Tsinghua-Princeton-CI Summer School
July 19-25, 2016

Lectures on Dynamics of Combustion Waves in Premixed Gases

Professor Paul Clavin

Aix-Marseille Université
ECM & CNRS (IRPHE)

Lecture XIV
Nonlinear dynamics of shock waves. Triple point and Mach stem formation.

Lecture 14: Nonlinear dynamics of shock waves
Mach stem formation

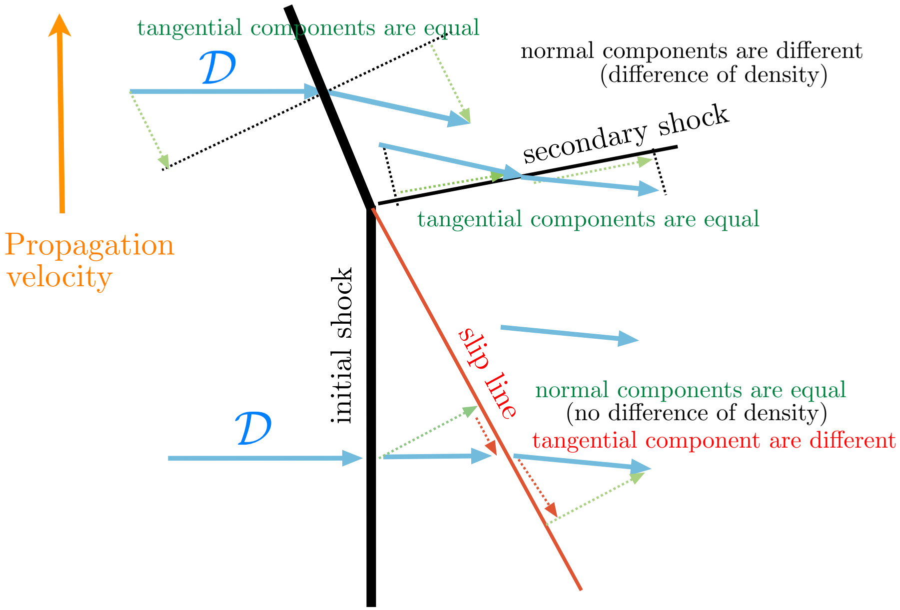
XIV-1) Introduction.
Recent experimental and DNS results

What is a Mach stem ?

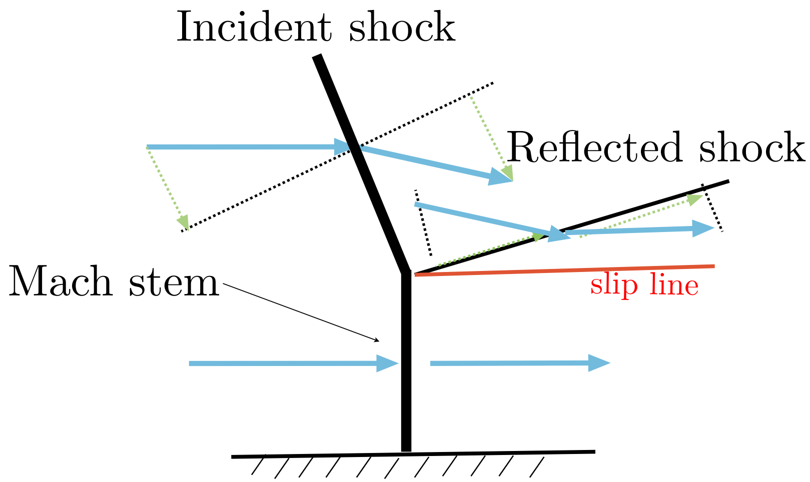
Triple point = 3 shock waves + 1 slip line (degenerescence of shear layer)

called also contact line discontinuity (Courant Friedrichs 1948)

First observed during the reflection of an oblique shock front incident an a wall



example of a triple point propagating in a uniform flow



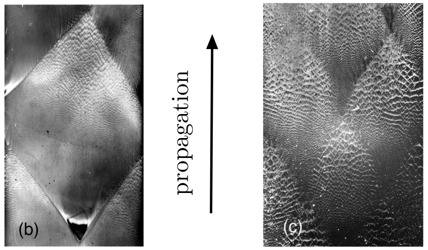
example of stationary triple point

Mach stems and cellular detonations

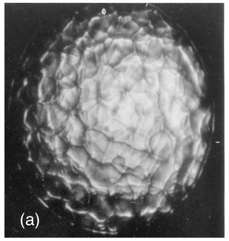
Experimental observations of the cellular structure of detonations

Transverse structures of gaseous detonations have been observed for a long time

Shchelkin Troshin (1965)



Joubert et al (2008)



Presles et al. (1987)

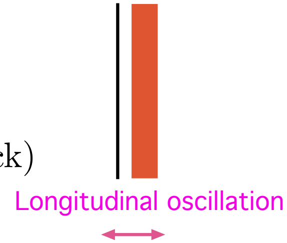
markings left on soot-coated foils on the walls

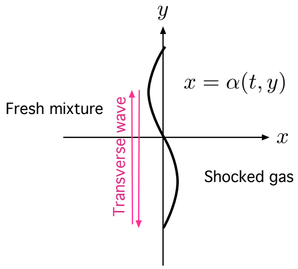
trajectory of triple points

visualisation of the cellular structures by optical methods

Underlying linear mechanisms

longitudinal oscillation of the complex shock reaction zone
see lecture XII
(Galloping detonation)
+
transverse oscillatory modes (normal modes of the lead shock)
see lecture XIII

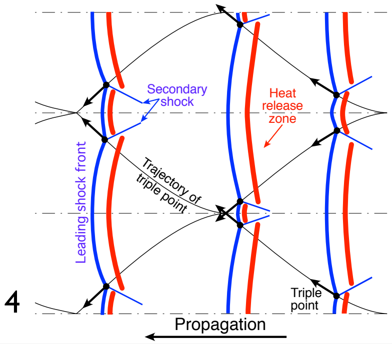




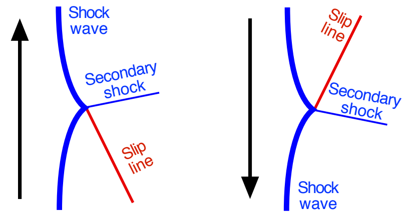
Nonlinear mechanisms

singularity of slope of the lead shock $\Rightarrow$ formation of Mach stems

trajectory of Mach stems $=$ makings

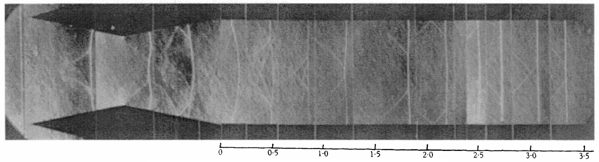


Mach stems propagating
in the transverse direction



Spontaneous formation of Mach stems on shock fronts

Schlieren experiments in shock tubes



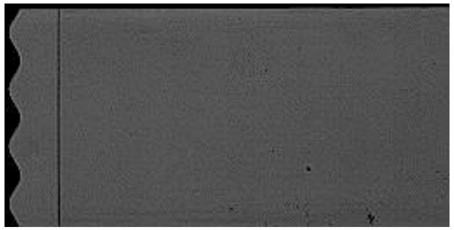
Freeman 1957 Lapworth 1959

Shock reflexion from a wavy wall

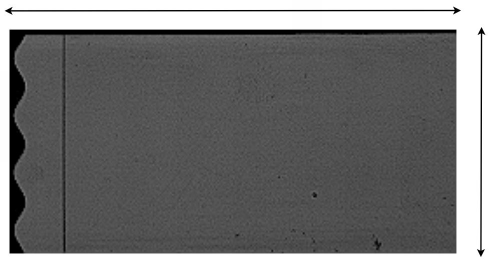
Briscoe Kovitz 1968

Jourdan Houas 2011

Denet et al. 2015

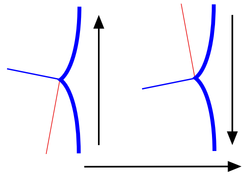


16 cm



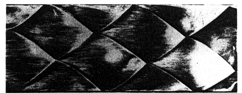
8 cm

Triple points



time between 2 consecutive frames: 8 μs

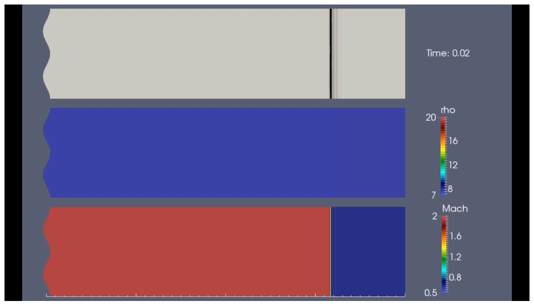
quite similar to the markings left by the transverse structure of cellular detonations



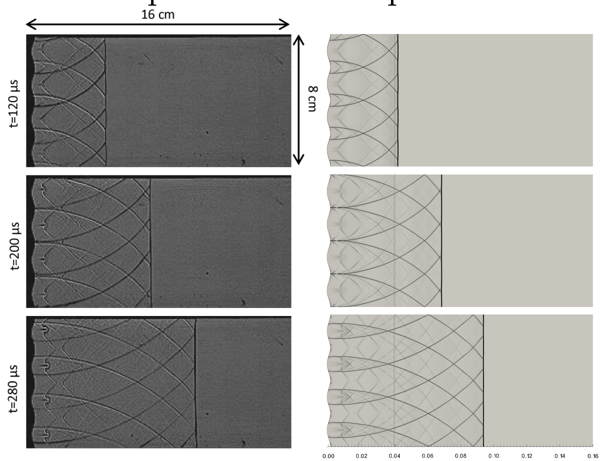
Shchelkin Troshin 1965

DNS in 2 dimension geometry

Lodato Vervisch 2015



Comparison with experiments



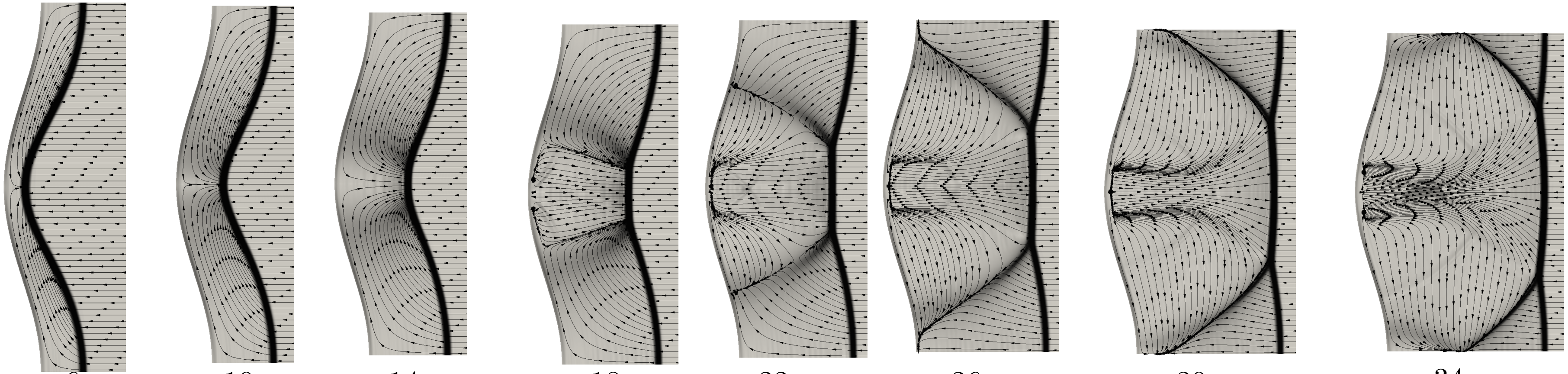
Denet et al 2015

Spontaneous formation of Mach stems

The incoming shock wave is not strong, $M_u = 1.5$ and the amplitude of wavy wall is small 1 mm

Immediately after reflection the wrinkled reflected shock has a smooth sinusoidal form

Singularity of slope is formed spontaneously at about 15 μs leading to triple points clearly observed as early as 20 μs

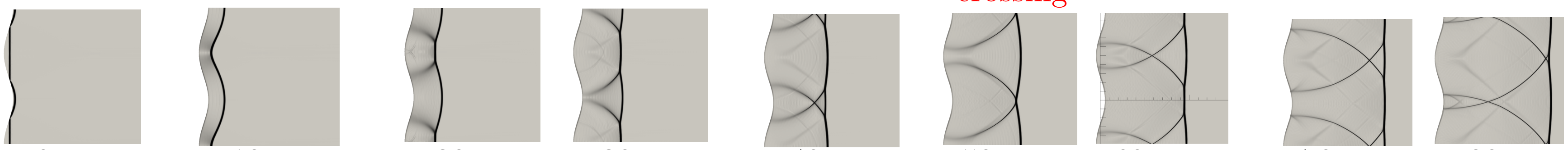


6 μs 10 μs 14 μs 18 μs 22 μs 26 μs 30 μs 34 μs

Lodato Vervisch 2015

Long-lived Mach stems on quasi-planar inert shock front

crossing



0 μs 10 μs 20 μs 30 μs 40 μs 50 μs 60 μs 70 μs 80 μs

Lodato Vervisch 2015

Sufficiently far from the wall the wall effect becomes negligible

The shock is quasi-planar with Mach stems propagating in the transverse direction crossing each other without deformation as solitons are known to do

XIV-2) Multidimensional dynamics of shock fronts

Analysis for strong shocks in the Newtonian limit

Linear dynamics

Distinguished limit

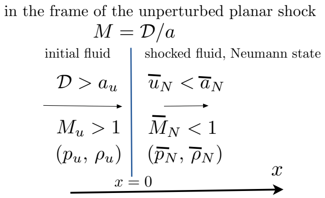
In order to simplify the presentation the analysis is performed for strong shock in the Newtonian limit

$$\overline{M}_u^2 \gg 1, \qquad (\gamma-1) \ll 1,$$

$$\overline{M}_u^2(\gamma-1) = O(1), \qquad \overline{M}_N^2 \approx (\gamma-1)/2 + 1/\overline{M}_u^2 \ll 1,$$

$$\boxed{\epsilon^2 \equiv M_N^2 \ll 1 \qquad \overline{M}_u = O(1/\epsilon), \qquad (\gamma-1) = O(\epsilon^2)}$$

$$\overline{u}_N/\overline{\mathcal{D}} \approx \epsilon^2, \qquad \overline{a}_N^2 \approx \overline{u}_N\overline{\mathcal{D}}, \qquad \overline{a}_N/a_u = O(1)$$



Rankine-Hugoniot relations (see p. 6 & p.9 lecture XIII)

$$\frac{\delta p_N}{\overline{p}_N} \approx -2\frac{\dot{\alpha}_t}{\overline{\mathcal{D}}}, \qquad \frac{\delta \rho_N}{\overline{\rho}_N} = -2\left(\frac{\overline{a}_u}{\overline{a}_N}\right)^2 \frac{\dot{\alpha}_t}{\overline{\mathcal{D}}},$$

$$(\delta u_N - \dot{\alpha}_t) = -\frac{[(\gamma-1)\overline{M}_u^2 - 2]}{2\overline{M}_u^2}\dot{\alpha}_t, \qquad \delta w_N \approx \overline{\mathcal{D}}\alpha'_y,$$

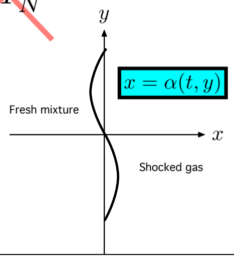
where for simplicity some unimportant ϵ^2 terms have been omitted in δp_N and δw_N

Quasi-isobaric approximation of the flow in the shocked gas

$$\pm 2S\cancel{\overline{M}_N}\sqrt{1+S^2} = S^2\left(1 + \cancel{\overline{M}_u^{-2}}\right) + 1 \qquad S \equiv \frac{\sigma}{\overline{a}_N|k|}\frac{1}{\sqrt{1-\cancel{\overline{M}_N^2}}} \quad \Rightarrow \quad \boxed{\sigma \approx \pm \mathrm{i}\overline{a}_N|k|}$$

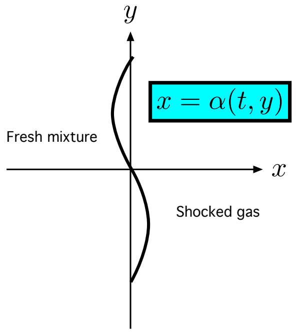
Dispersion relation see p. 10 lecture XIII

$$\omega \approx \overline{a}_N k, \qquad \boxed{\partial^2\alpha/\partial t^2 - \overline{a}_N^2\partial^2\alpha/\partial y^2 = 0}$$



Wave equation

$$\boxed{\epsilon^2 \equiv M_N^2 \ll 1 \qquad \overline{M}_u = O(1/\epsilon), \qquad (\gamma-1) = O(\epsilon^2)}$$



$$\overline{u}_N/\overline{\mathcal{D}} \approx \epsilon^2, \qquad \overline{a}_N^2 \approx \overline{u}_N \overline{\mathcal{D}}, \qquad \overline{a}_N/a_u = O(1)$$

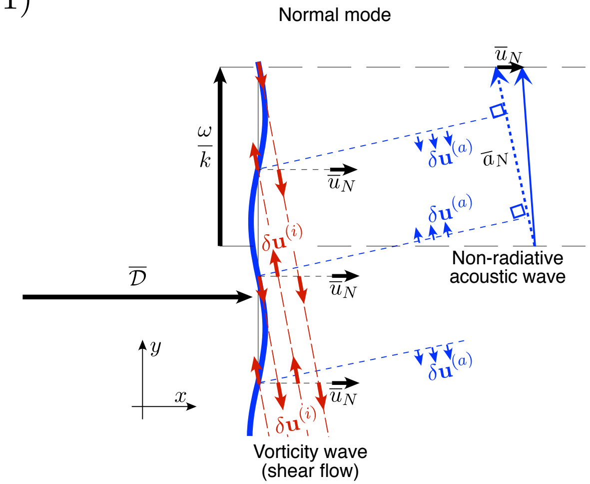
$$\omega \approx \overline{a}_N k, \qquad \boxed{\partial^2\alpha/\partial t^2 - \overline{a}_N^2 \partial^2\alpha/\partial y^2 = 0}$$

Wave equation

$$\omega \approx \overline{a}_N |k| \qquad \dot{\alpha}_t = O(\overline{a}_N \alpha'_y)$$

$$\delta p = \tilde{p}_N \exp\left(\mathrm{i}l_\pm x + \mathrm{i}ky + \sigma t\right) \qquad il_\pm = \pm il,$$

(see p.4 lecture XIII)



$$(l/|k|)\sqrt{1-\overline{M}_N^2} = \overline{M}_N \Omega + \sqrt{\Omega^2 - 1} > 0 \qquad \Rightarrow \qquad \boxed{l/|k| = O(\epsilon)}$$

$$\overline{M}_N = \epsilon \qquad \Omega \approx \omega/(\overline{a}_N|k|) \approx 1$$

the acoustic waves propagates in a direction quasi-parallel to the front

$$\tilde{u}^{(a)} = -\frac{\mathrm{i}l_\pm \overline{u}_N}{\sigma + \mathrm{i}l_\pm \overline{u}_N} \frac{\tilde{p}_N}{\overline{\rho}_N \overline{u}_N} \mathrm{e}^{\mathrm{i}l_\pm x}, \qquad \delta u^{(a)} = O(\epsilon \delta p_N/\overline{\rho}_N \overline{a}_N) \quad \delta u^{(a)} = O(\epsilon^2 \delta p_N/\overline{\rho}_N \overline{u}_N)$$

see p. 4 lecture XIII

$$\tilde{w}^{(a)} = -\frac{\mathrm{i}k\overline{u}_N}{\sigma + \mathrm{i}l_\pm \overline{u}_N} \frac{\tilde{p}_N}{\overline{\rho}_N \overline{u}_N} \mathrm{e}^{\mathrm{i}l_\pm x}, \qquad \delta w^{(a)} = O(\delta p_N/\overline{\rho}_N \overline{a}_N) \quad \delta w^{(a)} = O(\epsilon \delta p_N/\overline{\rho}_N \overline{u}_N)$$

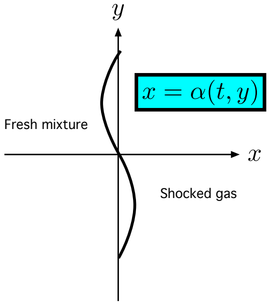
$$\Rightarrow \boxed{\begin{aligned} \delta u^{(a)} &= O(\epsilon^2 \dot{\alpha}_t) \\ \delta w^{(a)} &= O(\epsilon \dot{\alpha}_t) \end{aligned}}$$

Rankine-Hugoniot:
(see p. 6 & p.10 lecture XIII)

$$\delta p_N/\overline{p}_N \approx -2\dot{\alpha}_t/\overline{\mathcal{D}} \Rightarrow \boxed{\delta p_N/(\overline{\rho}_N \overline{u}_N) = O(\dot{\alpha}_t)}$$

weak acoustic wave

$$\overline{p}_N \approx \overline{\rho}_N \overline{a}_N^2 \approx \overline{\rho}_N \overline{u}_N \frac{\overline{a}_N}{\overline{M}_N} \qquad \frac{\overline{a}_N}{\overline{M}_N} = \frac{\overline{a}_N/a_u}{\overline{M}_u \overline{M}_N} \overline{\mathcal{D}} = O(\overline{\mathcal{D}})$$

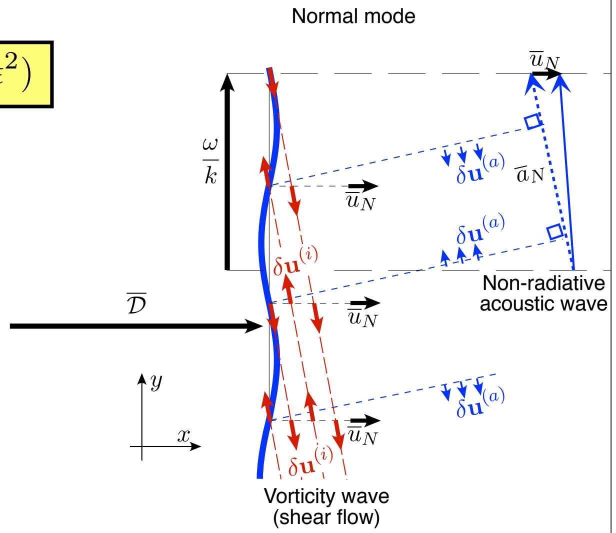


$$\boxed{\epsilon^2 \equiv M_N^2 \ll 1 \qquad \overline{M}_u = O(1/\epsilon), \qquad (\gamma - 1) = O(\epsilon^2)}$$

$$\overline{u}_N/\overline{\mathcal{D}} \approx \epsilon^2, \qquad \overline{a}_N^2 \approx \overline{u}_N \overline{\mathcal{D}}, \qquad \overline{a}_N/a_u = O(1)$$

$$\omega \approx \overline{a}_N k, \qquad \boxed{\partial^2\alpha/\partial t^2 - \overline{a}_N^2 \partial^2\alpha/\partial y^2 = 0}$$

Wave equation



Rankine-Hugoniot:

(see p. 6 & p.10 lecture XIII)

$$(\delta u_N - \dot{\alpha}_t) = -\frac{[(\gamma-1)\overline{M}_u^2 - 2]}{2\overline{M}_u^2}\dot{\alpha}_t,$$

$$\Rightarrow \quad \boxed{\delta u_N \approx \dot{\alpha}_t \qquad \delta w_N \approx \overline{\mathcal{D}}\alpha'_y = O(\dot{\alpha}_t/\epsilon)}$$

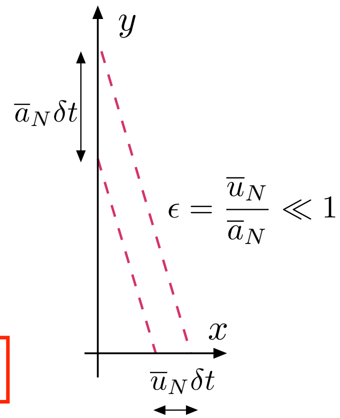
$$\delta u^{(i)}|_{x=0} = \delta u_N \approx \dot{\alpha}_t \qquad \delta w^{(i)}|_{x=0} = \delta w_N \approx \overline{\mathcal{D}}\alpha'_y$$

$$\Rightarrow \quad \boxed{\begin{array}{c} |\delta u^{(a)}/\delta u^{(i)}| = O(\epsilon^2) \\ |\delta w^{(a)}/\delta w^{(i)}| = O(\epsilon^2) \end{array}}$$

the acoustic waves are negligibly smaller than the vorticity wave

$$\boxed{|\delta u^{(i)}/\delta w^{(i)}| = O(\epsilon)}$$

the vorticity wave is a shear flow quasi-parallel to the front
propagating at a subsonic velocity in the normal direction



$$\left(\frac{\partial}{\partial t} + \overline{u}_N \frac{\partial}{\partial x}\right)\mathbf{u}^{(i)} = 0 \;\Rightarrow\; \delta u^{(i)} = \dot{\alpha}_t(y, t - x/\overline{u}_N) \qquad \delta w^{(i)} = \overline{\mathcal{D}}\alpha'_y(y, t - x/\overline{u}_N)$$

$$\frac{\partial u^{(i)}}{\partial x} + \frac{\partial w^{(i)}}{\partial y} = 0 \;\Rightarrow\; -\frac{1}{\overline{u}_N}\frac{\partial^2\alpha}{\partial t^2} + \overline{\mathcal{D}}\frac{\partial^2\alpha}{\partial y^2} = 0 \qquad \boxed{\overline{u}_N \overline{\mathcal{D}} \approx \overline{a}_N^2 \quad \Rightarrow \quad \text{Wave equation}}$$

A subsonic wave that is sufficiently tilted yields a trace on the front that is sonic

Weakly nonlinear analysis

Clavin (2013)

Nonlinear Euler equations

$$\frac{\partial u}{\partial t}+\overline{u}_N\frac{\partial u}{\partial x}=\mathcal{U}-\frac{1}{\rho}\frac{\partial p}{\partial x},\qquad \frac{\partial w}{\partial t}+\overline{u}_N\frac{\partial w}{\partial x}=\mathcal{W}-\frac{1}{\rho}\frac{\partial p}{\partial y},$$

$$-\mathcal{U}\equiv\delta u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial y},\qquad -\mathcal{W}\equiv\delta u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial y},$$

$$\mathcal{U}=O(\varepsilon\partial u^{(i)}/\partial t)\qquad \mathcal{W}=O(\varepsilon\partial w^{(i)}/\partial t)$$

where $\boxed{\delta u\equiv u-\overline{u}_N\approx u^{(i)} \quad w\approx w^{(i)}}$

$\boxed{u^{(i)}=\dot{\alpha}_t(y,t-x/\overline{u}_N) \quad w^{(i)}=\overline{\mathcal{D}}\alpha'_y(y,t-x/\overline{u}_N) \quad \overline{\mathcal{D}}\overline{u}_N\approx\overline{a}_N^2 \quad \ddot{\alpha}_{tt}=\overline{a}_N^2\alpha''_{y^2}}$

where $\varepsilon\equiv|\dot{\alpha}_t|/\overline{u}_N=O(|\alpha'_y|\overline{a}_N/\overline{u}_N)$ that is $\boxed{\varepsilon=O(|\alpha'_y|/\epsilon)}$

The weakly nonlinear approximation is valid when the nonlinear terms $\mathcal{U}$ and $\mathcal{V}$ are small (compared with the linear terms)

This is the case for small amplitudes of the wrinkles $\boxed{|\alpha'_y|\ll\epsilon}$ where $\epsilon\equiv\overline{M}_N=\overline{u}_N/\overline{a}_N$ namely for $\varepsilon\ll 1$

Perturbation analysis for $\varepsilon\ll 1$

$$\mathcal{U}\approx\frac{1}{2}\frac{\partial H}{\partial x},\qquad \mathcal{W}\approx-\frac{1}{2}\frac{\overline{\mathcal{D}}}{\overline{u}_N}\frac{\partial H}{\partial y},$$

where $H\equiv[-\dot{\alpha}_t^2(y,t-x/\overline{u}_N)+\overline{a}_N^2\alpha_y'^2(y,t-x/\overline{u}_N)].$

progressive wave: $\dot{\alpha}_t=\pm\overline{a}_N\alpha'_y \quad\Rightarrow\quad H=0,\quad \mathcal{U}=0,\quad \mathcal{V}=0$

no first order correction term coming from the Reynolds tensor

the shear wave $u^{(i)}=\dot{\alpha}_t(y,t-x/\overline{u}_N)$ $w^{(i)}=\overline{\mathcal{D}}\alpha'_y(y,t-x/\overline{u}_N)$ is an exact solution of the Euler equations for $p=0$

the first order correction terms should come from the boundary conditions at $x=\alpha(y,t)$ (Rankine-Hugoniot)

Limiting the attention to the nonlinear corrections of order $\boxed{\varepsilon \equiv |\alpha'_y|/\epsilon}$ the Rankine-Hugoniot conditions yield

mass $\quad \rho_N(u_N - \partial\alpha/\partial t - w_N \partial\alpha/\partial y) = \rho_u(\overline{\mathcal{D}} - \partial\alpha/\partial t)$

p.5 lecture IV

tangential momentum $\quad \delta w_N = (\overline{\mathcal{D}} - \overline{u}_N)\partial\alpha/\partial y$

longitudinal momentum $\quad \dfrac{p_N}{p_u} = \dfrac{2\gamma M_u^2 - (\gamma - 1)}{(\gamma+1)} \approx M_u^2$

where $\quad M_u = \dfrac{(\overline{\mathcal{D}} - \dot\alpha_t)}{a_u\,(1+\alpha_y'^2)^{1/2}}$

p.7 lecture X

$$\Rightarrow \quad \boxed{\begin{array}{l} x = \alpha(y,t): \quad p = p_N, \quad u = u_N, \quad w = w_N \\ p_N/\overline{p}_N \approx 1 - 2\dot\alpha_t/\overline{\mathcal{D}}, \quad u_N - \overline{u}_N \approx \dot\alpha_t + \overline{\mathcal{D}}\alpha_y'^2, \quad w_N \approx \overline{\mathcal{D}}\alpha'_y \end{array}}$$

the only nonlinear term that yields a correction of order $\varepsilon \equiv |\alpha'_y|/\epsilon$

$$|\dot\alpha_t| = O(\overline{a}_N \alpha'_y), \quad \overline{\mathcal{D}}/\overline{a}_N = O(1/\epsilon) \quad \Rightarrow \quad \overline{\mathcal{D}}|\alpha_y'^2/\dot\alpha_t| = O(|\alpha'_y|/\epsilon)$$

The shift of the front position also introduces quadratic terms

$$x = 0: \quad \delta u \equiv u_f(y,t) \approx \delta u_N - \alpha u'_x, \quad w \equiv w_f(y,t) \approx w_N - \alpha w'_x$$

The nonlinear equations for the wrinkles is obtained from the incompressible condition

$$-\overline{u}_N^{-1}\partial u_f/\partial t + \partial w_f/\partial y = 0$$

$H = 0 \quad \Rightarrow \quad$ the nonlinear terms coming from shift of the front position do not contribute

$$-\overline{u}_N \partial(\alpha u'_x)/\partial t + \partial(\alpha w'_x)/\partial y = 0$$

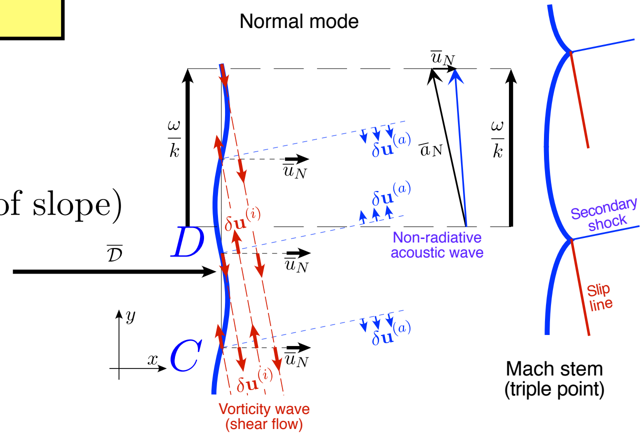
$$-\frac{1}{\overline{u}_N}\frac{\partial u_N}{\partial t} + \frac{\partial w_N}{\partial y} = 0 \quad \Rightarrow \quad \boxed{\frac{\partial^2\alpha}{\partial t^2} - \overline{a}_N^2 \frac{\partial^2\alpha}{\partial y^2} + \overline{\mathcal{D}}\frac{\partial}{\partial t}\left(\frac{\partial\alpha}{\partial y}\right)^2 = 0} \qquad (\overline{a}_N^2 \approx \overline{\mathcal{D}}\overline{u}_N)$$

nonlinear correction of order ε, $\quad \overline{\mathcal{D}}\alpha_y'^2/|\dot\alpha_t| \approx (\overline{\mathcal{D}}/\overline{a}_N)|\alpha'_y| \approx \varepsilon$

Mach stem formation

nonlinear correction of order ε, $\quad \overline{\mathcal{D}}\alpha_y'^2/|\dot\alpha_t| \approx (\overline{\mathcal{D}}/\overline{a}_N)|\alpha_y'| \approx \varepsilon$

$$\boxed{\frac{\partial^2\alpha}{\partial t^2} - \overline{a}_N^2\frac{\partial^2\alpha}{\partial y^2} + \overline{\mathcal{D}}\frac{\partial}{\partial t}\left(\frac{\partial\alpha}{\partial y}\right)^2 = 0}$$



Two timescales problem: wavelength of the wrinkles

short time $\tau_s \equiv L/\overline{a}_N$ (period of oscillation)

long time $\tau_l \equiv \tau_s/\varepsilon$ (for the formation of a singularity of slope)

Non-dimensional form $\quad \mathrm{t} \equiv t/\tau_s, \quad \mathrm{y} \equiv y/L, \quad \mathcal{A} \equiv \alpha/(\varepsilon\epsilon L)$

$$\frac{\partial^2\mathcal{A}}{\partial \mathrm{t}^2} - \frac{\partial^2\mathcal{A}}{\partial \mathrm{y}^2} + \varepsilon\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial\mathcal{A}}{\partial \mathrm{y}}\right)^2 = 0$$

small nonlinear correction term producing a singularity at finite (but long) non-dimensional time $1/\varepsilon$, $\mathrm{t} = O(1/\varepsilon)$

that is at the long timescale $t = O(\tau_l)$

so that $\mathcal{A}$ may be considered to depend on two reduced time variables t and $\mathrm{t}' \equiv \varepsilon\mathrm{t}$,
$$\begin{cases} \dfrac{\partial}{\partial \mathrm{t}} \to \dfrac{\partial}{\partial \mathrm{t}} + \varepsilon\dfrac{\partial}{\partial \mathrm{t}'} \\ \dfrac{\partial^2}{\partial \mathrm{t}^2} \to \dfrac{\partial^2}{\partial \mathrm{t}^2} + 2\varepsilon\dfrac{\partial}{\partial \mathrm{t}}\dfrac{\partial}{\partial \mathrm{t}'} + \varepsilon^2\dfrac{\partial^2}{\partial \mathrm{t}'^2} \end{cases}$$

Considering a simple progressive wave $\mathrm{y}' = \mathrm{y} \pm \mathrm{t}$

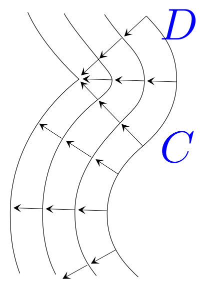
and looking for a solution in the form $\mathcal{A}(\mathrm{y},\mathrm{t}) = A(\mathrm{y}',\mathrm{t}')$ one gets
$$2\varepsilon\frac{\partial^2 A}{\partial \mathrm{t}\partial \mathrm{t}'} + \varepsilon^2\cancel{\frac{\partial^2 A}{\partial \mathrm{t}'^2}} + \varepsilon\frac{\partial}{\partial \mathrm{t}}\left(\frac{\partial A}{\partial \mathrm{y}'}\right)^2 + \varepsilon^2\cancel{\frac{\partial^2}{\partial \mathrm{t}'}\left(\frac{\partial A}{\partial \mathrm{y}'}\right)^2} = 0$$

leading order $\quad 2\dfrac{\partial A}{\partial \mathrm{t}'} + \left(\dfrac{\partial A}{\partial \mathrm{y}'}\right)^2 \approx 0$

Burgers equation for $\quad B(y',t') \equiv \partial A/\partial y'$ $\quad \boxed{\partial B/\partial \mathrm{t}' + B\partial B/\partial \mathrm{y}' = 0}$

known to produce a singularity after a finite time see pp 3-4 lecture X

Geometrical construction for the slip line and the secondary shock



Collapse of points C and D by a Huygens like construction

$\dot\alpha_t \approx \overline{\mathcal{D}}\alpha_y'^2 \qquad -\dot\alpha_t/(1+\alpha_y'^2)^{1/2} = \overline{\mathcal{D}}$

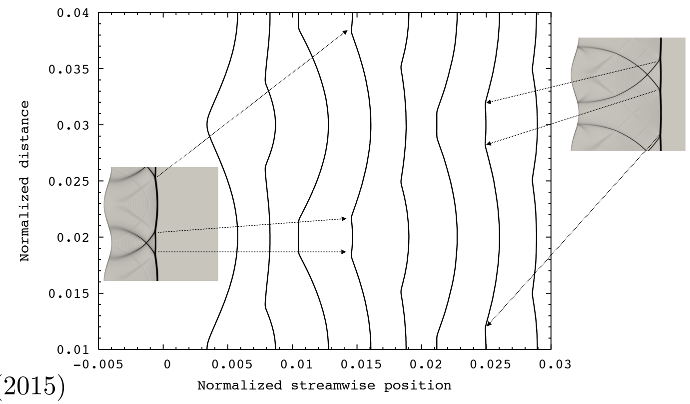
$\dot\alpha_t \approx \overline{\mathcal{D}}\alpha_y'^2/2$

different only by a numerical factor 1/2

Numerical solution of the model equation and comparison with experiments

$$\frac{\partial^2 \alpha}{\partial t^2} - \overline{a}_N^2 \frac{\partial^2 \alpha}{\partial y^2} + \overline{\mathcal{D}} \frac{\partial}{\partial t} \left(\frac{\partial \alpha}{\partial y} \right)^2 = 0$$

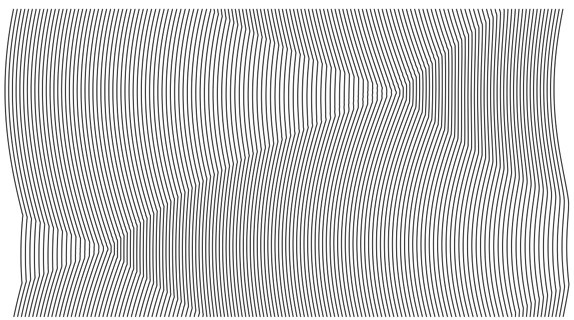
Initial condition: sinusoidal shock front of small amplitude



Lodato Vervisch (2015)

The formation of corners is clearly observed

Good agreement with experiments and DNS



Denet (2014)

The trajectory of the corners looks quite similar
to the traces left on the wall by a cellular detonation

XIV-3) Shock-vortex interaction

Formulation

Clavin (2013)

strong shock + weak vortex

$$\epsilon^2 \equiv M_N^2 \ll 1 \qquad \overline{M}_u = O(1/\epsilon), \qquad (\gamma - 1) = O(\epsilon^2)$$

Consider a cylindrical and very subsonic vortex of diameter L and turnover velocity v_e, $\boxed{v_e/\overline{a}_u \ll \epsilon}$ $(v_e \ll \overline{a}_u/\overline{M}_u)$

Interaction time $\tau_{int} = L/\overline{\mathcal{D}} \ll$ turnover time $L/v_e \Rightarrow$ frozen flow $u_e(\mathbf{r})\ w_e(\mathbf{r})$ + small disturbances of the front

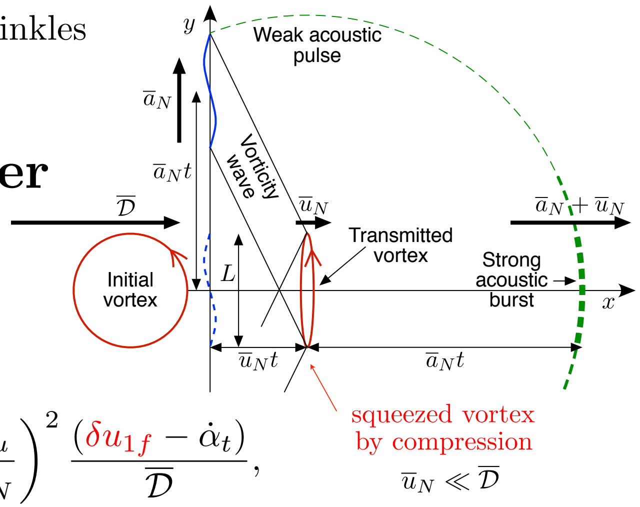
The disturbances of the front during the crossover can be described by a linear analysis

Interaction time $\tau_{int} = L/\overline{\mathcal{D}} \ll$ propagation time in the transverse direction of the wrinkles $L/\overline{a}_N$

After the interaction time, $t > \tau_{int}$, the wrinkled shock front propagates in a quiescent medium

2 timescales: short crossover and longer transverse propagation of the wrinkles

the crossover provides the initial conditions



Linear analysis of the crossover

Similar analysis but with an upstream flow

$$\delta u_1(\mathbf{r},t), \quad \delta w_1(\mathbf{r},t), \quad \delta p_1(\mathbf{r},t)$$

Rankine-Hugoniot (generalization of the relations p. 6)

(the subscript f denotes the value at the shock front of the upstream flow)

$$\frac{\delta p_N}{\overline{p}_N} - \frac{\delta p_{1f}}{\overline{p}_u} \approx 2\frac{(\delta u_{1f} - \dot{\alpha}_t)}{\overline{\mathcal{D}}}, \qquad \frac{\delta \rho_N}{\overline{\rho}_N} - \frac{\delta \rho_{1f}}{\overline{\rho}_u} = 2\left(\frac{\overline{a}_u}{\overline{a}_N}\right)^2 \frac{(\delta u_{1f} - \dot{\alpha}_t)}{\overline{\mathcal{D}}},$$

$$(\delta u_N - \dot{\alpha}_t) = \frac{[(\gamma-1)\overline{M}_u^2 - 2]}{2\overline{M}_u^2}(\delta u_{1f} - \dot{\alpha}_t), \quad \delta w_N \approx \overline{\mathcal{D}}\alpha'_y + \delta w_{1f},$$

$$\delta u_{1f}(y,t) = u_e|_{x=-\overline{\mathcal{D}}t}, \quad \delta w_{1f}(y,t) = w_e|_{x=-\overline{\mathcal{D}}t}, \quad \delta p_{1f}(y,t) = p_e|_{x=-\overline{\mathcal{D}}t}$$

vortex

Acoustic burst

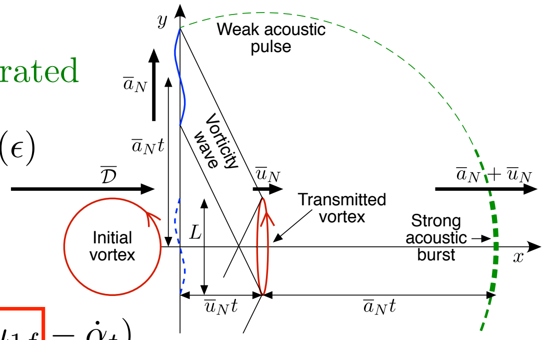
Acoustic in the shocked gases (Doppler neglected for simplicity) $0 < t < \tau_{int}$

$$\frac{\partial u^{(a)}}{\partial t} \approx -\frac{1}{\overline{\rho}_N}\frac{\partial p}{\partial x}, \qquad \frac{\partial w^{(a)}}{\partial t} \approx -\frac{1}{\overline{\rho}_N}\frac{\partial p}{\partial y}, \qquad \frac{\partial^2 p}{\partial t^2} \approx \overline{a}_N^2\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right)$$

$$x=0, \quad 0<t<\tau_{int}: \qquad \partial/\partial t = O(\overline{\mathcal{D}}/L), \qquad \partial/\partial y = O(1/L)$$

A quasi-planar and longitudinal pressure burst of transverse extension L is generated

$$\partial p/\partial x \approx (1/\overline{a}_N)\partial p/\partial t \approx (\overline{\mathcal{D}}/\overline{a}_N)\delta p/L \approx \epsilon^{-1}(\partial p/\partial y) \Rightarrow |\delta w^{(a)}|/|\delta u^{(a)}| = O(\epsilon)$$



Rankine-Hugoniot $\quad \delta p_{1f}/\overline{p}_u$ is negligible $\quad \dfrac{\delta p_{1f}/\overline{p}_u}{v_e/\overline{\mathcal{D}}} \approx \dfrac{(v_e/\overline{a}_u)^2}{v_e/\overline{\mathcal{D}}} \approx \dfrac{(v_e/\overline{a}_u)}{\epsilon} \ll 1$

$$v_e/\overline{a}_u \ll \epsilon: \qquad \delta p_N/\overline{p}_N \approx 2(\delta u_{1f} - \dot{\alpha}_t)/\overline{\mathcal{D}}$$

$\delta u^{(a)} \approx \delta p/(\overline{\rho}_N \overline{a}_N)$

$$v_e/\overline{a}_u \ll \epsilon, \ 0<t<\tau_{int}: \quad \delta u^{(a)}|_{x=0} \equiv \delta u_N^{(a)} \approx 2\boxed{(\overline{a}_N/\overline{\mathcal{D}})(\delta u_{1f}} - \dot{\alpha}_t)$$

Vorticity wave (transmitted vortex)

$\delta u_N \approx \dot{\alpha}_t$
$$\delta u^{(i)}|_{x=0} \equiv \delta u_N^{(i)} = \delta u_N - \delta u_N^{(a)} \approx \dot{\alpha}_t - 2\boxed{(\overline{a}_N/\overline{\mathcal{D}})\delta u_{1f}}$$

$\delta w_N \approx \overline{\mathcal{D}}\alpha'_y + \delta w_{1f}$
$$\delta w^{(i)}|_{x=0} \equiv \delta w_N^{(i)} = \delta w_N - \delta w_N^{(a)} \approx \overline{\mathcal{D}}\alpha'_y + \delta w_{1f} - \delta w_N^{(a)}$$

Vorticity wave $\quad \delta u^{(i)} = \delta u_N^{(i)}(y, t - x/\overline{u}_N), \quad \delta w^{(i)} = \delta w_N^{(i)}(y, t - x/\overline{u}_N)$

$$\frac{\partial \delta u^{(i)}}{\partial x} = -\frac{1}{\overline{u}_N}\frac{\partial \delta u^{(i)}}{\partial t} = O\left(\frac{\overline{\mathcal{D}}}{\overline{u}_N}\frac{\delta u^{(i)}}{L}\right) = \boxed{O\left(\frac{\delta u^{(i)}}{\epsilon^2 L}\right)}$$

Incompressibility $\quad \partial\delta u^{(i)}/\delta x + \partial \delta w^{(i)}/\delta y = 0 \qquad |\delta u^{(i)}|/|\delta w^{(i)}| = O(\epsilon^2)$ The vorticity wave is quasi-parallel to the front

To leading order
$$\boxed{(\overline{a}_N/\overline{\mathcal{D}})\delta u_{1f}} = O(\epsilon v_e) \Rightarrow (\overline{a}_N/\overline{\mathcal{D}})\boxed{\partial \delta u_{1f}/\partial x} \approx v_e/(\epsilon L) \gg \boxed{\partial \delta w_{1f}/\partial y} = O(v_e/L)$$

$$\boxed{v_e/\overline{a}_u \ll \epsilon, \ 0<t<\tau_{int}: \quad \dot{\alpha}_t \approx 2(\overline{a}_N/\overline{\mathcal{D}})\delta u_{1f}, \quad \alpha(y,t) = 2\frac{\overline{a}_N}{\overline{\mathcal{D}}}\int_{-\overline{\mathcal{D}}t}^{0} \mathrm{d}x \frac{u_e(x,y)}{\overline{\mathcal{D}}}}$$

longitudinal component of the vortex velocity

Wrinkles of very small amplitude are left on the shock front by the vortex

$$|\alpha| = O(\epsilon^2 L v_e/\overline{a}_u), \qquad |\alpha'_y| = O(\epsilon^2 v_e/\overline{a}_u)$$

XIV-4) Shock-turbulence interaction

Strong shock propagating in a weakly turbulent flow

Composite solution for a single vortex

During crossover

$$v_e/\overline{a}_u \ll \epsilon,\ 0 < t < \tau_{int}: \quad \dot{\alpha}_t \approx 2(\overline{a}_N/\overline{\mathcal{D}})\delta u_{1f}, \quad \alpha(y,t) = 2\frac{\overline{a}_N}{\overline{\mathcal{D}}}\int_{-\overline{\mathcal{D}}t}^{0} \mathrm{d}x \frac{u_e(x,y)}{\overline{\mathcal{D}}}$$

beginning of interaction $< t <$ end of interaction : $\dfrac{\partial^2 \alpha}{\partial t^2} \approx 2\dfrac{\overline{a}_N}{\overline{\mathcal{D}}}\dfrac{\partial \delta u_{1f}}{\partial t}$

valid during a short lapse of time of order $L/\overline{\mathcal{D}}$

After crossover

$$\frac{\partial^2 \alpha}{\partial t^2} - \overline{a}_N^2 \frac{\partial^2 \alpha}{\partial y^2} + \overline{\mathcal{D}}\frac{\partial}{\partial t}\left(\frac{\partial \alpha}{\partial y}\right)^2 \approx 0$$

involves a time scale of evolution $L/\overline{a}_N$ longer than $L/\overline{\mathcal{D}}$

$\epsilon = \overline{a}_N/\overline{\mathcal{D}} \ll 1$

Composite equation

frozen velocity field $u_e(x,y)$

$$\frac{\partial^2 \alpha}{\partial t^2} - \overline{a}_N^2 \frac{\partial^2 \alpha}{\partial y^2} + \overline{\mathcal{D}}\frac{\partial}{\partial t}\left(\frac{\partial \alpha}{\partial y}\right)^2 = 2\frac{\overline{a}_N}{\overline{\mathcal{D}}}\frac{\partial \delta u_{1f}}{\partial t}$$

$$\delta u_{1f}(y,t) = u_e|_{x=-\overline{\mathcal{D}}t}$$

short living forcing term

taking advantage of the two different time scales

Model equation

Extension to 3 dimensions

$$\times\left(\frac{L\overline{\mathcal{D}}}{\overline{a}_N^3}\right) \qquad \frac{\partial^2\alpha}{\partial t^2} - \overline{a}_N^2\Delta\alpha + \overline{\mathcal{D}}\frac{\partial|\nabla\alpha|^2}{\partial t} = 2\frac{\overline{a}_N}{\overline{\mathcal{D}}}\frac{\partial\delta u_{1f}}{\partial t}$$

where $\delta u_{1f}(y,z,t) = u_e(x,y,z,t)|_{x=-\overline{\mathcal{D}}t}$

forcing term varying on the on the length scale L and on time scale $L/\overline{\mathcal{D}}$

non-dimensional form

$$\eta \equiv y/L, \qquad \zeta \equiv z/L, \qquad \tau \equiv \overline{a}_N t/L, \qquad \phi \equiv \alpha/(\epsilon L) \qquad \epsilon \equiv \overline{a}_N/\overline{\mathcal{D}}$$

$$\boxed{\frac{\partial^2\phi}{\partial\tau^2} - \Delta\phi + \frac{\partial|\nabla\phi|^2}{\partial\tau} = \frac{\partial\psi(\eta,\zeta,\tau/\epsilon)}{\partial\tau}} \quad \text{where} \quad \psi \equiv 2\left(\frac{\delta u_{1f}}{\overline{a}_N}\right) \qquad |\psi| = O(v_e/\overline{a}_N)$$

ψ is a small term varying on the short (reduced) time scale ϵ and on the (reduced) length scale unity

$\partial\psi/\partial\tau$ is a small (reduced) forcing term fluctuating rapidly $\quad |\partial\psi/\partial\tau| = O(v_e/(\epsilon\overline{a}_u)) \quad (v_e/\overline{a}_u \ll \epsilon)$

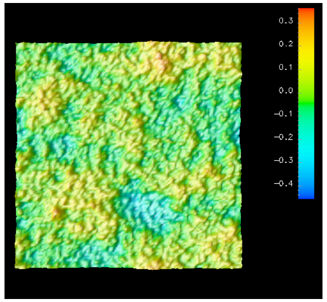
Numerical results

Denet (2015)

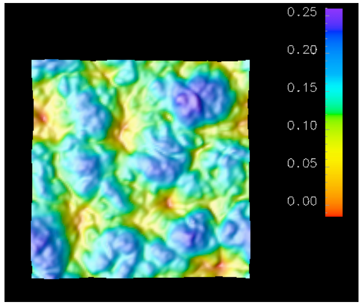
The characteristic cell size of the patterns at the shock front is much larger than the integral scale of the turbulence

The size of the patterns looks to grow with time
Saturation by the box size ?

length scale of the turbulence at the shock front



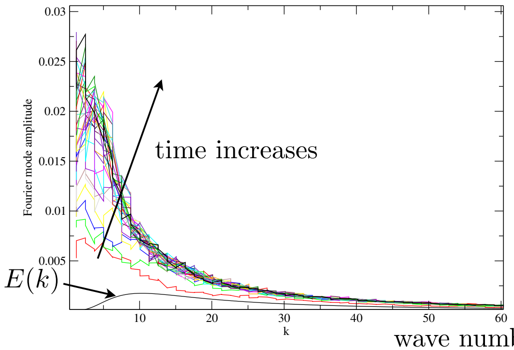
wrinkles of the shock front at time $\tau = 4$ after starting the interaction



Spectral analysis of the pattern size

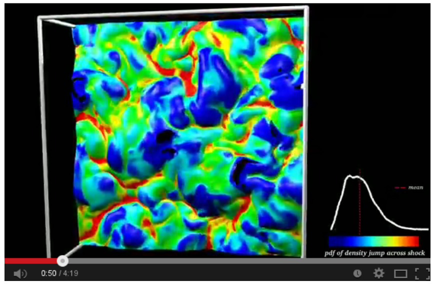
Evolution of the spectra of the wrinkles of the front shock

The size of the pattern increases with time

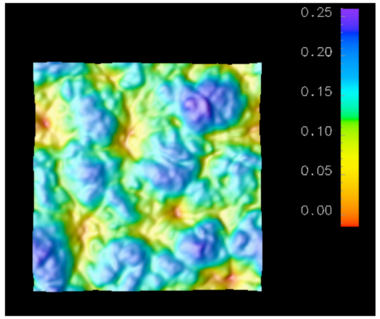


Denet (2015)

Comparison with DNS

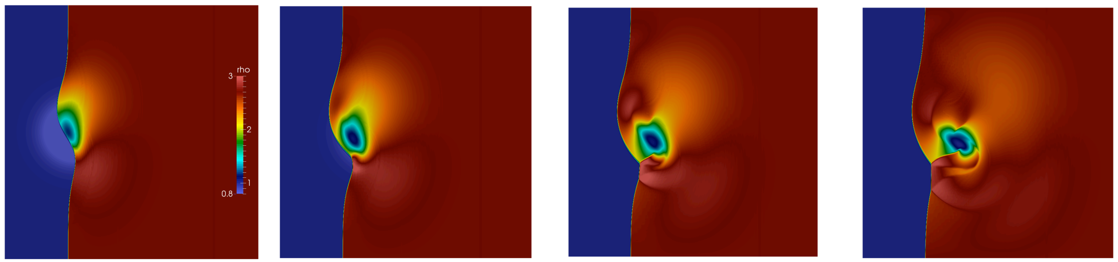


DNS *Larson et al. (2013)*



Model equation *Denet (2015)*

DNS shock-vortex interaction Vervisch Lodato (2015)



$\overline{\mathcal{D}}/a_u = 2, \qquad v_e/a_u = 0.8, \qquad \gamma = 1.4$

Two Mach stems are observed as in the model equation