On the elasticity of an inertial liquid shock

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A drop of low viscosity hitting a solid may bounce back, provided that the material is highly hydrophobic. As a model case of such a situation, we consider here the case of a very hot solid. Then, as discovered by Leidenfrost, a thin layer of vapor sustains the drop, preventing any contact with the substrate. Thrown on such a solid, a drop rebounds, and we discuss here the elasticity of the shock. Two very different cases are described: at a large velocity, the higher the impact velocity, the poorer the elasticity; at a small velocity, a quasi-elastic regime is evidenced. The frontier between both domains is set by a Weber number, which compares the kinetic and surface energies of the drop, of order unity.
I. Introduction

In 1756, two hundreds years before the foundation of the Journal of Fluid Mechanics, Johan Leidenfrost, a German physician from Duisburg, published a summary of his discoveries on the physics of drops (Leidenfrost, 1756). He described in particular the remarkable behavior of volatile liquids, when deposited on substrates whose temperature is significantly larger than their boiling point (Gottfried 1966, Baumeister 1973). For water on copper or iron at a temperature of about 200°C (or more), it is observed that drops are extremely mobile and quickly roll off the substrate if any slope is present. On a curved surface (as the inside of a spoon, in order to keep the drop trapped), the liquid does not boil and lasts very long (typically a few minutes for millimetric drops), despite the high temperature of the substrate (Bell, 1967).

Leidenfrost correctly interpreted these experiments. He understood that a vapor film forms in-between the drop and the substrate, allowing the liquid to levitate (Goldshтик 1985). Because of the absence of contact, bubble nucleation is inhibited (no boiling), and the existence of a cushion of air dramatically reduces the friction with the substrate. Since a gas is a good insulator, the heat transfer is not that efficient in the film, which induces a relatively slow evaporation for the liquid (Berenson 1961, Wachters 1966, Michiyoshi 1978, Zhang 1991, Chandra 1994). By placing a candle behind the drop and looking through the base, Leidenfrost saw a sheet of light of a thickness of about 100 μm (a hair diameter), allowing him not only to prove the existence of the vapor film, but also to measure its (correct) typical thickness (Biance 2003).

The shape of the drops is also worth being described. While large centimetric drops are flattened by gravity, small millimetric droplets are quasi-spherical, which was understood (by Young and Laplace) 50 years after Leidenfrost’s observations as a consequence of the cohesion of liquids: a drop has a surface energy proportional to its area, whose minimum corresponds to a spherical shape. The surface energy per unit area is called the surface tension and denoted as $\gamma$. For a Leidenfrost water drop, evaporation maintains the drop temperature at 100°C, so that the surface tension is fixed at 58 mN/m (this condition of constant temperature should limit Marangoni effects in the drop). This allows us to specify when gravity can be neglected: while the surface energy of a drop of size $R$ scales as $\gamma R^2$, its gravitational energy is proportional to $\rho g R^4$, denoting $\rho$ as the liquid density and $g$ as the
acceleration of gravity. Hence, gravity is negligible provided that the drop is smaller than the so-called capillary length, $\kappa^1 = (\gamma/\rho g)^{1/2}$. For water at 100°C, $\kappa^1$ is 2.5 mm; for less cohesive liquids (such as light oils or liquid nitrogen), it can be as low as 1 mm.

Apart from these classical static properties, such non-adhesive Leidenfrost drops have a remarkable dynamic characteristics: if they impact a surface, they bounce back, as solid spheres do (Richard 2000a, Karl 2000). This does not happen with “usual” drops; then, the kinetic energy is dissipated by viscosity as the drop spreads on its substrate (Rein 1993), in particular owing to the moving contact lines close to which viscous losses are enhanced. For Leidenfrost drops, there is no contact line, and the kinetic energy efficiently converts in surface energy (the drop deforms as it hits the solid) and then in kinetic energy again, allowing the system to behave as an elastic spring. This directly tells how the contact time of a bouncing drop scales with the different parameters (Richard 2002): the liquid surface tension $\gamma$ is the stiffness of this spring and $\rho R^3$ is its mass, so that the natural response time scales as $(\rho R^3/\gamma)^{1/2}$, of the order of a few milliseconds for a millimetric drop – a time negligible compared to the lifetime of the globule.

Here we discuss the elasticity of this kind of liquid shock. We shall see that unlike what happens for solid shocks, the elasticity is very sensitive to the impact velocity. It is found in particular that the shock is all the more elastic since the impact speed is small. In the limit of very small velocities, the use of Leidenfrost drops leads to a regime of quasi-elastic rebounds. For larger velocities, the shock can be much less elastic; in addition, elasticity is lost above a critical radius (whatever the impact velocity), for which the weight dominates the surface effects. These series of experiments are interpreted using a minimal model.

II. Characteristics of a liquid impact

Figure 1 shows a typical sequence of events, observed as a water drop hits a silicon plate whose temperature (300°C) is much larger than the corresponding boiling point. Here, the drop has a radius $R = 1$ mm, and it is released from a height $H = 3.2$ cm, so that the impact speed $V = \sqrt{2gH}$ is about 0.8 m/s. The pictures are taken with a high-speed camera (1000 frames per second), using back-lighting to improve the contrast.
Rebound of a millimetric water drop hitting a steel plate heated at 300°C. The interval between two pictures is 1.8 ms. The drop radius is 1 mm, and the impact velocity is 80 cm/s. The corresponding Weber number (Eq. 1) is about 10, meaning that large deformations are observed during the impact, leading to strong oscillations.

As it impacts the solid, the drop deforms: it first spreads until it transiently forms a kind of non-wetting puddle, as it reaches its maximum extension. Then, it retracts and elongates in the vertical direction: the globule is highly deformed at take off (after a “contact” time of 11 ms, in figure 1). As a consequence, it strongly vibrates as it rises in air; then, it reaches its maximum height, whose position can be measured. We deduce from such sequences the restitution coefficient $e = V' / V$ of the shock, defined as the ratio between the velocities of the center of mass after and before impact. In the example in Fig. 1, $e$ is 0.6, showing a modest elasticity (the final altitude of the drop is only 36% of the one from which it was released).

The contact time $\tau$ is defined as the interval between the moment when the drop reaches the plate and the one when it leaves it. Measurements of $\tau$ are shown in Figure 2 as a function of the impact velocity (for a fixed drop radius $R = 1.06$ mm), and as a function of drop radius (for a fixed impact velocity of 0.7 m/s). The results are very close to what can be observed for drops bouncing off super-hydrophobic substrates (Richard 2002). On the one hand, the contact time is quite independent of the impact speed; it very slightly decreases with it, which might be related to the non-linear regime of oscillation for strong deformations. On the other hand, it rapidly increases with the drop size, as $R^{3/2}$ (a law drawn with a full line). As stressed in the introduction, the contact time should be of the order of the oscillation time. This quantity was calculated by Rayleigh, for a drop freely oscillating in air (Rayleigh 1879). For
the simplest (quadrupolar) mode of oscillation, this time is $\pi(\rho R^3/2\gamma)^{1/2}$, which exhibits the different scalings observed in Fig. 2. However, the numerical coefficient is slightly different: the Rayleigh coefficient $\pi/\sqrt{2}$ is about 2.2, while we deduce from the experiments a coefficient of $2.65 \pm 0.10$. Courty et al. recently showed that the period of oscillation of a non-wetting drop is increased because of the presence of a plate below, compared to a free drop (Courty 2005). This might explain the slight disagreement found on the coefficient.

![Figure 2](image1.png)

Contact time $\tau$ of a drop bouncing on plate heated at 280°C. a. In a first series of data, $\tau$ is plotted as a function of the impact velocity $V$ (for a fixed drop radius $R = 1.06$ mm), and found to be quite insensitive to $V$ (multiplying $V$ by a factor of about 7 makes $\tau$ decrease by only 15%). b. In a second series, $\tau$ is observed (in a log-log plot) to increase rapidly with the drop radius $R$, as $R^{1/2}$ (drawn with a full line). Writing $\tau = \alpha (\rho R^3/\gamma)^{1/2}$, we deduce from the fit a numerical coefficient $\alpha = 2.65 \pm 0.20$.

We also stressed the existence of a well-defined state of maximal extension. We denote the radius of this transient puddle as $R_M$, and its thickness as $\delta$. We shall first consider the strong
deformation regime ($R_M \gg \delta$), which will occur if the kinetic energy of the impinging drop is much larger than its surface energy. The Weber number $We$ compares these two energies:

$$We = \frac{\rho V^2 R}{\gamma}$$  \hspace{1cm} (1)

A strong deformation at impact corresponds to $We$ larger than unity. The maximum radius should generally be a function of the Weber number. We could assume that the shock just converts the kinetic energy (of the order of $\rho R^3 V^2$) in surface energy (of the order of $\gamma R_M^2$ in the limit considered here), which would yield (Chandra 1991):

$$R_M \sim R We^{1/2}$$  \hspace{1cm} (2)

But energy conservation is never obvious in this kind of systems, in particular because of the existence of internal flows during the contact time. We rather propose that the drop, as it hits the solid, is subjected to an acceleration which scales as $V^2/R$, since the velocity decreases from $V$ to 0 on a distance of about $R$ (Clanet 2004). This acceleration is typically 100g, in impact experiments, so that the drop will be (transiently) flattened, in this reinforced gravity field. The thickness $\delta$ of a non-wetting gravity puddle is proportional to a rescaled capillary length $\kappa = (\gamma/\rho g)^{1/2}$, where $g$ must be replaced by $V^2/R$. Together with the volume conservation ($R^3 \sim \delta R_M^2$), this yields an extension:

$$R_M \sim R We^{1/4}$$  \hspace{1cm} (3)

This scaling appears to be different from the one arising from energy conservation: in the limit of large $We$, the drop is expected to be more contracted than predicted by Eq. 2, by a factor of $We^{1/4}$ (1.8 to 3.2, for $We$ between 10 and 100). Of course, this description can only hold provided that the drop size $R$ is larger than the “dynamic” capillary length ($\gamma R/\rho V^2)^{1/2}$ (only these drops will be flattened), which brings us back to $We \gg 1$. 

6
Figure 3

Maximum radius $R_M$ of a water drop impacting a plate heated at $T = 280^\circ$C, as a function of the Weber number $We$ characterizing the shock and defined by Eq. 1. Drops have a radius $R = 1.02$ mm, and the Weber number is varied by changing the impact velocity. The diagram has logarithmic scales, and the full line shows the slope $\frac{3}{4}$ suggested by Eq. 3. Writing $R_M = \alpha We^{1/4}$, we deduce from the fit a numerical coefficient $\alpha = 1.1 \pm 0.1$.

Fig. 3 shows that the extension observed for the drop (and obtained owing to a variation of the impact velocity) follows a behavior compatible with the scaling proposed in Eq. 3 (the exponent deduced from Fig. 3 is $0.30 \pm 0.05$). It is useful to define the maximum extension of the drop, because it defines the state from which the drop will retract and take off.

III. Elasticity of the shock: observations

We focus here on the elasticity of the shock. Our measurements consisted of filming rebounds for various liquids (water, ethanol, acetone, and liquid nitrogen), different drop radii $R$ (between $0.8$ mm and $2$ mm) and impact velocities $V$ (between $0.02$ m/s and $1$ m/s). The case of larger drops (of size approaching the capillary length $\kappa^{-1}$) will be discussed separately (see section V, and in particular Figure 8). The plates were systematically at a “high” temperature, that is, more than $100^\circ$C above the boiling point of the liquid. Before impact, the drop is spherical and we extracted from the movies the position of the bottom of the drop. The impact velocities were varied using different release heights, and the drop radii were changed using various hypodermic needles. The take-off speed $V'$ was deduced from the time $t$ after which the drop touched the solid again – during the flight, the drop is only subjected to gravity, so that we simply have $V' = gt/2$. 
We displayed in Figure 4a the variation of the restitution coefficient $e = V' / V$, as a function of the Weber number $We$ (defined in Eq. 1). It is first observed that all the data collapse in the same curve, showing that the Weber number is indeed the parameter which fixes the shock elasticity. At low Weber, $e$ is close to unity: the shock is quasi-elastic. At large $We$, the elasticity abruptly drops, to reach values as low as 0.2 for $We$ of about 30 (meaning that the drop rises after the shock at a height which is only 4% of the initial height). This is our main finding: unlike solid shocks, the elasticity of liquid shocks (which imply strong
deformations of the whole globule of matter) is extremely sensitive to the impact speed. Displaying the function $e(We)$ in a log-log plot (Fig. 4b) shows that the data for $We > 1$ can be described by the scaling $e \sim We^{-1/2}$.

![Figure 5](image)

Restitution coefficient for millimetric water drops hitting a super-hydrophobic surface. As in Figure 4, a loss of elasticity is observed as increasing the impact speed. But contrasting with Leidenfrost drops, the elasticity sharply decreases at small impact velocity, for which a sticking transition is observed. These data also emphasize that the drop behaviour is quite similar for water (empty symbols) and mixture of water and glycerol twice more viscous (full symbols).

These behaviors partly differ from what can be observed for drops impacting a super-hydrophobic surface (that is, a micro-textured hydrophobic surface on which contact angles are typically of the order of 160°). Then, as seen in Figure 5, a strong decrease of the elasticity (compatible with a variation of $e$ as $1/V$) is also observed for “large” impact velocities (corresponding, again, to large $We$). However, drops are found to stick to their substrates ($e = 0$) for moderate impact velocities ($V < 10$ cm/s). This can be interpreted as the result of the existence of a small adhesion force in this case: the kinetic energy of the drop becomes too low, so that pinning of the line on the surface textures allow the drop to remain stuck. This implies that smaller drops (of larger ratio surface/volume) will stick more easily than large ones, which was indeed observed (Richard 2000b). As a consequence, the elasticity is maximum for some intermediate velocity, for which the restitution coefficient can (in this case) be as large as 0.85 – a value significantly lower than reported for Leidenfrost drops in Fig. 4. This also suggests that the minimal velocity above
which a rebound can be seen could be taken as a criterion of super-hydrophicity: the smaller this velocity, the better the surface – the limit being a hot plate for which this velocity seems to be zero.

IV. Poorly elastic shocks

1. Liquid springs

As sketched in Figure 6, we propose to model a bouncing drop as a spring. The simplest object which can be imagined for this purpose is a spring of initial length \( l_0 \), stiffness \( k \) with two masses \( m/2 \) attached at its ends. The rebound can be decomposed in two phases: first, the drop spreads owing to its kinetic energy, so that it (partially) stores it in surface energy (as a compressed spring does in elastic energy); later, the drop transfers its surface energy in translational kinetic energy (allowing it to take off) and in oscillatory kinetic energy. Similarly, a compressed spring will oscillate as it takes off. We shall justify in the final discussion (section V.3) why usual sources of dissipation can be neglected in our model.

**Figure 6**

Comparison between a spring consisting of two attached masses, with an initial length \( l_0 \), a stiffness \( k \) and compressed to a height \( \delta \) and a bouncing drop of initial radius \( R \), stiffness \( \gamma \) and compressed to a height \( \delta \).
Let us describe in a more detailed way the spring case. Newton’s equation of motion can be written for both masses, whose positions are denoted as $x_1$ (at the top) and $x_2$ (at the bottom):

$$\frac{1}{2} m \frac{d^2 x_1}{dt^2} = -\frac{1}{2} mg - k(x_1 - x_2 - l_0)$$  \hspace{1cm} (4)

for the first mass and:

$$\frac{1}{2} m \frac{d^2 x_2}{dt^2} = -\frac{1}{2} mg - k(x_2 - x_1 + l_0) + F$$  \hspace{1cm} (5)

for the second one, denoting $F$ as the reaction of the solid. At the beginning, and as long as the force acting on mass 2 is negative (and balanced by $F$), we just have $x_2 = 0$. The first equation can be integrated, taking as initial conditions an imposed compression $\delta$, and a zero velocity. Thus we find:

$$x_1(t) = l_0 - \frac{mg}{2k} + (\delta - l_0 + \frac{mg}{2k}) \cos \omega t$$  \hspace{1cm} (6)

where $\omega$ is the natural pulsation of the mass:

$$\omega = \sqrt{\frac{2k}{m}}$$  \hspace{1cm} (7)

The bottom mass 2 leaves the substrate if the force which acts on it is positive ($k(x_1 - l_0) - mg/2 > 0$). Replacing $x_1$ by the quantity calculated above, we find the time $t_0$ after which the mass takes off. It reads:

$$-mg + k(\delta - l_0 + \frac{mg}{2k}) \cos \omega t_0 = 0$$  \hspace{1cm} (8)

This equation only admits solution if the absolute value of $\cos \omega t_0$ is smaller than unity. Since we also have $\delta < l_0$, we finally find:
\[ \delta < l_0 - \frac{mg}{k} \]  

(9)

The spring must be compressed enough to overcome its weight. If this criterion is fulfilled, we can calculate the speed at take off. Then we have \( x_2(t_o) = 0 \) and (from Eq. 6) \( \frac{dx}{dt}(t_o) = -\omega (\delta - l_0 - \frac{mg}{2k}) \sin \omega t_o \). In the limit of a compressed spring \( \delta \ll l_0 \), and denoting \( x(t) \) as the position of the center of mass of the spring, we find for the speed of take off:

\[ V' = \frac{dx}{dt}(t_o) = \frac{\omega l_0}{2} \sqrt{\left(1 - \frac{g}{\omega^2 l_0}\right) \left(1 - \frac{3g}{\omega^2 l_0}\right)} \]  

(10)

If, in addition, the weight is negligible compared to the elastic energy typically stored in the spring \( (mg \ll kl_0) \), we simply get:

\[ V' = \frac{\omega l_0}{2} \]  

(11)

These two limits \( \delta \ll l_0 \) and \( mg \ll kl_0 \) are satisfied in our experiments: the size of the spring is the drop radius (or diameter); the compression due to impact is the drop thickness \( \delta \) as it reaches its maximum extension (given by Eq. 3, which together with volume conservation yields \( \delta \sim R We^{-1/2} \)); and the stiffness of the spring is the surface tension \( \gamma \). Thus these two conditions can be written for a bouncing drop \( We > 1 \) and \( R < \kappa^{-1} \), respectively, which are both fulfilled in this section. More generally, the results found for a spring should hold for a drop: the main difference between both systems is the way the mass is distributed in space, which should affect the numerical coefficients but not the scaling laws. Eq. 11 can be used for evaluating the speed after take off:

\[ V' \sim V_o = \sqrt{\frac{\gamma}{\kappa R}} \]  

(12)

This is quite a surprising result: the speed of take off \( V' \) is independent of the impact speed \( V \) (for \( We > 1 \), i.e. in the regime of large deformation), so that the restitution coefficient \( e = V'/V \) should decrease with \( V \), as:
This result is in good agreement with the results displayed in Figures 4 and 5. This shows a contrario that the speed after take off is the same whatever the impact speed. For a millimetric drop, this speed is about 25 cm/s, in agreement with the value deduced from Eq. 12. With such a speed, the drop rises to a height of about 3.5 mm (as it can be observed in Fig. 1). Since \( V' \) does not depend on the compression of the drop, the detail of the model providing the rate of compression (Eqs 2 or 3, for example) does not impact the result. This result also helps understanding the difference of these kinds of shock, compared to shocks of solid marbles. In the latter case, the impact object mainly deforms close to its contact with the solid on which it bounces, and it stores its kinetic energy in (volumetric) elastic energy. Then, the restitution coefficient increases with the speed \( V \) at impact, before saturating (and very slightly decreasing) at very large \( V \) (Falcon 1998). It seems that the energy transferred (and later lost) in vibrational modes is negligible, compared to the energy dissipated in elastic waves at the substrate surface and in plastic deformation in the impacting body.

2. Conditions for a rebound

The mass spring analogy also helps understanding what is the criterion for observing a rebound. We stressed earlier that the elastic force must overcome the weight in order to take off (Eq. 9). In our case, the compression is the height of the drop after it has been squeezed by the impact: the drop is all the more compressed since the impact is violent (Fig. 3). Thus, we expect from Eq. 9 that a rebound can only occur provided that the speed is large enough. For \( We > 1 \), Eq. 3 can be considered as a law for the compression; using volume conservation, it yields \( \delta \sim R \, We^{-1/2} \). Then, replacing the different terms in Eq. 9 (i.e. taking \( l_0 \sim R \), \( k \sim \gamma \) and \( m \sim \rho R^3 \)) yields as a criterion of bouncing:

\[
V > V_c = \frac{V_o}{(1 - R^2 \kappa^2)}
\]  

(14)

where \( V_o \) is defined by Eq. 12, and is about 25 cm/s for a millimetric water drop. For small droplets \( (R \ll \kappa^{-1}) \), Eq. 14 indicates that drops will only bounce if \( V > V_o \), that is, \( We > 1 \), which is the limit considered in this section. But it can be different if drops get larger: as \( R \)
approaches the capillary length $\kappa^{-1}$, the velocity $V_c$ diverges, meaning that drops which satisfy
the condition $We > 1$ will not necessarily bounce back. Note that this result does not depend
on the choice of the model for spreading: using Eq. 2 for evaluating the compression of the
drop leads to a relation similar to Eq. 14, yet with a different diverging behavior as $R$
approaches $\kappa^{-1}$ (then, we find: $V > V_c = \sqrt{\frac{\rho \omega}{(1-R^2 \kappa^2)^3}}$).

![Figure 7](image)

Our device for varying the size of the bouncing drop: a volume of water (radius $R$) is deposited on a slightly
inclined plate. As the gate is opened, the drop falls and bounces on the ground. All the elements are made of
steel and heated, so that the drop is everywhere in a Leidenfrost situation.

For testing these ideas, we performed the experiment sketched in Figure 7. A steel block with
a slightly inclined (by 1 or 2$^\circ$) top plate is heated above the Leidenfrost temperature $T_L$.
Liquids are kept trapped on the top plate thanks to a steel gate, which is similarly brought
above $T_L$. As the gate is opened, the drop falls down till it reaches another flat plate also
above $T_L$. Movies of the fall and the (possible) rebound were made for different liquids
(water, ethanol, acetone and liquid nitrogen). In each case, the size of the initial globule was
varied, and the velocity of impact kept as constant as possible, between 42 and 51 cm/s. This
variation together with the uncertainty on the measurement of $V'$ generate some dispersity in
the results displayed in Figure 8, where the restitution coefficient $e$ is plotted as a function of
the drop radius.
Restitution coefficient of the shock for a drop of varying radius $R$ falling from a given height $H = 1$ cm, as a function of $R$ for different liquids (○: liquid nitrogen; ▲: ethanol; ×: acetone; □: water). In each case, an abrupt decrease is observed around a critical radius $R_c$ marked with a vertical line.

For each liquid, it is observed that $\varepsilon$ dramatically decreases when approaching some critical size $R_c$ for the drop. This size is quite comparable for ethanol and acetone, but significantly smaller for liquid nitrogen, and larger for water. For drops larger than $R_c$, the shock is inelastic ($\varepsilon = 0$): a “large” drop cannot bounce, despite the (quasi) absence of friction for these non-wetting systems.

Critical radius $R_c$ above which a Leidenfrost drop does not bounce, as a function of the capillary length associated with the liquid (○: liquid nitrogen; ▲: ethanol; ×: acetone; □: water), as deduced from Fig. 8.

We plotted in Figure 9 the critical radius $R_c$ as a function of the capillary length. It is observed that both quantities are proportional to each other, with a constant of proportionality close to unity ($R_c \kappa = 1.0 \pm 0.1$). This result agrees with Eq. 14, which predicts that bouncing should not occur (whatever the impact velocity) for drops of the order of (or larger than) $\kappa^{-1}$. This
result might be summarized in a very simple way: a drop larger than the capillary length is subjected to gravity, which makes it flatten. Thus, such a drop loses its elasticity, which makes it unable to bounce back. Such large drops were just observed to oscillate on the plate, after hitting it; these oscillations were damped by viscosity, yet sometimes reappeared spontaneously, as reported by Yoshiyasu (1996) or by Strier (2000). More generally, the loss of elasticity could arise from other factors, such as viscosity or adhesion. However, transitions to sticking in these cases remain to be described.

V. Quasi-elastic shocks

1. Phenomenology

It was also observed in Fig. 4 that a rebound at a small Weber number is characterized by a very high restitution coefficient. Let us first stress that such impacts are difficult to achieve: \( We \ll 1 \) implies for a millimetric drop an impact velocity smaller than 25 cm/s, corresponding to a drop released from a height smaller than 3 mm. Thus we chose to study series of rebounds: a given drop is released from a centimetric height, and its successive rebounds are monitored as a function of time. This allows us to reach (after a few rebounds, and provided that the substrate is extremely horizontal) impact velocities of the order of 0.1 m/s, or even smaller. However, this does not avoid the vibrations of the drop (which oscillates after each rebound, as described in section II), which will be found to affect the rebound itself.

![Figure 10](image_url)

Restitution coefficients observed for series of successive shocks experienced by water drops falling from an initial height of 16 mm; \( n \) designates the number of shocks. Whatever the plate temperature, the restitution coefficient tends towards a value very close to unity (quasi-elastic regime), allowing a very large number of successive rebounds (here several tens, but we often observed several hundreds).
Figure 10 shows the restitution coefficient of a water drop bouncing off hot plates, as a function of the number of rebounds. Very numerous rebounds are observed: for similar experiments, we monitored up to 1000 successive rebounds, which emphasizes the quasi-elasticity of the shocks. In Figure 10, the restitution coefficient is indeed observed to tend after about 10 shocks towards a value which is very close to unity, whatever the plate temperature (provided it is larger than the Leidenfrost temperature). Hence the drop constantly rises to the same height (independent of the position from which it is released), which is of the order of 2 mm for a drop of radius $R = 1$ mm. Note also in Figure 10 that the restitution coefficient can be larger than unity; this is partially related to the uncertainty on the measurement, but values larger than 1 might indeed exist, because of the oscillations of the drop (which store some energy), and possibly because of the energy gained close to the hot plate (close to which the drop slightly evaporates). Figure 11 shows the successive positions of a drop on one period of this stationary elastic regime.

![Figure 11](image)

Sequence of pictures showing the rebound of a water Leidenfrost drop in the quasi-elastic regime. The drop radius is $R = 1$ mm, the impact velocity is $V = 7$ cm/s, which yields a Weber number $We = 0.1$. The interval between two pictures is 2.5 ms.

The drop modestly deforms during the contact ($We << 1$). Its size remains constant during the period, showing that evaporation can be neglected at this time scale (this allows the drop to bounce several hundreds times). The period in Figure 11 is $17.5 \pm 1.0$ ms, a time negligible compared to the lifetime of this Leidenfrost drop, if deposited (and trapped) on the hot plate, namely, 40 s (at 250°C). (This lifetime is observed to remain of the same order for a bouncing drop.) But most importantly, it can be noticed in Figure 11 that the vibrations of the drop fit with its rebound: the drop is spherical at impact and take off; it flattens twice, namely during the contact and when it reaches its maximum height. It thus seems that the oscillations are synchronized with the motion of the center of mass.
We recorded the mean height of the drop \( h \) (that is, the position of its center of mass) together with its equatorial diameter \( d \), as a function of time, for several successive oscillations (and rebounds). Figure 12a shows that both curves are periodic, without significant damping, and that the period is indeed the same. For a better visualization, we displayed in Figure 12b a zoom on two periods, together with a sketch of the successive shapes of the drop within a period. The drop makes half an oscillation on the plate, and one and a half in air. The respective durations of these two regimes are 10 ms and 25 ms, from which we deduce that the period of oscillation is different when the drop is contacting the solid (then, the period is 20 ms) and when it is free (then, the period is 16.7 ms), as commented in Courty (2005).

\[
\begin{align*}
\text{Figure 12a} \\
\text{Height} \ h \text{ of the center of mass of a water drop} \ (R = 1.25 \text{ mm}) \text{ in the quasi-elastic regime: no appreciable decrease of the height is observed on more than 30 successive rebounds. We superimposed in the same plot the equatorial diameter} \ d \text{ of the drop: the vibrations generated by the impact (and seen in Figure 11) are observed to be in phase with the flight.}
\end{align*}
\]

\[
\begin{align*}
\text{Figure 12b} \\
\text{Zoom on two periods of Figure 12a. We sketched below the shape at the drop for different moments of its flight.}
\end{align*}
\]
Considering that the flight time is of the order of the oscillation time, we can deduce a simple formula for the height of the drop $h_c$ in the regime of multiple rebounds. Taking the Rayleigh time $\pi(\rho R^2/\gamma)^{1/2}$ of oscillation of a free drop (Rayleigh, 1879), and considering that the time of the flight $t$ (related to the height by the relation $h = gt^2/8$) is 1.5 the oscillation time of a drop, we find a relation between $h_c$ and the drop radius:

$$h_c = \frac{9\pi^2}{64} \frac{R^3}{\kappa^{-2}}$$

(15)

We tested this relation, and plot our results in Figure 13. The domain of variation is quite limited for the drop radius. On the one hand, we are limited by the capillary length $\kappa^{-1}$, as shown in section IV.2. On the other hand, the drop size cannot be smaller than a fraction of a millimeter for making the observation feasible. On the whole, a fair agreement is found, without any adjustable parameter.

![Figure 13](image)

Height for which multiple rebounds are observed as a function of the drop radius. The data are compared with Eq. 15, drawn in full line.

2. Conditions for observing a quasi-elastic rebound

We tried to characterize the conditions which favor a quasi-elastic rebound. We first compared shocks for which the restitution coefficient $e$ could not be distinguished from unity, with shocks for which it was found to be significantly smaller. We displayed in Figure 14 an interesting difference between both cases: the drop diameter (normalized by its value without oscillation) is plotted as a function of time during the shock: it thus in both cases increases
from unity to a value slightly larger (1.2 and 1.4), before decreasing back to unity (when the drop takes off). The origin of time is chosen for the maximum extension. For each series of data (symbols), we superimposed (thin lines) curves and their time-symmetric. It is found that only the quasi-elastic rebound (squares) is time-reversible, which stresses the phase locking between the two sources of oscillation (drop vibration, parabolic flight).

![Figure 14](image)

Equatorial diameter of an impacting drop (normalized by its diameter before impact) as a function of time, for a drop in the quasi-elastic regime (squares) and for a drop for which the oscillation and flight periods differ (circles). The origin of time is chosen at the maximal extension. It is observed that only the drop deformation is time-reversible, in the quasi-elastic regime only.

A natural question, at this point, is how elastic (or less elastic, as shown in Fig. 4) regimes may be generated. We stressed above (Figure 10) that the elastic regime spontaneously sets after a short transient. It happens in about 70 % of the experiments where we followed successive rebounds, and always exhibits the different characteristics reported above (synchronization of the oscillation with the flight, time-reversibility). This suggests that the phase of the drop influences the rebound, and we thus tried to evidence the role of this phase.

![Figure 15](image)

Our phase convention for a drop subjected to a quadrupolar oscillation.
We chose for the phase of the drop oscillation the convention sketched in Figure 15. Note that the two spherical states ($\phi = \pi/2$ and $\phi = 3\pi/2$) differ by the internal motion of the liquid: the equatorial diameter tends to contract in the phase $\phi = \pi/2$, while it tends to expand in the phase $\phi = 3\pi/2$. Our experiment consisted of monitoring impacts for similar drops having a similar velocity, yet a different phase at impact. The phase was varied using the device drawn in Figure 16: a plate pierced with a thin hole (diameter comparable with that of the drop) is displayed between the point from which the drop is released and the plate. Because of the presence of this hole, the drop is forced to elongate (and then vibrates), so that tuning the height of the pierced plate allowed us to vary the phase at impact. The impact was recorded and the restitution coefficient of the shock deduced from the record.

![Figure 16](image)

**Figure 16**

Experimental set-up for tuning the phase of a drop at impact. A plate pierced with a hole of millimetric diameter (and heated to be non-wettable) is placed between the height from which the liquid is release and the bottom plate on which the rebound is observed. Crossing the hole elongates the drop, which later oscillates. Varying the position of the pierced plate thus allows us to vary the phase at impact.

Despite its apparent simplicity, this experiment is extremely delicate to perform at very small impact velocities (where the elastic regime is observed), because this corresponds to small heights, which makes it impossible to display the pierced plate. We thus were forced to make this experiment with various heights (of the order of one centimeter), corresponding to impact velocities between 20 and 80 cm/s. We carefully measured the restitution coefficients for about 100 shocks in this interval of impact velocities, allowing us to determine a mean value $\bar{e}$ of $e$ for each velocity. The results are displayed in Figure 17, where we normalized for each shock the measured restitution coefficient by its mean value at this velocity. This explains
why in our results the quantity $e/\bar{e}$ can be larger than unity. The uncertainty on the
determination of the phase at impact is 0.3 rd. We performed (on average) 5 measurements
per interval of 0.3 rd, which provides the uncertainty on the coefficient of restitution.

![Graph](image)

Figure 17
Restitution coefficient of a drop bouncing off a hot plate as a function of its phase at impact (defined in Figure
15). The restitution coefficient is normalized by its mean value at a given impact velocity. Each data corresponds
to at least five experiments, and the corresponding error bars are indicated.

It seems that the phase does affect the elasticity of the shock: the data can be fitted by a
sinusoidal function, whose maximum is observed for $\phi = 1.4\pi$, close to $3\pi/2$. The effect is
small, yet significant: the relative increase in measured restitution coefficient between $\phi = \pi/2$
and $3\pi/2$ is about 40 %, larger than the uncertainty on the measurement ($\pm 10\%$). It turns out
that the maximum elasticity is found to be reached for the phase ($\phi = 3\pi/2$) observed for
impacting drops in the quasi-elastic regime. The system thus locks in the state of minimal
dissipation. By definition, the phase $\phi = 3\pi/2$ is the one for which the drop is spherical,
coming back from an oblate configuration. In the reference frame of the drop, the velocity of
the bottom of the drop is directed towards the top, and it is maximal in this phase. Since the
drop is moving at a velocity $V$, this phase thus corresponds to the minimal velocity of the
bottom of the drop. On the other hand, the phase $\phi = \pi/2$ has a maximal velocity, and was
observed to be the less elastic. It seems that the local velocity at impact might influence the
elasticity, which is all the better since this velocity is small.
We can evaluate this minimal velocity. In the regime of small deformation, we assume that there is energy conservation (Okumura, 2003): at impact, the drop stores its kinetic energy in surface energy. Denoting $\varepsilon_o$ as the increase of radius in the oblate state ($\phi = \pi$, see Fig. 15), the conservation of energy dimensionally writes: $\rho R^3 v^2 \sim \gamma \varepsilon_o^2$, which gives $\varepsilon_o (\varepsilon_o \sim R W e^{1/2})$. The elongation of the drop radius can be written: $\varepsilon = \varepsilon_o \sin(\omega t - 3\pi/2)$, where $\omega$ is the Rayleigh pulsation of a free drop $(\omega \sim (\gamma \rho R^3)^{1/2})$. As the drop impacts, the bottom moves up at a velocity $\varepsilon_\omega\omega$, that is, of the order of $V$, the impact velocity of the center of mass. Hence, the drop touches the solid with a minimized velocity, which accounts for a minimized energy loss during the shock.

3. Discussion

Periodic rebounds are also observed for solid marbles bouncing off vibrated plates. Depending on the pulsation $\omega$ and amplitude $A$ of the vibrations, the marble may bounce, or not. There also, there is a correlation between the time of flight and the period of vibration. In our case, we must first write an equation for the shock – as discussed above, the restitution coefficient varies with the phase, and with the impact velocity: $e = e_o (1 - kA/R \sin\phi) (b + cV)$, where $A$ is the amplitude of the oscillation and $b$, $c$ and $k$ coefficients. Neglecting the variation of the restitution coefficient with the velocity (which yields a second order correction), we get a relationship between the take-off and impact velocities:

$$ V' = V e_o (1 - kA/R \sin\phi) \quad (16) $$

The phase at take-off is $\pi/2$. Denoting the time of flight (between two impacts) as $\tau$, we thus get for the phase at impact:

$$ \phi = \omega \tau + \frac{\pi}{2} \quad (17) $$

As it flies, the drop is only subjected to gravity; in the quasi-elastic regime, it falls down on the plate at the same velocity $V$ as for the previous shock, which writes:

$$ V = V' - g\tau \quad (18) $$
Since the trajectory of the bottom of the drop $z_b$ is: $z_b = -\frac{gr^2}{2} + V't + A \cos(\omega t + \pi/2)$, we obtain a third equation at the contact:

$$ -\frac{1}{2} g \tau^2 + V' \tau - A \sin \omega \tau = 0 \quad (19) $$

Denoting $x = \omega \tau$ and $a = A/R$, the solution of these equations verifies:

$$ (1-e_o) \frac{x^3 R^2 \kappa^2}{16} + \frac{e_o}{2} a \sin 2x + \frac{e_o R^2 \kappa^2}{16} \ kx^2 \cos x + (1-e_o) \sin x = 0 \quad (20) $$

where $\kappa$ is the inverse of the capillary length. Eq. 20 can be solved numerically ($x$ being the unknown), which was done for $k = 1$ (the solutions hardly depend on the value of this parameter). The solutions are displayed in Figure 18, as a function of $a$, the reduced amplitude of the oscillation, for a water drop at 100°C of radius $R = 1.1$ mm and for $e_o = 0.9$.

![Figure 18](image)

Numerical solution of Eq. 20, as a function of the amplitude of the oscillation. For the resolution, we have chosen $R = 1.1$ mm and $e_o = 0.9$. For each family of solution, there is one stable branch (in full line), and one unstable branch (in dotted line).

It is observed that periodic solutions exist provided that the amplitude of the oscillation is larger than a threshold $a_c = (1-e_o)/e_o$, i.e. about 0.1 in the example of Fig. 18. These solutions tend towards $\pi$, $3\pi$, $5\pi$ at small amplitude, which seems in good agreement with our experimental results: the periodic regime described above corresponds to $x = 3\pi$, and we observed drops oscillating twice during the flight, corresponding to $x = 5\pi$. A different choice
for \( k \) would slightly shift the threshold amplitude \( a_c \). Note also that only half the branches are stable in Fig. 18, as indicated in the figure (the stability is deduced from the evolution of the trajectory after perturbing the time of flight). It is thus possible to observe periodic regimes – and, as found experimentally, it might be not the case, which is found to be due to a too small amplitude. For most observations, the amplitude was found to be about \( 0.15R \), in qualitative agreement with the results in Figure 18. Because of the modest value of this amplitude, \( x \) should be close to its minimal value of \( 3\pi \), as indeed found in the experiments.

We assumed in this description that the amplitude of the oscillation remains constant all along the sequence, which was observed experimentally. This raises the question of dissipation and energy input in these processes, which we finally discuss.

The oscillations might principally be damped by viscosity. The typical time associated with this damping scales as \( \rho R^2/\eta \), and is found to be about 3s for a millimetric drop of water (whose viscosity falls to about 0.3 mPa s at 100°C). It is much longer than a typical time of flight (i.e. a few milliseconds), allowing us to understand the negligible role of the liquid viscosity in these experiments.

Conversely, the drop can gain some momentum during the impact because of its evaporation (by a quantity \( \delta m \)), which might sustain the rebounds. The corresponding gain of energy scales \( \delta m V^2 \). Denoting \( \tau \) as the contact time of the drop (see Fig. 2), and \( \tau_L \) as its lifetime, we expect \( \delta m \) to be of the order of \( m\tau/\tau_L \); for \( \tau = 10 \) ms, \( \tau_L = 40 \) s and \( R = 1 \) mm, we find \( \delta m \) of about \( 10^{-9} \) kg. On the other hand, the loss of translational kinetic energy due to impact can be written \( mV^2(1-e^2) \). Taking for \( e \) about 0.9, that is, a typical value observed on a cold superhydrophobic surface in the regime of high elasticity, the ratio between these two energies scales as \( \delta m/m(1-e^2) \). This is typically \( 10^{-3} \), suggesting that the main cause of elasticity, in this system, is the absence of pinning on the hot solid, rather than drop evaporation. The elasticity could also be lowered by the presence of the vapor film, which was treated in the paper as a passive medium allowing a non-wetting situation. The viscous force associated with the spreading of the drop (roughly) scales as \( \eta V/h_f R_M^2 \) (with \( \eta \) and \( h_f \) the vapor film viscosity and thickness), and thus the energy dissipated by viscosity as \( \eta V/h_f R_M^3 \). For large Weber numbers, equation (3) gives \( R_M \) (\( R_M \sim R We^{1/4} \)). We can thus compare the energy lost by viscosity to the kinetic energy of the impinging drop, which writes: \( \eta V/h_f We^{3/4} \). For
standard values of the different parameters (in particular taking $h_f$ of the order of 100 $\mu$m), we find about $10^{-4}$ for the first dimensionless group $\eta_d/\rho V h_f$, which implies a negligible viscous loss (in the vapor film) during the spreading stage. This partially justifies why we could ignore dissipation in our spring model.

VII. Conclusion

We reported in this paper a series of experiments evidencing the rebounds of liquid droplets of low viscosity hitting very hot plates, in the so-called Leidenfrost situation – a question of practical importance in cooling and deposition processes. We focussed on the elasticity of the shock, and mainly found that two regimes of bouncing can be distinguished, depending on the Weber number.

When the inertia of the falling droplet is large compared to its surface tension (high Weber number), the rebound is all the less elastic since the impact speed is large. This considerably differs from solid elastic shocks at similar velocities, for which the elasticity weakly depends on the impact velocity. This difference is related to the loss of energy associated with the shock of a liquid, which has two main causes: a dissipation during the spreading at the impact, and the partition of the energy between drop oscillations and translation at take-off. The higher the velocity, the larger the part of energy which is transferred in oscillations, and thus the less elastic the shock. In a similar vein, it is found that elasticity will be lost if considering large drops: then, the impact is found to be inelastic; owing to gravity, drops larger than the capillary length cannot restore enough translational energy to take off, and thus remain stuck on the hot plate where they oscillate without bouncing back.

For shocks at a small Weber number, we reported the existence of a quasi-elastic regime of rebounds: a drop may bounce hundreds of times, constantly coming back to the same (millimetric) height. Then, it is observed that there is a strong correlation between the oscillations of the drop and the rebound: the sequence of oscillations exactly fits with the drop trajectory, so that time reversibility is obeyed. The effect of the drop oscillations at the first impact is experimentally addressed, and it suggests that a bouncing drop maximizes its elasticity by adjusting its flying time to its oscillations. A successively bouncing drop reaches a resonant state where the energy loss is minimized.
As mentioned in the past, the bouncing of a drop is very similar to its oscillation. In this frame, one can notice the strong similitude between the behaviour of a drop on an oscillating plate and a Leidenfrost drop (Yoshiyasu, 1996). If the frequency and the amplitude are adequate, a small drop on a non wetting oscillating plate bounces periodically. If the size of the drop is increased (puddles), the drop remains stuck to the plate because of gravity (as for large Leidenfrost drop impact) and oscillates, experiencing star shapes (spontaneously oscillating Leidenfrost stars may also be observed (Strani, 1983; Strier, 2000)). We proposed in some (favourable) cases qualitative explanations for the observed phenomena, but we have the feeling that many questions remain, often related to energy conservation (or loss) in these systems: it might be worth describing how viscosity modifies our picture; it would also be very useful to understand if the quasi-elastic regime is specific of the Leidenfrost situation: would a drop in zero-wetting bounce similarly on a cold solid (i.e. without any evaporation)?

Owing to a residual adhesion, it should be very difficult to answer this question experimentally. Numerical simulations, well-adapted to these slippery states of water where the absence of any contact lines simplifies the approach (Renardy, 2003) might be there very useful. They would also contribute to show how the frequency locking is found by the system. The same kind of approach would be very precious to confirm and make more quantitative our observation on the dependence of the restitution coefficient as a function of the oscillation phase of the impacting drop.

References


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