ESUCB 2011

Characterization of growing bone : curvature and anisotropy.

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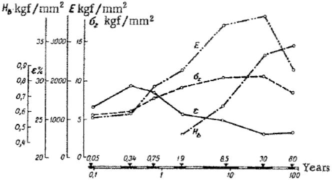
Outline

1. Characteristics of growing bone
   - Geometry
   - Anisotropy
   - FGM
2. Bone model
   - Anisotropy & tube
   - US characterization
3. Results
   - Plate or tube
   - Influence of soft tissue
4. Conclusion

Mechanical behaviour : what about literature ?

- Few studies about the properties of human growing bone :

  - Mechanical properties
    Currey and Butler (JB&JS, 1975) : stiffness and toughness of children bone 30% lower than adult bone.
    Vinz (Mechanics Of Composite Materials, 1975)



  - Microstructural properties
    Kerley (American Journal of Physical Anthropology, 1965)

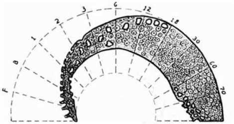


Fig. 2a Cross-sectional age changes in the femoral cortex.

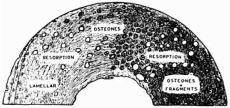
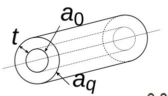


Fig 2b The life cycle of bone.

  - Berteau et al. *Elastic values of fibula children bone autotransplants* ESUCB 2011
    (Mon 20 June 2011, 10 :45-11 :00)

Specifity of growing bone : geometry

Long bone ≈ tubular geometry



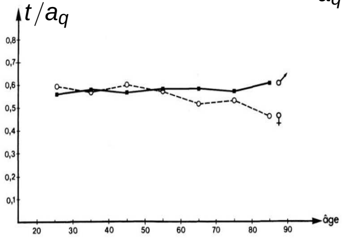
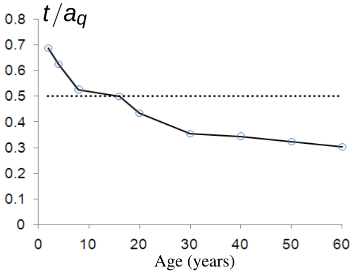


FIG. 6. — Evolution de l'indice cortico-diaphysaire moyen de l'humérus et du fémur, en fonction de l'âge, par classes décennales.



Bergot & Bocquet (1976)

Carter & Beaupré (2001)

Hypothesis

- Lack of data in literature on healthy children bone

  $\Rightarrow$ hypothesis on anisotropy.

  growing bone = less organized bone : orthotropy vs transverse isotropy.

  Ex. plexiform bone = immature bone in quickly growing animals $\Rightarrow$ orthothropic (Katz, 1984)

Young Elder

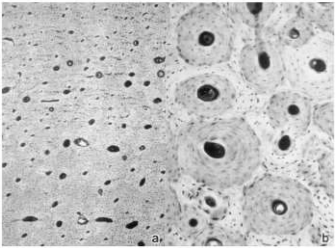
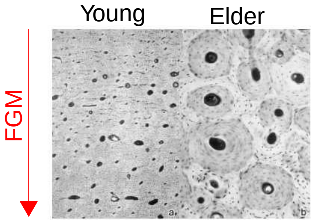


Fig. 4. A light microscopic image of Villanueva's bone stain from a young aged sample (a) and from an elder aged sample (b). Various osteons appear in the elder, but not in the young.

Watanabe et al. (1998)

Hypothesis

FGM=Functionally Graded Material = determinant of aging and pathology ?



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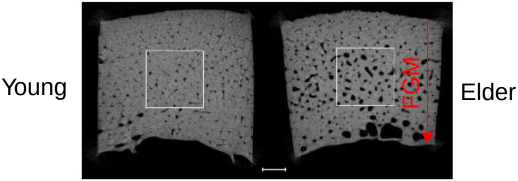


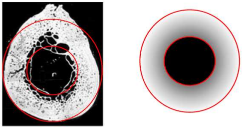
Figure 2. Two-dimensional cross-sectional micro-computed tomography images from samples A (left) and B (right) acquired at 10 μm spatial resolution. The region of interest represents the region of interest (2.5 mm × 2.5 mm) targeted for skeletonization and quantitative analysis. Scale bar = 1 mm.

Cooper et al. (2003)

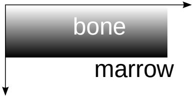
How to model bone ?

Clinical application → *in-vivo* model

- Geometry
  - plate
  - cylinder
  - tube
  - realistic section
- Boundary conditions
  - vacuum(free-free)
  - fluid (soft tissue and marrow) : perfect fluids or viscous fluids
- Properties of bone
  - elasticity/visco-elasticity
  - linear/non-linear elasticity
  - isotropy/anisotropy
  - homogeneity/heterogeneity



Baron Ultrasonics 2011.



Baron et al. CRM (2008) ;
Baron et al. JASA (2010).

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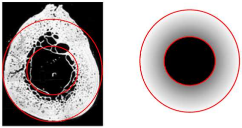

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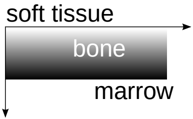

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Characterization of an anisotropic tube

Anisotropy
(> transverse isotropy)
+
cylindrical geometry
+
FGM

→ Classical methods unefficient (no analytical solution)

→ Stroh's formalism (Peano series expansion of the matricant)

Wave equation in Fourier domain : $\frac{d}{dr}\eta(r) = \mathbf{Q}(r)\eta(r).$

with $\eta(r) = (u_r, u_\theta, u_z, \imath r\sigma_{rr}, \imath r\sigma_{r\theta}, \imath r\sigma_{rz})^T$ and $\mathbf{Q} = \mathbf{Q}(\mathbf{C}, k, \omega)$.

Analytic solution : Peano expansion of the matricant

$$\mathbf{M}(r, r_0) = \mathbf{I} + \int_{r_0}^{r} \mathbf{Q}(\xi)d\xi + \int_{r_0}^{r} \mathbf{Q}(\xi)\int_{r_0}^{\xi} \mathbf{Q}(\xi_1)d\xi_1 d\xi + \ldots,$$

$\mathbf{M}(r, r_0)$ is a propagator matrix :

$$\eta(a_q) = \mathbf{M}(a_q, a_0)\eta(a_0)$$

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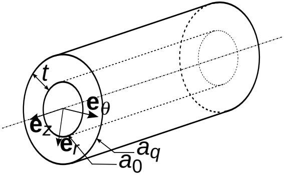
Tube in vacuum → eigenmodes



- axial waves
  - longitudinal modes $L(0,m)$
  - torsionnal modes $T(0,m)$
  - flexural modes $F(n,m)$
- circumferential waves

Characterization of an anisotropic tube

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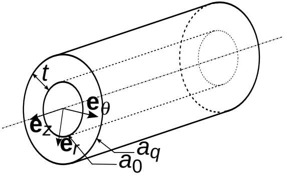

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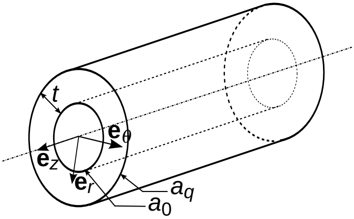
Plate vs. tube

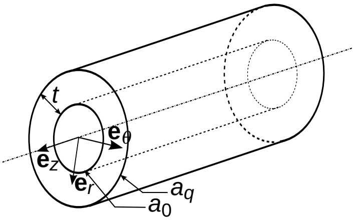


$t/a_q < 0.5$ → Plate - SH and Lamb waves

$t/a_q > 0.5$ → Tube - axial waves

Cortical bone : anisotropic tube.

Plate vs. tube



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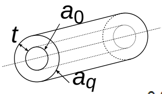

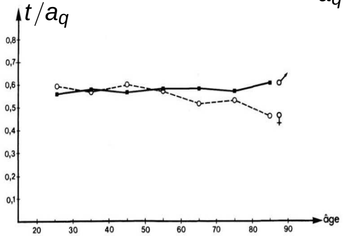

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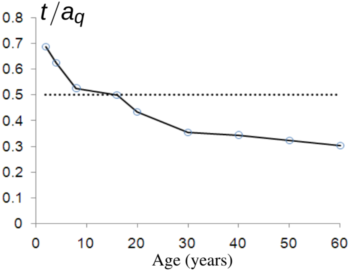


Bergot & Bocquet (1976)

Carter & Beaupré (2001)

Bone properties

Orthotropic material

$$\begin{pmatrix} 18 & 9.98 & 10.1 & 0 & 0 & 0 \\ 9.98 & 20.2 & 10.7 & 0 & 0 & 0 \\ 10.1 & 10.7 & 27.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6.23 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.61 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.52 \end{pmatrix}$$

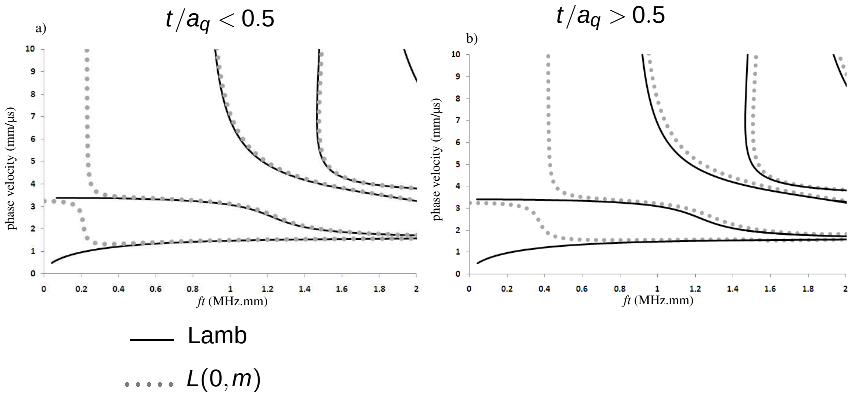
Van Burskirk & Ashman, 1981

Long bone : plate or tube

- Dispersion curves of propagation modes :
  ___ plate
  ..... tube
- 2 ratios : $t/a_q = 0.4 < 0.5$ et $t/a_q = 0.6 > 0.5$
- modes
  - longitudinal
  - torsional
  - flexural

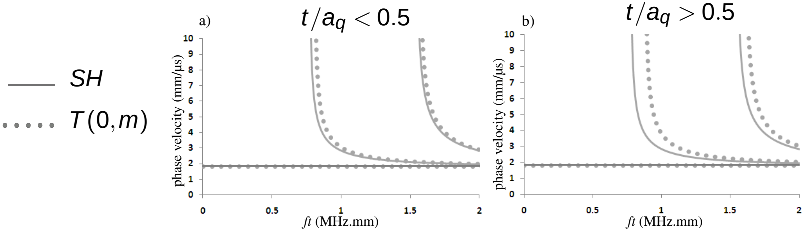
Long bone : plate or tube

longitudinal modes



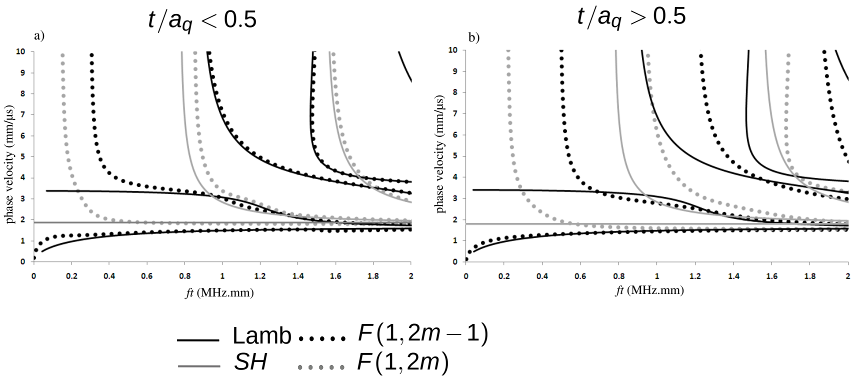
Long bone : plate or tube

torsional modes



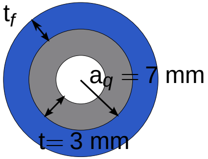
Long bone : plate or tube

flexural modes



Soft Tissue

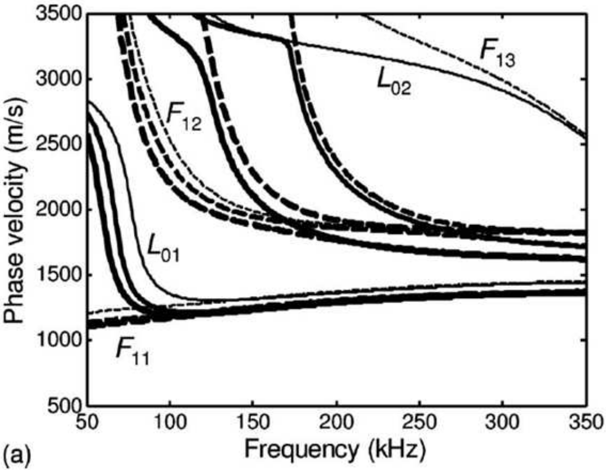
Isotropic bone tube coated with water shell (Moilanen et al., JASA 2008)



$t_f = 5$ mm

$t_f = 3$ mm

$t_f = 1$ mm



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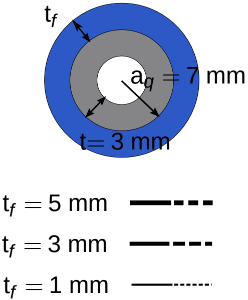
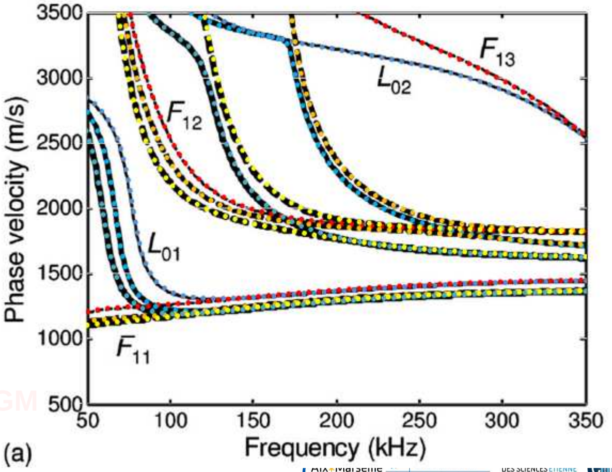
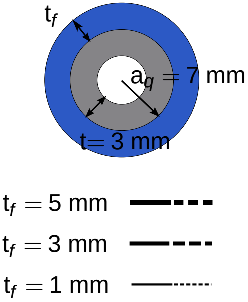


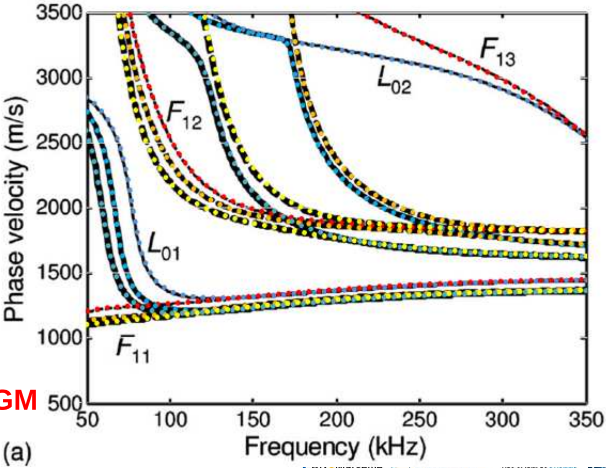


Orthotropic and FGM

Soft Tissue

Isotropic bone tube coated with water shell (Moilanen et al., JASA 2008)





(a)

Orthotropic and FGM

Soft Tissue

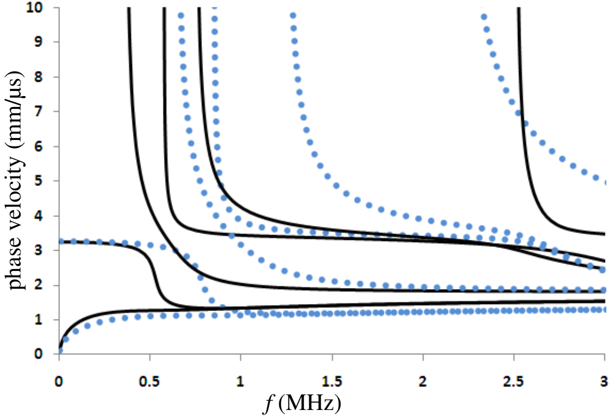
Orthotropic bone tube

free-free (black solid lines) and coated with water shell (blue dots)

vacuum

fluid layer

$t_f = 3$ mm



Conclusion and perspectives

- Analytical solution (Peano expansion of the matricant - Stroh's formalism)
  $\Rightarrow$ Guided waves propagation in an **orthotropic** functionnally graded tube coated by a perfect fluid layer

- Further work
  Fill the tube $\Rightarrow$ problem of infinite values for $r << 1$.

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Thank you !

Equation

$$
\mathbf{Q}(r) = \frac{1}{r}\begin{pmatrix}
-\frac{c_{12}}{c_{11}} & -\imath n\frac{c_{12}}{c_{11}} \\
-\imath n & 1 \\
-\imath k_z r & 0 \\
\imath\left(\gamma_{12} - r^2\rho\omega^2\right) & -n\gamma_{12} \\
n\gamma_{12} & \imath n^2\gamma_{12} + \imath r^2\left(k_z^2 c_{44} - \rho\omega^2\right) \\
k_z r\gamma_{23} & \imath n k_z r\left(\gamma_{23} + c_{44}\right)
\end{pmatrix} \cdots
$$

$$
\cdots \begin{pmatrix}
-\imath k_z r\frac{c_{13}}{c_{11}} & -\frac{\imath}{c_{11}} & 0 & 0 \\
0 & 0 & -\frac{\imath}{c_{66}} & 0 \\
0 & 0 & 0 & \frac{\imath}{c_{55}} \\
-k_z r\gamma_{23} & \frac{c_{12}}{c_{11}} & -\imath n & -\imath k_z r \\
\imath n k_z r\left(\gamma_{123} + c_{44}\right) & -\imath n\frac{c_{12}}{c_{11}} & -1 & 0 \\
\imath n^2 c_{44} + \imath r^2\left(k_z^2\gamma_{13} - \rho\omega^2\right) & -\imath k_z r\frac{c_{13}}{c_{11}} & 0 & 0
\end{pmatrix}
$$

with

$$
\gamma_{12} = c_{22} - \frac{c_{12}^2}{c_{11}}\ ;\ \gamma_{13} = c_{33} - \frac{c_{13}^2}{c_{11}}\ ;\ \gamma_{23} = c_{23} - \frac{c_{12}c_{13}}{c_{11}}
$$

Propriétés de l'os

	c_{11} (GPa)	c_{13} (GPa)	c_{33} (GPa)	c_{44} (GPa)	c_{66} (GPa)	ρ (g.cm^{-3})
C_M	25.9	11.1	29.6	5.5	4.4	1.753
C_m	11.8	5.1	17.6	3.3	2.2	1.66

TABLE: Les valeurs minimale et maximale $[C_m, C_M]$ de chaque propriété avec $c_{12} = c_{11} - 2c_{66}$. A noter que la correspondance entre les directions de l'espace et la notation indicielle est $1 \leftrightarrow r; 2 \leftrightarrow \theta; 3 \leftrightarrow z$.

Profil linéaire

$$C(r) = C_m + (C_M - C_m)(r - a_0)/(a_q - a_0),$$

L'os : plaque ou tube

- Courbes de dispersion des modes de propagation :

  —— plaque

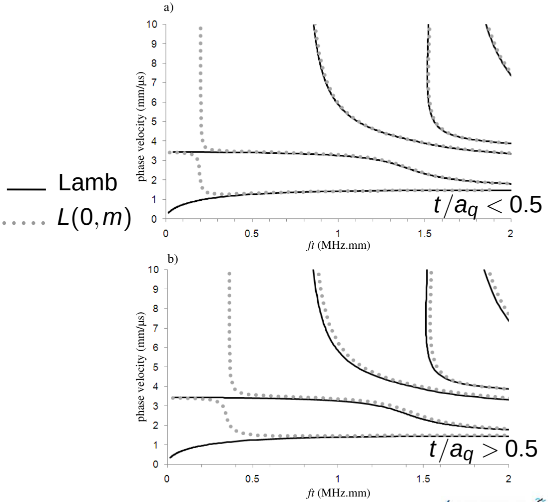
  ..... tube

- 2 rapports : $t/a_q = 0.4 < 0.5$ et $t/a_q = 0.6 > 0.5$

- modes
  - longitudinaux
  - de torsion
  - de flexion

L'os : plaque ou tube

modes longitudinaux

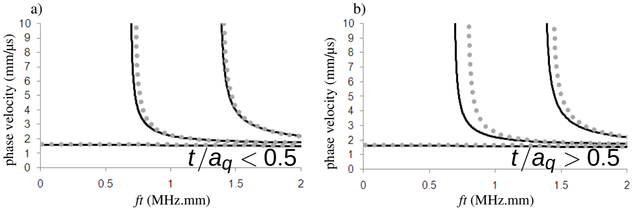


L'os : plaque ou tube

modes de torsion

—— *SH*

······ $T(0,m)$



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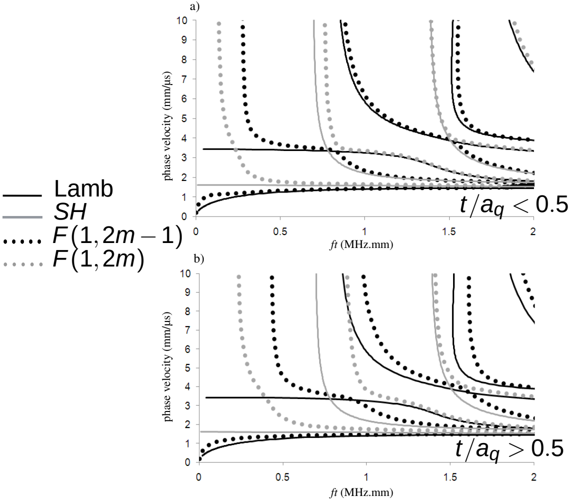
cnrs

L'os : plaque ou tube

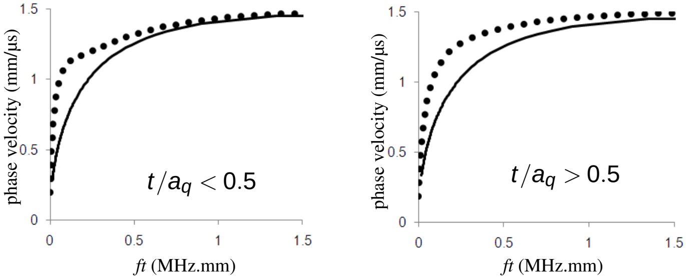
modes de flexion



Estimation de l'épaisseur corticale : A_0 vs $F(1,1)$

Dans Moilanen et al., UMB (2007) : estimation de l'épaisseur corticale
A_0 vs $F(1,1)$

——— A_0
• • • • • $F(1,1)$



Baron, Ultrasonics (2011)