Investigation of the porous network as a determinant of the overall stiffness of cortical bone: Mori-Tanaka model vs. ultrasound propagation

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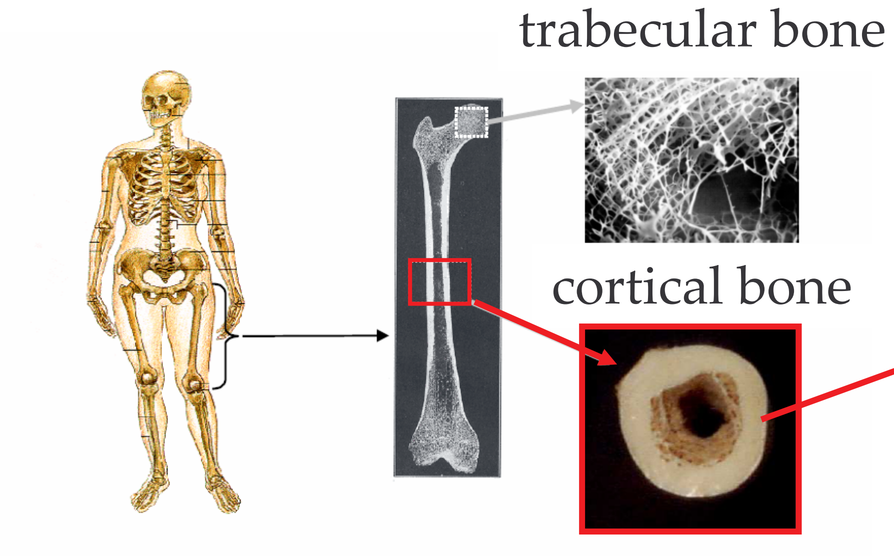
A biological porous medium : cortical bone

Two types of bone



Cortical bone = compact bone

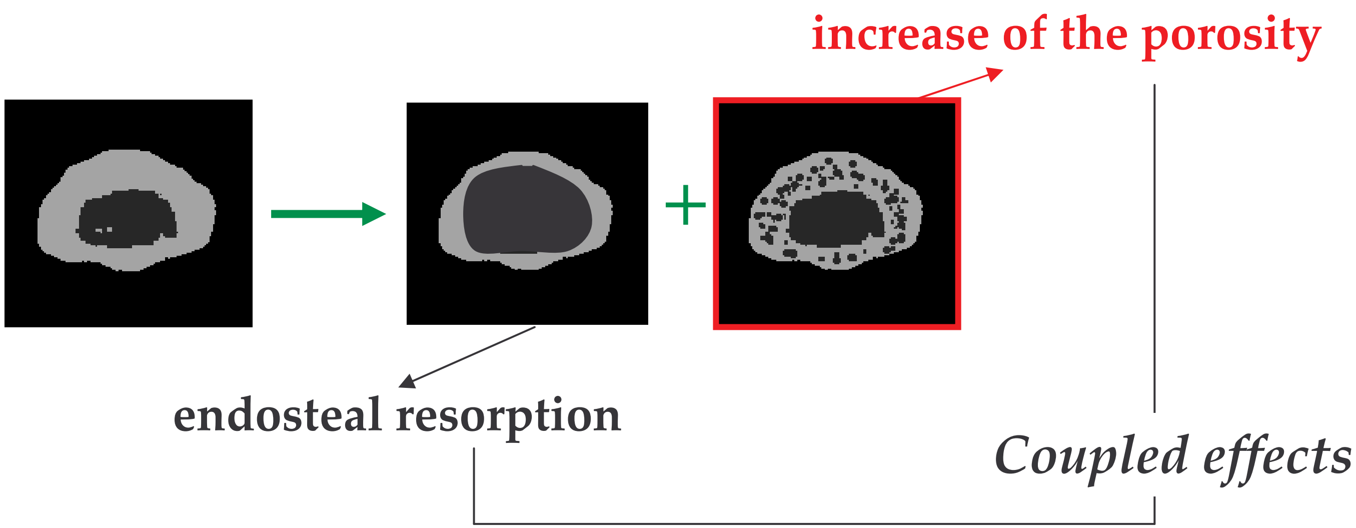
Porosity <15%

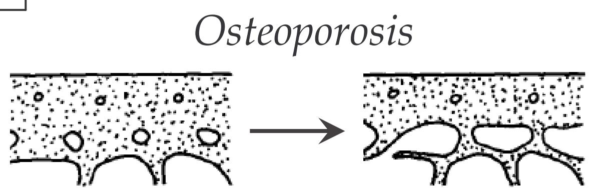
80% of the bone mass

Osteoporosis effects on cortical bone

Osteoporosis *: disease in which the density and quality of bone is reduced, increasing the risk of fracture* (IOF 2008)

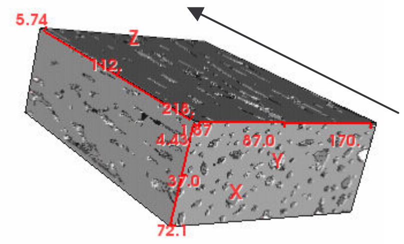
<u>Two main effects on cortical bone</u> :





Background

Bone axis (axis 3)



Porous network - Synchrotron
(Bossy, 2004)

Porous network in cortical bone :

microscopic porosity (lacunae)

+

mesoscopic porosity (50-200 μm)

(Haversian canals and resorption cavities)

Porosity effect:

- ↘ overall stiffness ;
- preferential orientation of pores ⇨ anisotropy of bone.

One of the determinants of the elasticity of cortical bone

(Dong and Guo 2004, Augat et al. 2006, Baron et al. 2007)

Ultrasound propagation

Previous results

2-step modelling

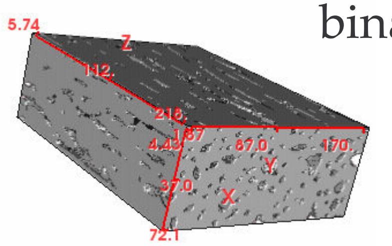
Porous medium & **Propagation of plane bulk waves**

Cortical bone = biphasic medium

water (perfect fluid) : ρ_w; c_w

+ bone matrix (isotropic) : ρ_0; c_{ij}^0

Synchrotron 3D reconstruction (20 µm) + binarization



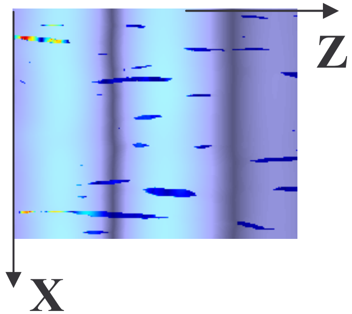
FDTD simulation ;

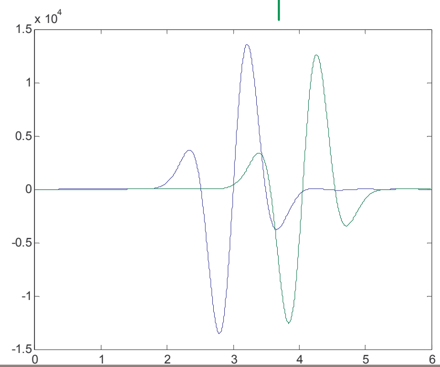
Transverse and longitudinal plane bulk waves along the 3 principal directions

(X : radial, Y : circonferential, Z : axial)

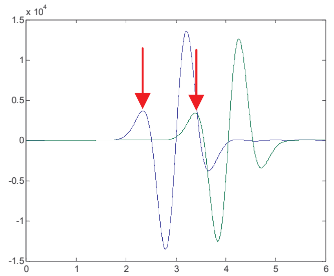
f = 1 MHz

$\lambda \approx 4$ mm





Previous results



Propagation velocity measurement

First maximum detection

$$\rho = \rho_0 \left(1 - p\right) + p\,\rho_w$$

Mixture law for the mass density

Effective diagonal stiffness coefficients $\boldsymbol{c_{ii}^{eff}}$

19 reconstructed samples = 19 porosity values of porosity p

$1\% < p < 15\%$

Evolution of the elasticity (c_{ii}^{eff} and anisotropy) of the effective medium vs. cortical porosity

Baron et al. *IEEE UFFC Proceedings 2006* ; **B**aron et al (*JASA 2007)*

Mori Tanaka model

Mori-Tanaka (MT) model

The Mori-Tanaka (Mori and Tanaka, 1973) estimation of the stiffness tensor is given by (Bornert, 2001)

porosity

Effective stiffness tensor

$$\mathbb{C}^{est} = \mathbb{C}_m + f_p(\mathbb{C}_p - \mathbb{C}_m) : \mathbb{L}_{MT}$$

Matrix stiffness **Pores stiffness**

$$\mathbb{L}_{MT} = \left\{\mathbb{I} + (1 - f_p)\mathbb{S}_p^m : [\mathbb{C}_m^{-1} : \mathbb{C}_p - \mathbb{I}]\right\}^{-1}$$

Eshelby tensor: interaction of the pores with the matrix.
Here, calculated for cylindrical pores and with a numerical method (Ghahremani, 1977)

Comparison
Mori Tanaka / US propagation simulation

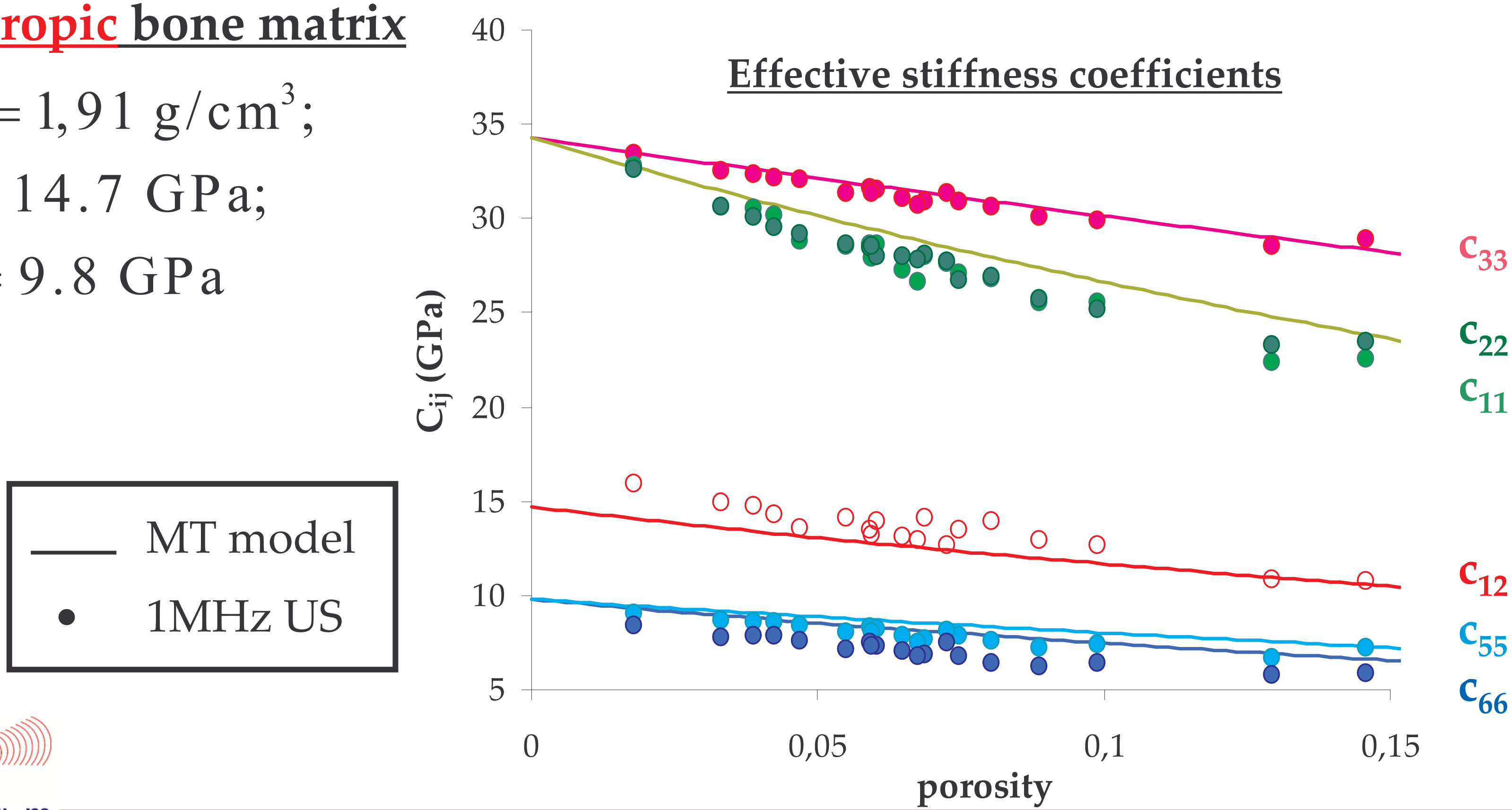
Water in pores

Isotropic bone matrix

$\rho_0 = 1{,}91\ \mathrm{g/cm^3};$

$\lambda = 14.7\ \mathrm{GPa};$

$\mu = 9.8\ \mathrm{GPa}$



Comparison
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Water in pores

Transversely isotropic bone matrix

$\rho_0 = 1.91 \text{ g/cm}^3;$

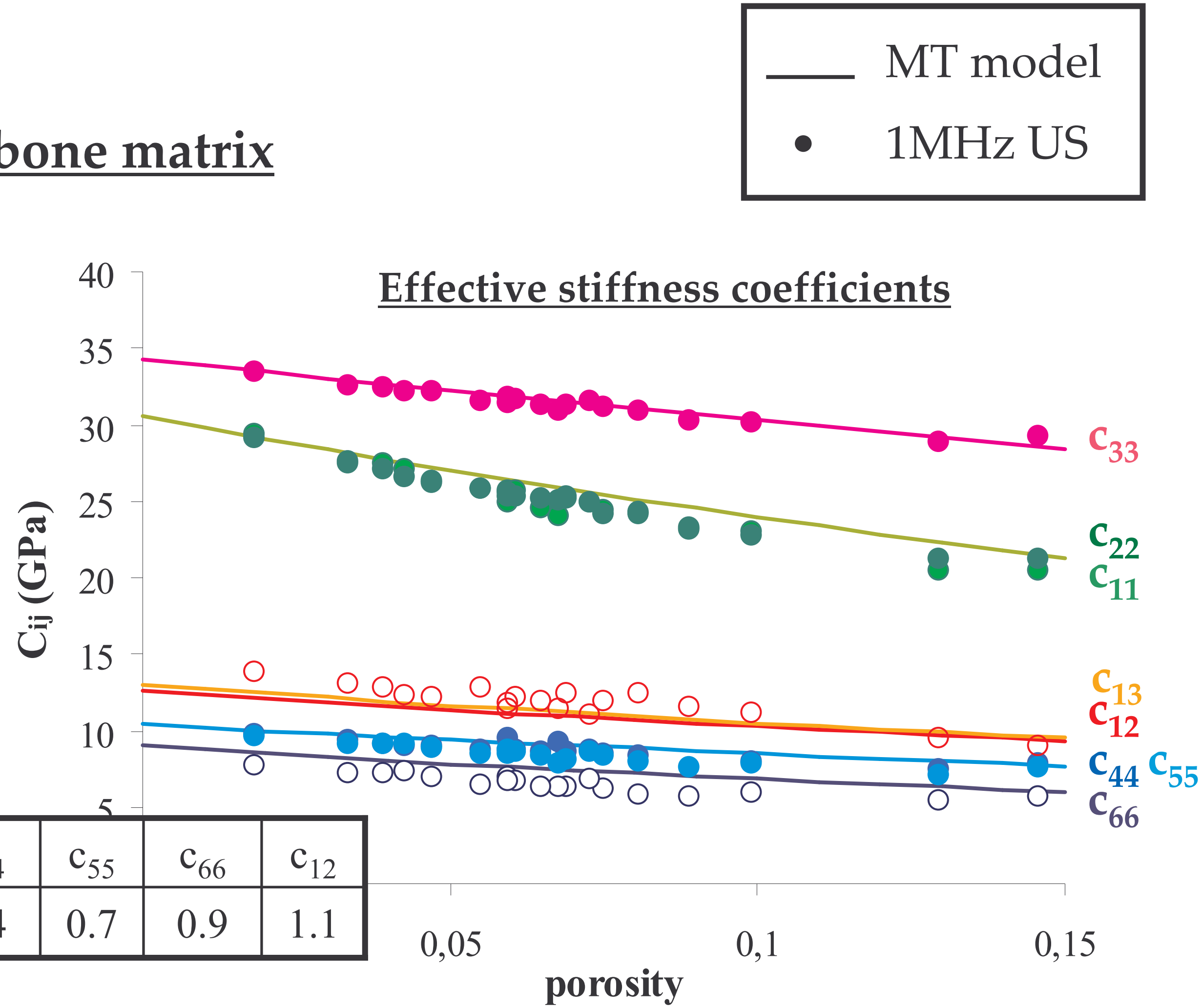
$c_{11}^0 = c_{22}^0 = 30.6 \text{ GPa};$

$c_{33}^0 = 34.3 \text{ GPa};$

$c_{13}^0 = 13 \text{ GPa};$

$c_{44}^0 = c_{55}^0 = 10.4 \text{ GPa};$

$c_{66}^0 = 9 \text{ GPa}.$



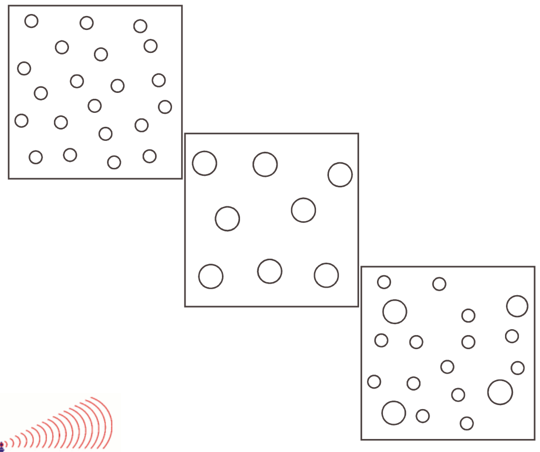
(GPa)	c_{11}	c_{22}	c_{33}	c_{44}	c_{55}	c_{66}	c_{12}
RMSE	1.0	0.9	0.3	0.4	0.7	0.9	1.1

Discussion

Interpretation of Mori-Tanaka (MT) model:

Idealized medium: random arrangement of elongated pores in a matrix

- Pores (more or less) aligned ;
- pore diameter << pore length ;
- solution calculated for a given volume fraction of water component (porosity), regardless of the pores shapes and distribution (as long as distribution is isotropic and random) ;
- « mean » pore shape = circular (random shape of pore cross-section) ;
- equivalence of mono- and poly-disperse configurations



- Small pores close to each others
- Large pores far from each others
- Polydisperse

Same volume fraction

Discussion : good comparison

Stiffness coefficients derived from 1MHz-ultrasound propagation well agree with those obtained with the Mori Tanaka model

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RMSE	1.0	0.9	0.3	0.4	0.7	0.9	1.1

➧ Quasi-static effective properties can be estimated with 1 MHz-ultrasound propagation to some extent.

➧ MT model assumptions relevant to interpret the relationship between porosity and effective properties determined with 1MHz-ultrasound propagation.

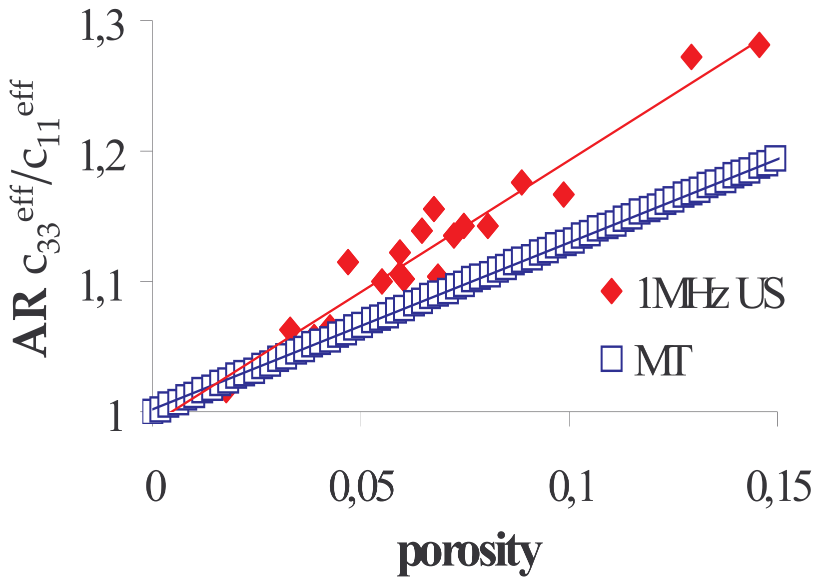
- influence of the specific size and shape distribution of pores on effective properties;
- results suggest that the specific pore distribution is not a major factor in the case of cortical bone ;
- nevertheless, the *detailed* effect of pore distribution on effective properties remains unknown

Discussion discrepancies

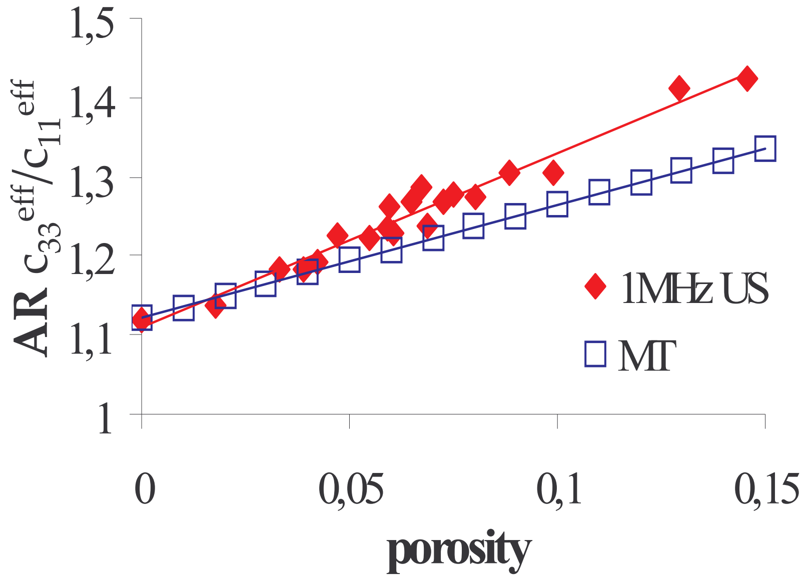
Anisotropy ratio (AR = $c_{33}^{eff}/c_{11}^{eff}$) vs. porosity

Different predictions of the anisotropy ratio

Isotropic bone matrix



Transversely isotropic bone matrix

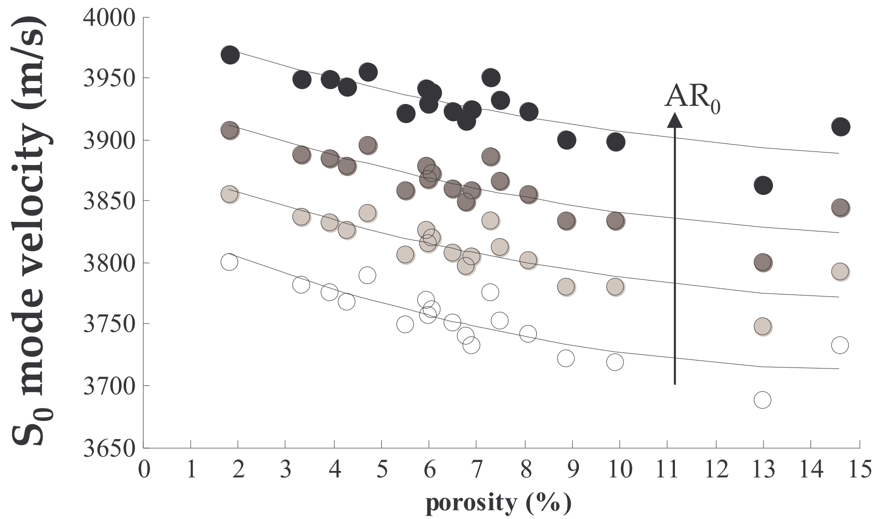


Perspectives

Other homegenization models have to be tested

- 1MHz-ultrasound propagation simulations on academic pores (Bossy et al. JASA, 2004)
- homogenization of periodic media (Crolet, 1993; Parnell and Grimal, 2008).

Prediction of the porosity effect on the velocities experimentally measured by axial transmission for bone evaluation



(Baron et al, IEEE 2006)

Thank you for your attention

Mori-Tanaka (MT) model

Characteristics:

• Quasi-static method of continuum mechanics;

• analytic or semi-analytic expressions to estimate the **effective stiffness tensor** of heterogeneous media = composite media = multiphase media.

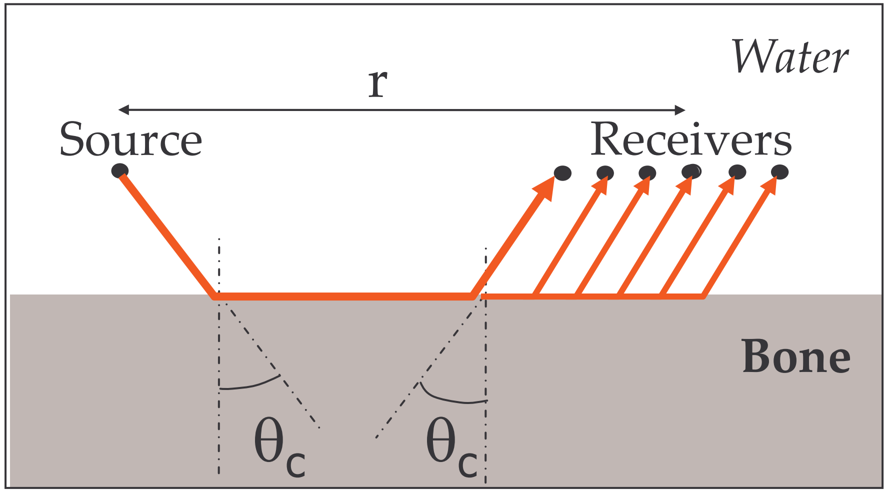
Model – hypotheses:

• Homogeneous strains in each phase of the composite medium;

• Strains in an ellipsoidal inclusion embedded in a fictitious infinite medium subjected to a homogeneous strain at infinity.

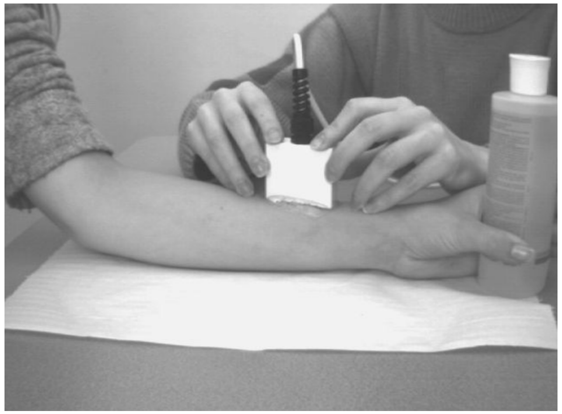
•Stiffness tensor of the fictitious medium equal to that of the matrix of the composite medium.

Axial transmission technique



- Skeletal site : multi-site
- Type of bone : cortical bone
- Frequency : 250kHz-2MHz
- Acoustic parameter : SOS
- Typical range of values : 3000-4000 m/s

Unilateral contact



Patent (WO/2003/099132)

Relevancy of the models

MT solution

Quasi-static method of continuum mechanics

↓

effective properties assessment by ultrasound propagation

Idealized pore shape randomly distributed in an anisotropic bone matrix

=

relevant model of physiologic reality

Comparison
Mori Tanaka / US propagation simulation

Water in pores

$\rho_w = 1\ \mathrm{g/cm^3};\quad c_w = 1.5\ \mathrm{mm/\mu s}\quad (\lambda = 2.25\ \mathrm{GPa};\ \mu = 0\ \mathrm{GPa})$

Transversely isotropic bone matrix

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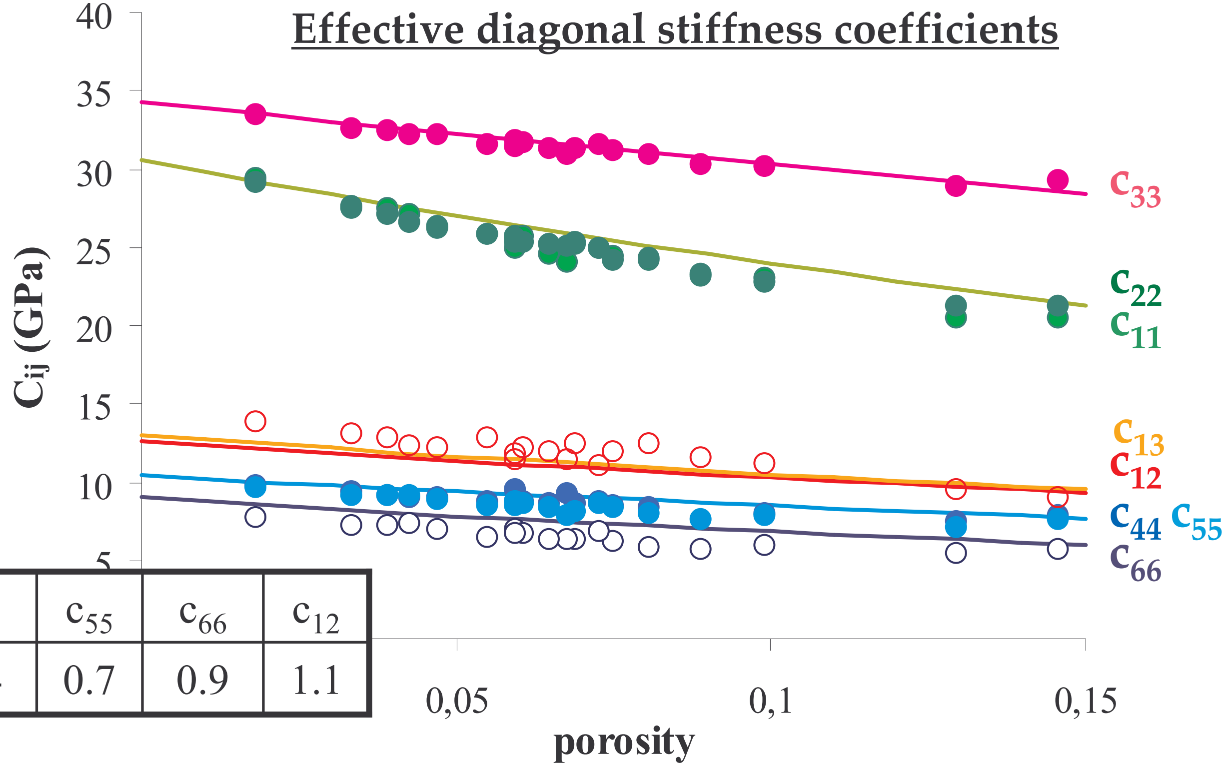

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MT model

<u>MT ≠ dilute model :</u> dilute approximation model = Volumic fraction of inclusion is less than a few %; it assumes that the inclusions do not « see » each others (strain field in inclusions as is they were alone in the infinite medium).

• Account for some interaction between the inclusions. In other words, inclusions do not « see » the macroscopic strain field but a local uniform field (same for all inclusions) which is the mean deformation of the matrix.

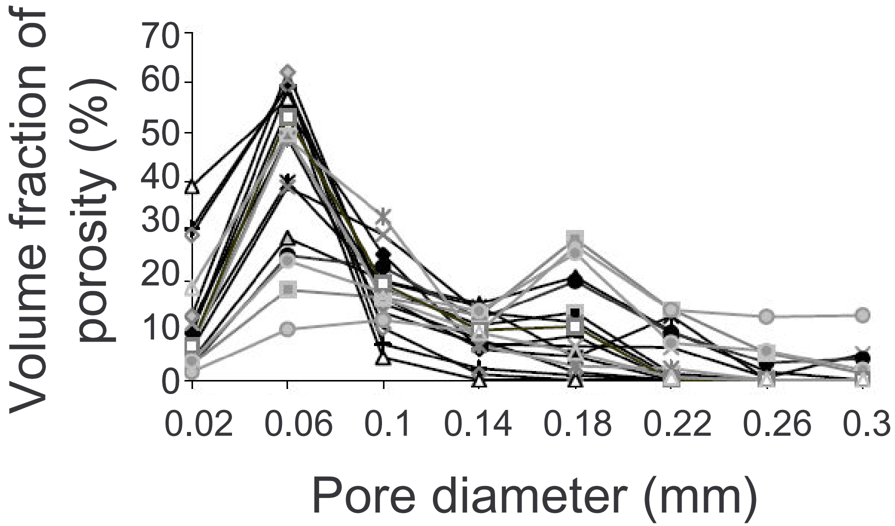
• inclusion strain = uniform contribution due to the neighbour inclusions and the matrix deformation;
• homogeneous strain field (and the same) in all the inclusions;
• the strain field in the matrix is heterogeneous.

MT model : for small concentrations, but not infinitesimal, up to 10-20% depending on phase contrast and required precision (Bornet, 2001).

Question

Which parameters are determinant of the evolution of cortical bone effective elasticity versus porosity?

- Poisson's ratio of the bone matrix (Baron et al. 2008);
- What about the porous network characteristics?



- Spatial pore distribution?
- Size pore distribution?
- Pore shape?

Micromechanical model

Mori-Tanaka (MT) model

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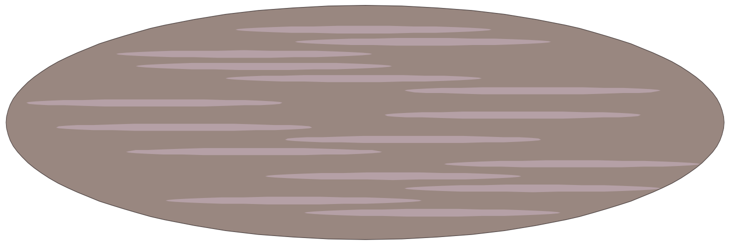

Single pore orientation + Eshelby problem for cylindrical inclusions