the flame velocity U_i at scale l_i satisfying additivity is $U_i^2 = U_j^2 + c\, v_{i,j}^2$

Pocheau 1994

Co-variance ? $\quad l_i > l_k > l_j, \quad v_{i,j}^2 = v_{i,k}^2 + v_{k,j}^2 \quad U_i^2 = U_j^2 + c\, v_{i,k}^2 + c\, v_{k,j}^2 = U_k^2 + c\, v_{i,k}^2$

co-variance ok $\quad U_i^2 = U_k^2 + c\, v_{i,k}^2$

$$\boxed{U_{tur}^2 = U_L^2 + c\, v^2}$$

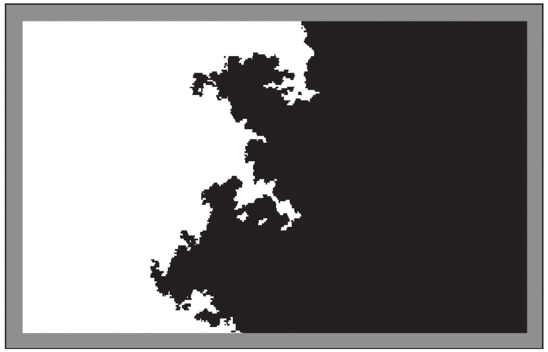
$v^2 \equiv \sum_{\forall n} v_n^2$ $\quad$ turbulence intensity

Not limited to a strong turbulence

The case $c = 1$ covers the known results at low and large turbulence intensity

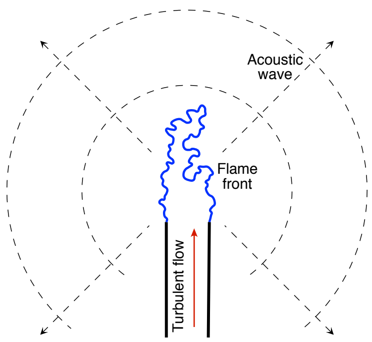
Reasonably good agreement with experiments

$$v/U_L = O(1), \quad l_I/l_K \approx 180$$



Pocheau 1996

IX-4) Turbulent combustion noise



wavelength $a/\omega \gg L$ size of the flame

Monopolar sound emission

Deformable (small) body with fluctuating volume $V(t)$

$$\mathbf{u} = \nabla\phi(\mathbf{r},t) \quad \text{acoustic potential } \phi(\mathbf{r},t) = -\frac{\dot{V}(t-a\,r)}{4\pi r} \qquad r \equiv |\mathbf{r}|, \quad \dot{V}(t) \equiv \mathrm{d}V/\mathrm{d}t$$

$$r \gg L: \quad v = (4\pi a r)^{-1}\ddot{V}(t - r/a), \qquad \ddot{V}(t) \equiv \mathrm{d}^2V(t)/\mathrm{d}t^2$$

Radiated flux of energy (intensity of sound) $I \equiv \rho a \left\langle v^2 \right\rangle$

$$I = (\rho/4\pi a)\left\langle \left(\mathrm{d}^2V/\mathrm{d}t^2\right)^2 \right\rangle$$

mass flow rate of burned gas in the lab frame

Sound generated by a turbulent flame $\mathrm{d}V/\mathrm{d}t = \dot{M}_b/\rho_b$

mass flow rate of fresh gas in the lab frame

$$\dot{M}_b = \rho_b \iint_S (\mathcal{D}_f + U_b)\mathrm{d}^2\sigma \qquad \dot{M}_u = \rho_u \iint_S (\mathcal{D}_f + U_L)\mathrm{d}^2\sigma \qquad \rho_u U_L = \rho_b U_b$$

normal flame velocity in the lab frame

elimination of $\mathcal{D}_f$ $\quad \mathrm{d}V/\mathrm{d}t = \dot{M}_b/\rho_b = \dot{M}_u/\rho_u + (U_b - U_L)S$ $\quad$ $\boxed{\mathrm{d}V/\mathrm{d}t = (U_b - U_L)S(t)}$

constant

$$I = (\rho/4\pi a)(U_b - U_L)^2 \left\langle (\mathrm{d}S/\mathrm{d}t)^2 \right\rangle \qquad \mathrm{d}\tilde{I}(\omega)/\mathrm{d}\omega = \frac{\rho}{4\pi a}(U_b - U_L)^2 \int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t} \left\langle \dot{S}(t)\dot{S}(0) \right\rangle$$

intensity of sound $\qquad$ power spectrum of sound

Strahle 1985

intensity of sound

$$I = (\rho/4\pi a)(U_b - U_L)^2 \left\langle (\mathrm{d}S/\mathrm{d}t)^2 \right\rangle$$

power spectrum of sound

$$\mathrm{d}\tilde{I}(\omega)/\mathrm{d}\omega = \frac{\rho}{4\pi a}(U_b - U_L)^2 \int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t} \left\langle \dot{S}(t)\dot{S}(0) \right\rangle$$

Stongly corrugated regime with a Kolmogorov cascade:

$$D_f = 4/3 \quad \Rightarrow \quad I \approx \frac{1}{4\pi}\left(\frac{T_b}{T_u} - 1\right)^2 (\rho \Delta V)\frac{v_I^4}{a l_I}$$

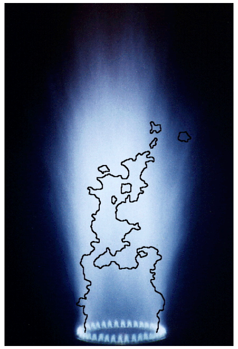
$\uparrow$ total volume of the flame brush

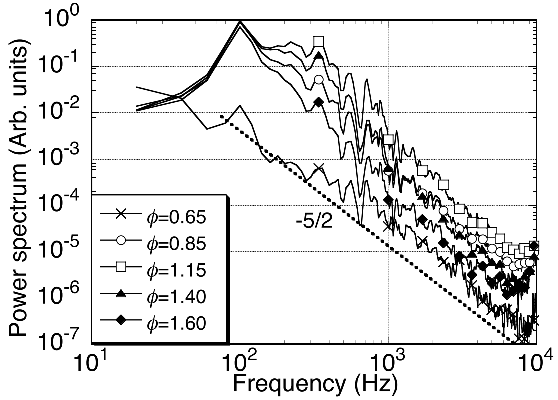
$$\mathrm{d}\tilde{I}(\omega) \propto \omega^{-5/2}\mathrm{d}\omega$$

Clavin Siggia 1991

in agreement with experiments on very large burners

(Abugov Obrezkov 1978)





Blowtorch noise

Combustion noise is two orders of magnitude higher

the noise is not resulting from the direct interaction of upstream turbulence on the flame front

amplification by the intrinsic flame instability is essential (Searby 2001)