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Lectures on Dynamics of Combustion Waves in Premixed Gases

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Lecture V
Thermo-diffusive phenomena

Lecture 5: Thermo-diffusive phenomena

5-1. Flame stretch and Markstein numbers

Passive interfaces

One-step flame model

The second Markstein number

5-2. Thermo-diffusive instabilities

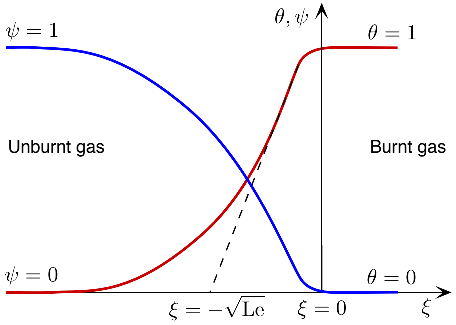
Planar flames for Le $\neq 1$

Jump conditions across the reaction layer

Linear equations and linear analysis

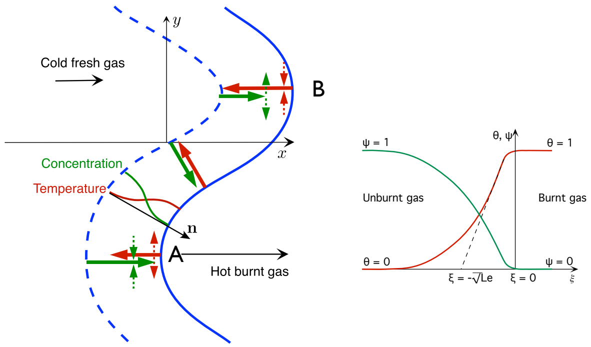
Cellular instability (Le < 1)

Oscillatory instability (Le > 1)

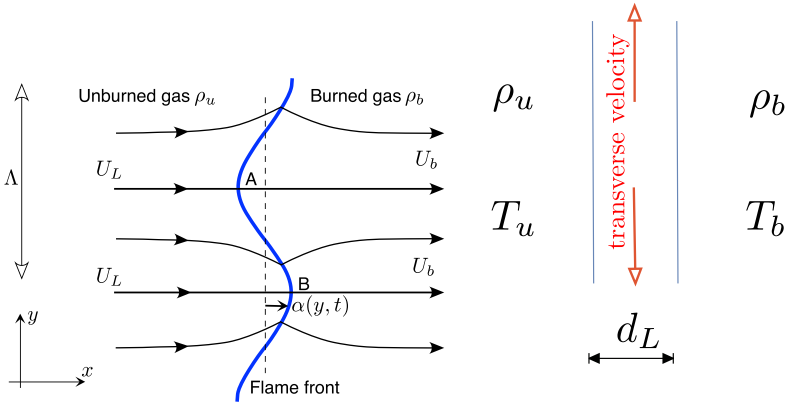


V-1) Flame stretch and Markstein numbers

Two mechanisms modify the inner flame structure



transverse diffusion



transverse convection

One-step model $\quad R \rightarrow P + Q$

A single scalar: the Markstein number $\mathcal{M}$

$$\boxed{(U_n^- - U_L)/U_L = -\mathcal{M}(\tau_L/\tau_s)}$$

U_n^- normal flame velocity in the fresh mixture

$1/\tau_s$ rate of stretch of flame surface

$$U_n^- = u_n^- - \mathcal{D}_f, \quad u_n^- \equiv \mathbf{n}_f . \mathbf{u}_f^-$$

Stretch rate, strain and curvature of a flame

Passive interface

element of surface area $\delta^2 s$ $\quad \frac{1}{\tau_s} = \frac{1}{\delta^2 s}\frac{\mathrm{d}\delta^2 s}{\mathrm{d}t}$ $\quad \mathrm{d}\mathbf{r}_f/\mathrm{d}t = \mathbf{u}^e(\mathbf{r}_f)$

element of volume $\delta^3 r = \delta^2 s\,\delta\zeta$ $\quad \frac{1}{\delta^3 r}\frac{\mathrm{d}}{\mathrm{d}t}\delta^3 r = \nabla.\mathbf{u}^e|_f$

coordinate normal to the front ζ continuity

$$\frac{1}{\delta^3 r}\frac{\mathrm{d}}{\mathrm{d}t}\delta^3 r = \frac{1}{\delta^2 s}\frac{\mathrm{d}}{\mathrm{d}t}\delta^2 s + \frac{1}{\delta\zeta}\frac{\mathrm{d}}{\mathrm{d}t}\delta\zeta$$

$$\mathrm{d}\delta\zeta/\mathrm{d}t = \mathbf{n}_f.\left[\mathbf{u}^e(\mathbf{r}_f + \delta\zeta\,\mathbf{n}_f) - \mathbf{u}^e(\mathbf{r}_f)\right]$$

first order correction in $d_L/L \ll 1$ $\quad \mathbf{u}^e\left(\mathbf{r}_f + \delta\zeta\,\mathbf{n}_f\right) \approx \mathbf{u}^e\left(\mathbf{r}_f\right) + \delta\zeta\ \mathbf{n}_f.\nabla\mathbf{u}^e$

$$\frac{1}{\delta\zeta}\frac{\mathrm{d}}{\mathrm{d}t}\delta\zeta = \ \mathbf{n}_f.\nabla\mathbf{u}^e.\mathbf{n}_f$$

$$\boxed{\frac{1}{\tau_s} \equiv \frac{1}{\delta^2 s}\frac{\mathrm{d}}{\mathrm{d}t}\delta^2 s = \nabla.\mathbf{u}^e|_f - \mathbf{n}_f.\nabla\mathbf{u}^e|_f.\mathbf{n}_f.}$$

Flame (first order correction) $d_L/\Lambda \ll 1$

$$\mathbf{u}^e(\mathbf{r}_f) = \mathbf{u}_f^- - U_L\mathbf{n}_f$$

$\mathbf{n}_f.\mathbf{n}_f = 1$

$\mathbf{n}_f.\nabla\mathbf{n}|_f.\mathbf{n}_f = 0$

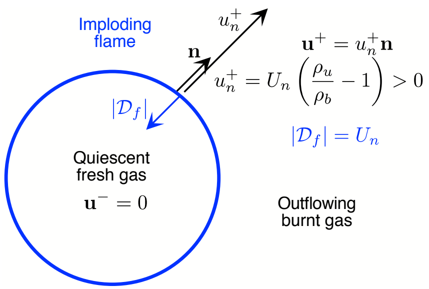
$$1/\tau_s = -U_L\nabla.\mathbf{n}_f + \nabla.\mathbf{u}^-|_f - \mathbf{n}_f.\nabla\mathbf{u}^-|_f.\mathbf{n}_f$$

Flame (first order correction) $d_L/\Lambda \ll 1$

$$1/\tau_s = -U_L\nabla.\mathbf{n}_f + \cancel{\nabla.\mathbf{u}^-}|_f - \mathbf{n}_f.\nabla\mathbf{u}^-|_f.\mathbf{n}_f$$

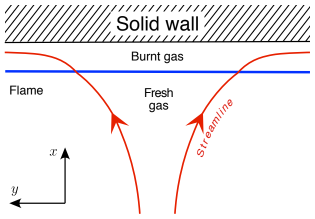
incompressibility

differential geometry $-\nabla.\mathbf{n}_f = 1/R \equiv (1/R_1 + 1/R_2)$



$$\boxed{1/\tau_s = U_L/R - \mathbf{n}_f.\nabla\mathbf{u}^-|_f.\mathbf{n}_f}$$

front curvature strain rate



study of structure of the one-step flame model $R \rightarrow P + Q$

Clavin, Williams 1982 Clavin, Garcia 1983

$$\boxed{(U_n^- - U_L)/U_L = -\mathcal{M}(\tau_L/\tau_s)}$$

reduced activation energy β Lewis number $\mathrm{Le} = D_T/D$

$\upsilon_b \equiv \rho_u/\rho_b > 1$ $\theta \equiv (T - T_u)/(T_b - T_u)$ $l \equiv \beta(\mathrm{Le} - 1)$ heat conductivity $\lambda(\theta)$

gas expansion $\Rightarrow$ hydrodynamics kinetics + diffusion

$$\boxed{\mathcal{M} = \frac{\upsilon_b}{\upsilon_b - 1}\mathcal{J} + \frac{l}{2}\frac{\mathcal{D}}{(\upsilon_b - 1)}}$$

lean hydrocarbon air mixtures $\mathcal{M} \approx 1 - 4$

$$\mathcal{J} = \int_0^1 \frac{(\upsilon_b - 1)\lambda(\theta)}{1 + (\upsilon_b - 1)\theta}\mathrm{d}\theta, \quad \mathcal{D} = -\int_0^1 \frac{(\upsilon_b - 1)\lambda(\theta)\ln\theta}{1 + (\upsilon_b - 1)\theta}\mathrm{d}\theta,$$

Clavin Garcia 1983

The second Markstein number

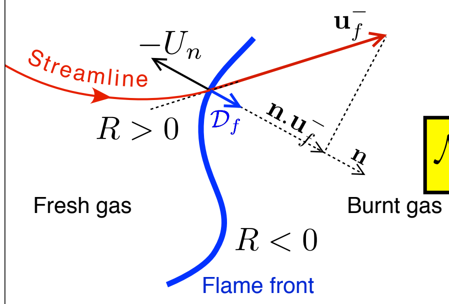
multiple-step flame model

$$\boxed{\mathcal{M}_{fc} \neq \mathcal{M}_{sr}}$$

Clavin Grana-Otero 2011

$$(U_n^- - U_L)/U_L = -\mathcal{M}_{fc}(d_L/R) + \mathcal{M}_{sr}(\tau_L \mathbf{n}_f.\nabla \mathbf{u}^-|_f.\mathbf{n}_f)$$

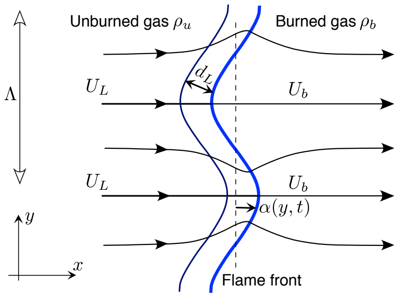
front curvature flow strain rate



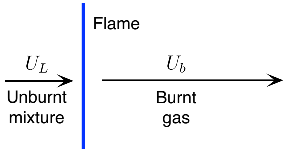
difficulty with the finite thickness:

$$\boxed{\mathcal{M}_{sr} \text{ varies with the position inside the flame structure}}$$

$$\mathcal{M}_{sr}(\tau_L \mathbf{n}_f.\nabla \mathbf{u}^-|_f.\mathbf{n}_f) = \text{cst.}$$



Markstein numbers in the burned gas

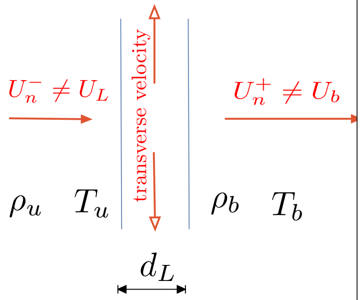


$$\frac{U_b}{U_L} = \frac{T_b}{T_u}$$

$$U_n^+ \equiv u_n^+ - \mathcal{D}_f \qquad u_n^+ \equiv \mathbf{n}_f.\mathbf{u}^+(\mathbf{r}_f)$$

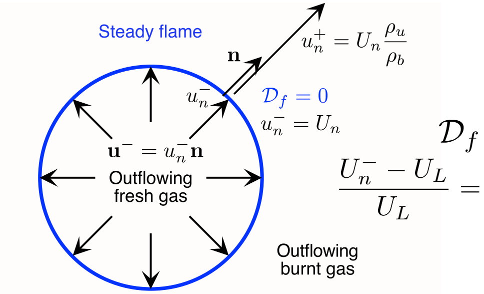
$$\frac{(U_n^+ - U_b)}{U_b} = -\mathcal{M}_{fc}^+ \frac{d_L}{R} + \mathcal{M}_{sr}^+ \tau_L \mathbf{n}_f.\nabla \mathbf{u}^+|_f.\mathbf{n}_f$$

$\rho_b U_n^- \neq \rho_b U_n^+$



$$\boxed{\mathcal{M}_{fc}^+ \neq \mathcal{M}_{fc}^- \qquad \mathcal{M}_{sr}^+ \neq \mathcal{M}_{sr}^-}$$

numerical and experimental data

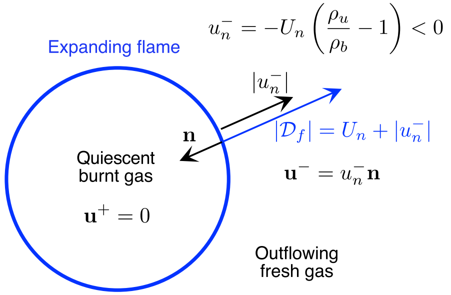


$$\mathcal{D}_f = 0, \quad U_n^- = u_n^-$$

$$\frac{U_n^- - U_L}{U_L} = -2(\mathcal{M}_{fc}^- - \mathcal{M}_{sr}^-)\frac{d_L}{R_f}$$

$$u_n^+ = 0 \quad U_n^+ = -\mathcal{D}_f$$

$$\frac{U_n^+ - U_b}{U_b} = -2\mathcal{M}_{fc}^+ \frac{d_L}{R_f}$$



V-2) Thermo-diffusive instabilities of planar flames

Sivashinsky 1977 Joulin Clavin 1979

instability mechanism $\neq$ hydrodynamic instability

equations $\cancel{\rho T = \rho_o T_o}$

$$\rho c_p \mathrm{D}T/\mathrm{D}t = \nabla.(\lambda \nabla T) + \sum_j Q^{(j)} \dot{W}^{(j)}(T,..Y_k..)$$

$$\rho \mathrm{D}Y_i/\mathrm{D}t = \nabla.(\rho D_i \nabla Y_i) + \sum_j \vartheta_i^{(j)} \mathsf{m}_i \dot{W}^{(j)}(T,..Y_k..),$$

+ ~~fluid mechanics~~

Thermo-diffusive flame model for a one-step kinetics $(\beta \gg 1)$

$$\theta \equiv \frac{T - T_u}{T_b - T_u} \in [0,1] \qquad \psi \equiv Y/Y_u \qquad \beta \equiv \frac{E}{k_B T_b}\left(1 - \frac{T_u}{T_b}\right) \qquad \frac{1}{\tau_{rb}} \equiv \frac{\mathrm{e}^{-E/k_B T_b}}{\tau_{coll}}$$

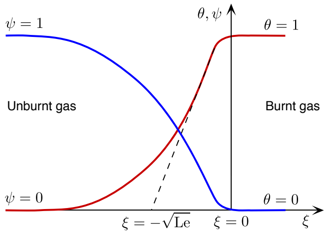
$\rho = \text{cst.}$

$$\boxed{\begin{array}{ll} \dfrac{\partial \theta}{\partial t} - D_T \Delta \theta = \dfrac{\psi}{\tau_{rb}} \mathrm{e}^{-\beta(1-\theta)} & \dfrac{\partial \psi}{\partial t} - D \Delta \psi = -\dfrac{\psi}{\tau_{rb}} \mathrm{e}^{-\beta(1-\theta)} \\ x = -\infty : \theta = 0, \psi = 1 & x = +\infty : \theta = 1, \psi = 0 \end{array}}$$

Planar flame for $\mathrm{Le} \equiv D_T/D \neq 1$

$$\xi \equiv \frac{x}{d_L(\mathrm{Le}=1)} \qquad \mu \equiv \frac{U_L}{U_L(\mathrm{Le}=1)} \; ?$$

$$\mu \frac{\mathrm{d}\theta}{\mathrm{d}\xi} - \frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} = \frac{\beta^2}{2} \psi \mathrm{e}^{-\beta(1-\theta)} \qquad \mu \frac{\mathrm{d}\psi}{\mathrm{d}\xi} - \frac{1}{\mathrm{Le}} \frac{\mathrm{d}^2\psi}{\mathrm{d}\xi^2} = -\frac{\beta^2}{2} \psi \mathrm{e}^{-\beta(1-\theta)}$$



reaction layer

$$-\frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} \approx \frac{\beta^2}{2} \psi \mathrm{e}^{-\beta(1-\theta)} \qquad \frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} + \frac{1}{\mathrm{Le}} \frac{\mathrm{d}^2\psi}{\mathrm{d}\xi^2} = 0$$

matching

$$\boxed{\mu = \sqrt{\mathrm{Le}}}$$

(first order reaction rate)

Flame temperature of curved flame for $\mathrm{Le} \neq 1$

$$\mathrm{Le} \neq 1 \quad \Rightarrow \quad \theta_f \neq 1$$

$$\beta \gg 1 \qquad \mathrm{Le} - 1 = O(1/\beta) \quad \Rightarrow \quad (\theta_f - 1) = O(1/\beta)$$

the thin reaction layer of curved flame is quasi-planar

$$-\frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} \approx \frac{\beta^2}{2}\psi \mathrm{e}^{-\beta(1-\theta)} \qquad \frac{\mathrm{d}^2\theta}{\mathrm{d}\xi^2} + \frac{1}{\mathrm{Le}}\frac{\mathrm{d}^2\psi}{\mathrm{d}\xi^2} = 0$$

ξ = non-dimensional normal coordinate

$$\theta = \theta_f - \Theta_1/\beta + .. \qquad \psi = -\Psi_1/\beta + .. \qquad \Theta_1 = \Psi_1/\mathrm{Le} \qquad \frac{1}{\beta^2}\frac{\mathrm{d}^2\Theta_1}{\mathrm{d}\xi^2} = \frac{1}{2}\mathrm{e}^{-\beta(1-\theta_f)}\Psi_1 \mathrm{e}^{-\Theta_1}$$

$\beta(\theta_f - 1) = O(1)$ $\quad$ $\mathrm{d}\theta/\mathrm{d}\xi|_{0+} = O(1/\beta)$ $\quad$ integration and matching $\quad$ $\mathrm{d}\theta/\mathrm{d}\xi|_{\xi=0-} \approx \mathrm{Le}^{1/2}\,\mathrm{e}^{-\beta(1-\theta_f)/2}$

jump conditions across the reaction layer

$$\mathrm{d}\theta/\mathrm{d}\xi|_{\xi=0-} = e^{-\beta(1-\theta_f)/2} \qquad \left[\frac{\mathrm{d}\theta}{\mathrm{d}\xi} + \frac{1}{\mathrm{Le}}\frac{\mathrm{d}\psi}{\mathrm{d}\xi}\right]_{0-}^{0+} = 0$$

valid at the leading order $\quad$ valid up to 1^{st} order

θ_f ?

Joulin Clavin 1979

Preheated zone

non-dimensional equations in the reference frame attached to the unperturbed flame

$$\xi \equiv x/d_L, \quad \eta = y/d_L, \quad \tau \equiv t/\tau_L \qquad \frac{\partial\theta}{\partial\tau} + \frac{\partial\theta}{\partial\xi} - \Delta\theta = 0, \quad \frac{\partial\psi}{\partial\tau} + \frac{\partial\psi}{\partial\xi} - \frac{1}{\mathrm{Le}}\Delta\psi = 0$$

boundary conditions: jump conditions and $\xi = -\infty \ : \theta = 0, \psi = 1, \qquad \xi = \infty \ : \theta = 1, \psi = 0.$

Linear equations

frame of reference attached to the reaction sheet (ζ, η, τ)

$$\zeta \equiv \xi - \mathrm{a}(\eta, \tau)$$

$\zeta = 0$: reaction sheet

$$\frac{\partial}{\partial \xi} = \frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial \eta} \to \frac{\partial}{\partial \eta} - \frac{\partial \mathrm{a}}{\partial \eta}\frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial \tau} \to \frac{\partial}{\partial \tau} - \frac{\partial \mathrm{a}}{\partial \tau}\frac{\partial}{\partial \zeta}$$

linearization

$$\theta = \overline{\theta}(\zeta) + \delta\theta, \quad \theta_f = 1 + \delta\theta_f, \quad \psi = \overline{\psi}(\zeta) + \delta\psi$$

external equations

$$\left[\frac{\partial}{\partial \tau} + \frac{\partial}{\partial \zeta} - \left(\frac{\partial^2}{\partial \zeta^2} + \frac{\partial^2}{\partial \eta^2}\right)\right]\delta\theta = \left(\frac{\partial \mathrm{a}}{\partial \tau} - \frac{\partial^2 \mathrm{a}}{\partial \eta^2}\right)\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}\zeta}$$

$$\left[\frac{\partial}{\partial \tau} + \frac{\partial}{\partial \zeta} - \frac{1}{\mathrm{Le}}\left(\frac{\partial^2}{\partial \zeta^2} + \frac{\partial^2}{\partial \eta^2}\right)\right]\delta\psi = \left(\frac{\partial \mathrm{a}}{\partial \tau} - \frac{1}{\mathrm{Le}}\frac{\partial^2 \mathrm{a}}{\partial \eta^2}\right)\frac{\mathrm{d}\overline{\psi}}{\mathrm{d}\zeta}$$

harmonic analysis
(normal modes)

$$\mathrm{a}(\eta, \tau) = \hat{\mathrm{a}}\mathrm{e}^{(\mathrm{i}\kappa\eta + \varsigma\tau)}, \quad \delta\theta_f(\eta, \tau) = \tilde{\theta}_f \hat{\mathrm{a}}\mathrm{e}^{(\mathrm{i}\kappa\eta + \varsigma\tau)}$$

$\varsigma \equiv \sigma\tau_L, \quad \kappa \equiv k d_L$

$$\delta\psi = \tilde{\psi}(\zeta)\hat{\mathrm{a}}\mathrm{e}^{(\mathrm{i}\kappa\eta + \varsigma\tau)} \qquad \delta\theta = \tilde{\theta}(\zeta)\hat{\mathrm{a}}\mathrm{e}^{(\mathrm{i}\kappa\eta + \varsigma\tau)}$$

ς ?
reduced linear growth rate

$$\left[\frac{\mathrm{d}}{\mathrm{d}\zeta} - \frac{\mathrm{d}^2}{\mathrm{d}\zeta^2}\right]\tilde{\theta}(\zeta) + \left(\varsigma + \kappa^2\right)\tilde{\theta}(\zeta) = \left(\varsigma + \kappa^2\right)\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}\zeta}$$

$$\left[\frac{\mathrm{d}}{\mathrm{d}\zeta} - \frac{1}{\mathrm{Le}}\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2}\right]\tilde{\psi}(\zeta) + \left(\varsigma + \frac{\kappa^2}{\mathrm{Le}}\right)\tilde{\psi}(\zeta) = \left(\varsigma + \frac{\kappa^2}{\mathrm{Le}}\right)\frac{\mathrm{d}\overline{\psi}}{\mathrm{d}\zeta}$$

boundary conditions: jump conditions and $\zeta = -\infty \ : \tilde{\theta} = 0, \tilde{\psi} = 0, \qquad \zeta = +\infty \ : \tilde{\theta} = 0, \tilde{\psi} = 0$

Analysis

external equations

$$\left[\frac{\mathrm{d}}{\mathrm{d}\zeta} - \frac{\mathrm{d}^2}{\mathrm{d}\zeta^2}\right]\tilde{\theta}(\zeta) + \left(\varsigma + \kappa^2\right)\tilde{\theta}(\zeta) = \left(\varsigma + \kappa^2\right)\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}\zeta}$$

$$\left[\frac{\mathrm{d}}{\mathrm{d}\zeta} - \frac{1}{\mathrm{Le}}\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2}\right]\tilde{\psi}(\zeta) + \left(\varsigma + \frac{\kappa^2}{\mathrm{Le}}\right)\tilde{\psi}(\zeta) = \left(\varsigma + \frac{\kappa^2}{\mathrm{Le}}\right)\frac{\mathrm{d}\overline{\psi}}{\mathrm{d}\zeta}$$

κ given
ς and θ_f ?

temperature in the external zones

particular solutions $\tilde{\theta} = \mathrm{d}\overline{\theta}/\mathrm{d}\zeta$ $\quad$ $\tilde{\psi} = \mathrm{d}\overline{\psi}/\mathrm{d}\zeta$

$$\overline{\theta}^{-} = \mathrm{e}^{\zeta} \quad \overline{\theta}^{+} = 1 \qquad \overline{\psi}^{-} = 1 - \mathrm{e}^{\mathrm{Le}\zeta} \quad \overline{\psi}^{+} = 0$$

boundary conditions

$$\zeta \rightarrow \pm\infty : \quad \tilde{\theta} = 0$$

$$\theta(\zeta = 0) = \theta_f \qquad \tilde{\theta}^{\pm} = \mathrm{d}\overline{\theta}^{\pm}/\mathrm{d}\zeta + \left(\tilde{\theta}_f - \mathrm{d}\overline{\theta}^{\pm}/\mathrm{d}\zeta\Big|_{\zeta=0}\right)\mathrm{e}^{r^{\pm}\zeta}$$

general solution to the homogeneous equation

$$r^2 - r - (\varsigma + \kappa^2) = 0 \qquad r^{\pm} = \frac{1}{2}\left[1 \mp \sqrt{1 + 4(\varsigma + \kappa^2)}\right]$$

mass fraction in the preheated zone zone $\quad$ $\zeta > 0 : \quad \tilde{\psi} = \overline{\psi} = 0$

$$\zeta \rightarrow -\infty : \quad \tilde{\psi} = 0$$

$$\psi(\zeta = 0) = 0$$

$$\tilde{\psi}^{-} = \mathrm{d}\overline{\psi}^{-}/\mathrm{d}\zeta - \left(\mathrm{d}\overline{\psi}^{-}/\mathrm{d}\zeta\Big|_{\zeta=0}\right)\mathrm{e}^{s^{-}\zeta}$$

$$\frac{1}{\mathrm{Le}}s^2 - s - \left(\zeta + \frac{\kappa^2}{\mathrm{Le}}\right) = 0 \qquad s^{-} = \frac{\mathrm{Le}}{2}\left[1 + \sqrt{1 + \frac{4}{\mathrm{Le}}\left(\varsigma + \frac{\kappa^2}{\mathrm{Le}}\right)}\right]$$

jump conditions

$$\mathrm{d}\theta/\mathrm{d}\xi|_{\xi=0-} = e^{-\beta(1-\theta_f)/2}$$

valid at the leading order

$$\left[\frac{\mathrm{d}\theta}{\mathrm{d}\xi}+\frac{1}{\mathrm{Le}}\frac{\mathrm{d}\psi}{\mathrm{d}\xi}\right]_{0-}^{0+}=0$$

valid up to 1^{st} order

ς and θ_f ?

asymptotic analysis $\beta \gg 1$:

$$\mathrm{Le}=1+l/\beta, \quad l\equiv\beta(\mathrm{Le}-1)=O(1)$$

$$\beta(1-\theta_f)=O(1) \qquad \beta\tilde{\theta}_f=O(1)$$

2^{nd} condition

$$\tilde{\theta}_f(r^+-r^-)=(s^--r^-)+(1-\mathrm{Le})$$

$$(r^+-r^-)=-\sqrt{1+4(\varsigma+\kappa^2)}$$

$$\mathrm{Le}\to 1:\quad s^--r^-\to 0$$

to leading order in small values of (Le-1) $= O(1/\beta)$

$$\boxed{\tilde{\theta}_f \approx \frac{(\mathrm{Le}-1)}{2}\left[\frac{1}{\sqrt{1+4(\varsigma+\kappa^2)}}-1+\frac{2\varsigma+4\kappa^2}{1+4(\varsigma+\kappa^2)}\right]}$$

linearized 1^{st}condition $\mathrm{d}\tilde{\theta}^-/\mathrm{d}\zeta|_{\zeta=0}=\beta\tilde{\theta}_f/2$ $\qquad 1-r^- = r^+ = \beta\tilde{\theta}_f/2$

valid to leading order using $\tilde{\theta}_f = O(1/\beta)$

$$\boxed{\beta\tilde{\theta}_f = 1+\sqrt{1+4(\varsigma+\kappa^2)}}$$

dispersion relation (ς,κ)

$$\boxed{-\frac{l}{2}\left[1-\sqrt{1+4(\varsigma+\kappa^2)}+2\varsigma\right]=\left[1-\sqrt{1+4(\varsigma+\kappa^2)}\right]\left[1+4(\varsigma+\kappa^2)\right]}$$

$$-\frac{l}{2}\left[1-\sqrt{1+4(\varsigma+\kappa^2)}+2\varsigma\right]=\left[1-\sqrt{1+4(\varsigma+\kappa^2)}\right]\left[1+4(\varsigma+\kappa^2)\right]$$

Cellular instability for $\mathrm{Le}\equiv D_T/D<1$

$$\kappa=0:\varsigma(\kappa)=0$$

Weakly curved limit. Small wavenumber expansion $\kappa\equiv kd_L\ll 1$ $\quad|\varsigma|\equiv|\sigma|\tau_L\ll 1$

$$\tilde{\theta}_f=(\mathrm{Le}-1)\kappa^2 \qquad \boxed{\varsigma=-(l+2)\kappa^2/2}$$

$l\equiv\beta(\mathrm{Le}-1)>-2$: $\sigma<0$ stable

$l\equiv\beta(\mathrm{Le}-1)<-2$: $\sigma>0$

unstable

$\mathcal{M}<0$

$$\frac{\partial\alpha}{\partial t}\propto -D_T\frac{\partial^2\alpha}{\partial y^2}$$

$$\boxed{\frac{\partial\alpha}{\partial t}=\left[\frac{\beta(\mathrm{Le}-1)+2}{2}\right]D_T\frac{\partial^2\alpha}{\partial y^2}}$$

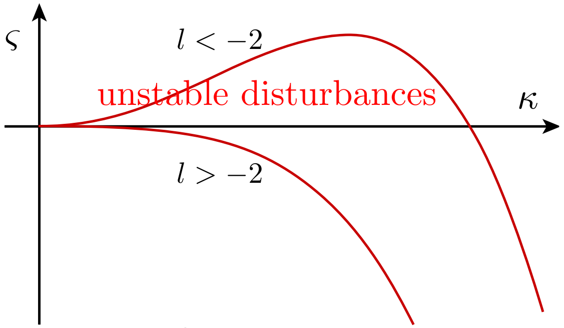
$$\frac{\partial\alpha}{\partial t}=\left[\frac{\beta(\mathrm{Le}-1)+2}{2}\right]D_T\left(\frac{\partial^2\alpha}{\partial y^2}+\frac{\partial^2\alpha}{\partial z^2}\right)=\left[\beta(\mathrm{Le}-1)+2\right]\frac{D_T}{R}, \qquad 2/R=1/R_1+1/R_2$$

$$\mathcal{M}=\beta(\mathrm{Le}-1)+2 \quad ⤨ \quad \mathcal{M}=\frac{\upsilon_b}{\upsilon_b-1}\mathcal{J}+\frac{l}{2}\frac{\mathcal{D}}{(\upsilon_b-1)}$$

$$\mathcal{J}=\int_0^1\frac{(\upsilon_b-1)\lambda(\theta)}{1+(\upsilon_b-1)\theta}\mathrm{d}\theta,\quad \mathcal{D}=-\int_0^1\frac{(\upsilon_b-1)\lambda(\theta)\ln\theta}{1+(\upsilon_b-1)\theta}\mathrm{d}\theta,$$

$$\varsigma=-(l+2)\kappa^2/2-8\kappa^4$$

Turing type of instability

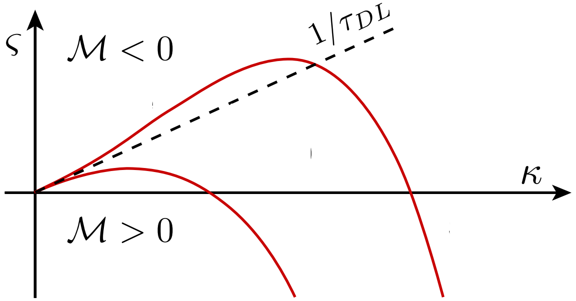


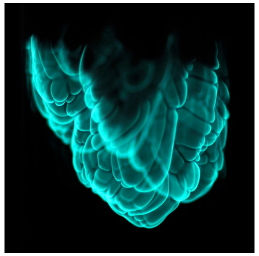
Zeldovich

Turing

Hydrodynamics + diffusion

Propane lean flame
$\mathcal{M} > 0$
hydrodynamic instability only



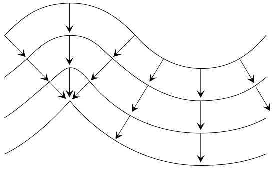


Propane rich flame
$\mathcal{M} < 0$
hydrodynamic + cellular instabilities
shorter wavelengths are unstable

Sivashinsky eq. 1977 $\dfrac{\partial \phi}{\partial \tau} - \mathcal{H}(\phi) - \triangle\phi + \dfrac{1}{2}|\nabla\phi|^2 = 0$

nonlinear equation

geometrical term: Huygens construction

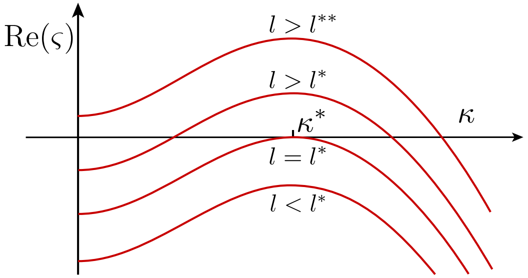


Oscillatory instability $\mathrm{Le} \equiv D_T/D > 1$

$\mathrm{Im}(\varsigma) \neq 0$

$l \equiv \beta(\mathrm{Le}-1) = l^* :\ \mathrm{Re}(\varsigma) = 0,\ \kappa^* \neq 0$

Poincaré-Andronov bifurcation $l^* \approx 10$



$l^{**} \approx 11$: planar pulsation. OK for solid combustion

effect of heat losses

Joulin Clavin 1979