Diamond Patterns in the Cellular Front of an Overdriven Detonation

P. Clavin and B. Denet
Institut de Recherche sur les Phénomènes Hors Equilibre, 49 rue Joliot Curie, BP 146, 13384 Marseille Cedex 13, France
(Received 14 June 2001; published 10 January 2002)

A nonlinear integral-differential equation describing the cellular front of an overdriven detonation is obtained by an analysis carried out in the neighborhood of the instability threshold. The analysis reveals both an unusual mean streaming motion, resulting from the rotational part of the oscillatory flow, and pressure bursts generated by the crossover of cusps representative of Mach stems propagating on the detonation front. A numerical study of the nonlinear equation exhibits the "diamond" patterns observed in experiments. An overall physical understanding is provided.

DOI: 10.1103/PhysRevLett.88.044502 PACS numbers: 47.54.+r, 47.20.Ky, 47.40.–x, 47.70.Fw

Gaseous detonations are supersonic combustion waves that have been extensively studied since their discovery 120 years ago. Experiments of the late 1950s [1] have established that the spatiotemporal structures of gaseous detonations differ notably from the dissipative structures of other nonequilibrium systems such as flames, crystal growth, Rayleigh-Benard convection, etc. Molecular transport may be neglected and detonations are governed by the Euler reactive equations (hyperbolic problem). One striking observation is the formation of Mach stems with triple point configurations traveling transverse to the front at nearly the sound speed in the burned gas. Their trajectories, as recorded on soot-coated foils, have a characteristic "diamond" or "fish scale" pattern [2] with cell sizes much larger than the thickness of the planar wave $\bar{d}$. The cell sizes are usually obtained from experimental data and are used in an empirical way for predicting critical conditions of detonation initiation with application to nuclear reactor safety [3]. The role of detonation cellular structure in astrophysics may also have an impact on the observed spectra of type Ia supernovae [4]. A challenging problem is to relate this pattern to the properties of the reactive mixture. Despite the advances in direct numerical simulations [5] and in nonlinear studies [6], the problem is still unresolved. Physical insights are elusive and there is no convincing explanation of either the size or the shape of the pattern. In order to provide an overall physical understanding, our approach in this Letter is to carry out a nonlinear study in the neighborhood of the instability threshold. Formation of cusps (representative of triple points) and pattern selection are addressed by an analysis of weakly unstable detonations, valid for general chemistry. The analysis reveals a new kind of front dynamics involving a mean streaming motion and time dependent patterns similar to those observed in experiments.

A one-dimensional detonation consists of an inert shock followed by a reacting flow which is subsonic (relative to the shock) with a local Mach number $\bar{M}$ ($\bar{M} \leq 1$) increasing with the distance from the shock. The flow becomes sonic at the end of the reaction zone of a self-sustained detonation (Chapman-Jouguet, C-J wave) while it remains subsonic when the wave is piston supported (overdriven waves), $\bar{M}_B < \bar{M}_{BCJ} = 1$ and $\bar{M}_u > \bar{M}_{uCJ}$ where $\bar{M}_u = \bar{u}_u/\bar{a}_u > 1$ is the propagation velocity reduced by the speed of sound in the initial mixture and subscripts u, B, and CJ identify conditions ahead of the shock (fresh mixture), in the burned gases and in a C-J wave, respectively. As the speed of the piston increases, $\bar{M}_u$ becomes larger and $\bar{M}_B$ gets smaller. The degree of overdrive is defined as $f = (\bar{M}_u/\bar{M}_{uCJ})^2$, $f \geq f_{CJ} = 1$, where $\bar{M}_{uCJ}$ is characteristic of the initial mixture [2]. In a perfect gas, three parameters characterize the propagation regimes, the ratio of specific heats $\gamma \equiv C_p/C_v$, $\bar{M}_u$ (or f), and the dimensionless heat release $q = \hat{Q}/C_p\bar{T}_N$ (reduced for convenience by the enthalpy of the compressed gas at the von Neumann spike, just downstream the inert shock, subscript N). Most of the observed detonations are strongly unstable with a moderate degree of overdrive ($f \approx 1$) and a strong shock ($\bar{M}_u^2 \gg 1$). The large density jump across the strong shock produces a large deflection of the streamlines which is essential to the dynamics of real detonations. With the heat release being small at the instability threshold [7], $q < 1$, the shock of a weakly unstable (or stable) C-J wave is weak, $\bar{M}_{uCJ}^2 \approx 1$. The dynamics of a weakly unstable detonation is closer to a real one when the leading shock is strong. This is the case only when $f \gg 1$. A nonlinear analysis of such weakly unstable and strongly overdriven regimes is presented in this Letter using the approximations $(\gamma - 1) \ll 1$ and $\bar{M}_u^2 \gg 1$. Weak instability occurs when q is as small as $(\gamma - 1)$ and $1/\bar{M}_u^2$. The perturbation analysis is then based on a small parameter $\varepsilon \equiv \bar{M}_N$, $\bar{M}_N^2 \approx (\gamma - 1)/2 + 1/\bar{M}_u^2$, typically $\gamma \approx 1.2$–1.3, $\bar{M}_u^2 \approx 30$ [2].

Defining the position of the wrinkled shock as $\hat{x} = \hat{\alpha}(\hat{t}, \underline{\hat{y}})$ with $\hat{t}$ denoting the time, $\hat{x}$ and $\underline{\hat{y}}$ the longitudinal and the transverse coordinates, one introduces the nondimensional coordinates $\xi = \int_{\hat{\alpha}}^{\hat{x}} \hat{\rho}(x', t)\, dx'/(\bar{\rho}_N \bar{d})$ (the shocked gases are in the region $\xi > 0$), $\underline{\eta} = \varepsilon \underline{\hat{y}}/\bar{d}$, $\tau = \hat{t}/\bar{\tau}_N$ where $\bar{\tau}_N \equiv \bar{d}/\bar{u}_N$ is the transit time of a fluid particle. Nondimensional variables are $u = \hat{u}/\bar{u}_N$, $\underline{v} = \varepsilon \underline{\hat{v}}/\bar{u}_N$, $p = \hat{p}/\bar{p}_N$, $T = \hat{T}/\bar{T}_N$, $\alpha = \hat{\alpha}/\bar{d}$, with $\hat{u}$, $\underline{\hat{v}}$, $\hat{p}$, $\hat{\rho}$, $\hat{T}$ denoting the longitudinal and transverse velocity, the pressure, the density, and the temperature (overbars are for the unperturbed state). The scaling of $\underline{\hat{y}}$ is controlled by the range of unstable disturbances, $\hat{\lambda} \approx \bar{d}/\varepsilon$ as explained later, and the scaling of $\hat{\underline{\nu}}$ results from the large jump of flow velocity across the wrinkled shock, $\bar{\rho}_N/\bar{\rho}_u \approx 1/\varepsilon^2$. The flow of the gas behind the leading shock satisfies the ideal gas law, the Euler equations, and the equations for conservation of mass, energy, and species. The boundary conditions are the Rankine-Hugoniot conditions at $\xi = 0_+$ and a boundedness condition at $\xi \to +\infty$ (the piston is assumed at infinity). The analysis is carried out at the second order approximation retaining terms like $\delta\alpha$, $\varepsilon^2\delta\alpha$, $(\delta\alpha)^2$ and neglecting higher order terms. The mathematical formulation is detailed elsewhere [8].

In the burned gas (where the reaction is complete and the unperturbed solution is uniform) the linear approximation of the flow is a superposition of an incompressible entropy-vorticity wave (superscript i) and an acoustic wave (superscript a), $\delta u = \delta u^{(i)} + \delta u^{(a)}$, $\delta\underline{\nu} = \delta\underline{\nu}^{(i)} + \delta\underline{\nu}^{(a)}$, $\delta p = \delta p^{(a)}$, $\delta p^{(i)} = 0$ where δb denote a small disturbance, $b = \bar{b} + \delta b$. Near the onset of the instability, $q = 0(\varepsilon^2)$, two main simplifications appear as follows.

(i) The flow differs only weakly from the one encountered in a strong inert shock ($q \ll 1$, $\bar{M}_u^2 \gg 1$).

(ii) Mach numbers are small everywhere across the detonation structure, $\bar{M}_N \leq \bar{M} \ll 1$.

Isobaric gas expansion due to heat release and compressibility effects may then be clearly separated. The leading order solution is that of an inert shock problem [9] and is controlled by the vorticity wave (oscillatory shear flow) $\delta u_0^{(i)} = \partial\alpha(\tau - \xi, \underline{\eta})/\partial\tau$, $\delta\underline{\nu}_0^{(i)} = \underline{\nabla}\alpha(\tau - \xi, \underline{\eta})$ with $\partial^2\alpha/\partial\tau^2 - \nabla^2\alpha = \underline{0}$. Pressure fluctuations at the shock are small, $\delta p(\xi = 0) \approx -2\varepsilon^2\dot{\alpha}_\tau$, and pressure gradients are even smaller, so that $(\delta u^{(a)}, \delta\underline{\nu}^{(a)})$ is smaller than $(\delta u^{(i)}, \delta\underline{\nu}^{(i)})$ by a factor ε^2. By introducing the splitting $u - \bar{u} = U + \delta u^{(a)}$, $\underline{\nu} = \underline{V} + \delta\underline{\nu}^{(a)}$, $p - \bar{p} = P + \delta p^{(a)}$ and subtracting out the acoustics, mass conservation and the entropy equation lead to the isobaric approximation of a low Mach number flow, the divergence of $(U, \underline{V})$ is balanced by the rate of gas expansion due to heat release qw,

$$d(U - \underline{V} \cdot \underline{\nabla}\alpha)/d\xi + \bar{u}\underline{\nabla} \cdot \underline{V} \approx q\delta w. \quad (1)$$

This equation is valid up to order $\varepsilon^2\delta\alpha$ and $(\delta\alpha)^2$. P and the nonlinear terms of qw are of a higher order and have been neglected in (1). Compressional heating is also negligible so that the flow of the entropy-vorticity wave $(\delta u_0^{(i)}, \delta\underline{\nu}_0^{(i)})$ and the temperature fluctuations at the shock are the only mechanisms perturbing the distribution of heat release rate. $\delta w(\xi, \tau, \underline{\eta})$ may then be computed in terms of $\delta\alpha(\tau, \underline{\eta})$ from the thermal and species balance, independently of P and U and without restriction to a specific chemical kinetics [10]. In the linear approximation and at a second order approximation, the transverse component of the nonacoustic part of the flow is given by the vorticity wave, $\delta\underline{V} = \delta\underline{\nu}^{(i)}$. δU is then obtained from an integration of (1) with respect to ξ from the shock $\xi = 0$ to any point $\xi > 0$. Matching δU with $\delta u^{(i)}$ in the burned gas requires that the sum of the terms in δU that are not varying with ξ when the reaction is complete ($\xi \gg 1$) must vanish. This leads to a compatibility condition yielding an evolution equation for the perturbed front $\delta\alpha$. The final result takes the form of an integral equation (2) involving two kernels $\bar{w}(\xi)$, $\int_0^{+\infty} \bar{w}(\xi')\,d\xi' = 1$, and $\bar{A}(\xi)$, $\int_0^{+\infty} \bar{A}(\xi')\,d\xi' = 0$, coming from δw. The function $\bar{w}(\xi)$ is the heat release rate distribution of the unperturbed wave and $\bar{A}(\xi) \equiv \varepsilon^2\bar{u}_u\delta\bar{w}(\xi)/\delta\bar{u}_u$ characterizes its sensitivity to the propagation velocity $\bar{u}_u$ [max $|\bar{A}(\xi)|$ is of order unity, $\bar{A}(\xi) = \beta(\gamma - 1)\bar{w}'_\theta(\xi)$ in the notations of [10]].

$$\frac{\partial^2\alpha}{\partial\tau^2} - c^2\nabla^2\alpha + N(\alpha) = -2(\varepsilon\sqrt{q})\frac{\partial}{\partial\tau}L(\alpha) + q\left(\frac{\partial^2}{\partial\tau^2}\int_0^{+\infty}\bar{A}(\xi)\alpha(\tau - \xi, \underline{\eta})\,d\xi + \nabla^2\int_0^{+\infty}\bar{B}(\xi)\alpha(\tau - \xi, \underline{\eta})\,d\xi\right), \quad (2)$$

$N = 0$ in the linear approximation, $c^2 \equiv 1 + 3(\gamma - 1)/2$, and $\bar{B}(\xi) \equiv \bar{w}(\xi) + \partial(\xi\bar{w})/\partial\xi$. Details are presented elsewhere [8]. The linear modes of shock waves [9] are recovered when $q = 0$. The instability of galloping detonations [11] is recovered in the planar case when $\bar{A} \neq 0$, while the instability to transverse disturbances may well occur when $\bar{A} = 0$, $\bar{B} \neq 0$. The transit time of the acoustic waves in both directions being shorter than $\bar{\tau}_N$ by a factor ε^2, the evolution time scale is governed by the vorticity wave, $\bar{\tau}_N$. The delay in the integral terms is introduced by the propagation of the entropy-vorticity wave. These integral terms of amplitude q are quasi-isobaric volume sources produced by the fluctuations of heat release rate. A resonance with the oscillatory modes of the leading shock, $\hat{\omega} \approx 2\pi\bar{a}_N/\hat{\lambda}$ [9], leads to an oscillatory instability of the shock-reaction complex with a frequency $\hat{\omega} \approx 1/\bar{t}_N$ and a range of most unstable wavelengths $\hat{\lambda} \approx \bar{d}/\varepsilon$, as it was anticipated in the scaling of $\underline{\eta}$. The growth rate σ of this isobaric instability is smaller than $\hat{\omega}$ by a factor q and is still positive for disturbances with small wavelengths $\hat{\lambda} \leq \bar{d}$, σ decreasing to zero with $\hat{\lambda}$. Compressibility effects are stabilizing. The acoustic waves propagating in the burned gases produce a negative feedback through a velocity coupling $\delta u^{(a)}(\xi = 0)$ taken into account by the boundary condition of δU solution of (1). As indicated by the presence of the Mach number ε, the first term in the right-hand-side of (2) (of amplitude $\varepsilon\sqrt{q}$) describes the stabilizing mechanism of sound waves. The linear operator $L(\cdot)$ is defined in the Fourier space, $\alpha(\tau, \underline{\eta}) = \sum\tilde{\alpha}(\tau, k)\exp(i\underline{k} \cdot \underline{\eta})$, as $\tilde{L}(\tilde{\alpha}) \equiv l(k)\tilde{\alpha}$ where $l(k)$ is a fairly complicated function of k satisfying $\mathrm{Re}[l(k)] > 0$ and $\lim_{k\to\infty} l(k) = k$, $k \equiv |\underline{k}|$ [8]. A Landau-Hopf (Poincaré-Andronov) bifurcation occurs by increasing $\sqrt{q}/\varepsilon$. In unstable situations, the acoustic effects may be neglected except at small wavelengths where the integral terms in (2) vanish, and $L(\alpha)$ is reduced to a damping rate proportional to k, $\tilde{L}(\tilde{\alpha}) \approx k\tilde{\alpha}$. An example of $\mathrm{Re}(\sigma(k))$ obtained from (2) in an unstable case is plotted in Fig. 1.

The nonlinear study proceeds in a similar way but requires one to investigate the pressure term $P(\tau, \xi, \underline{\eta})$ and to solve the Euler equations. Anticipating that the nonlinear equation takes the same form as (2) with a quadratic term $N(\alpha)$ added and recalling $q = 0(\varepsilon^2)$, the final amplitude of the wrinkles is found to be of order ε^2. At a second order approximation ε^4, the Reynolds stresses of the vorticity wave, $(\delta u_0^{(i)} \partial/\partial\xi + \delta\underline{v}_0^{(i)} \cdot \underline{\nabla})(\delta u_0^{(i)}, \delta\underline{v}_0^{(i)})$, have to be retained in the Euler equations, as well as $\delta\underline{v}_0^{(i)} \cdot \underline{\nabla}\delta\alpha$ in (1). Nonlinear contributions involving acoustics or qw introduce higher order terms. A Poisson equation for $P(\tau, \xi, \underline{\eta})$ is obtained in the burned gas $\xi \gg 1$,

$$(1/\varepsilon^2)\partial^2 P/\partial\xi^2 + \nabla^2 P = (\nabla^2 - \partial^2/\partial\xi^2) g(\tau - \xi, \underline{\eta}), \tag{3}$$

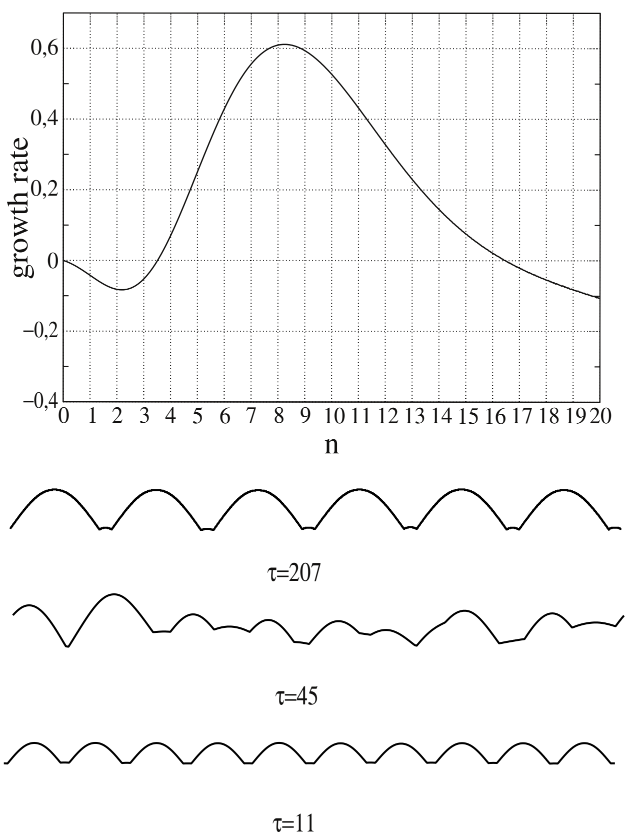

FIG. 1. Detonation with Arrhenius kinetics and an activation energy $E/R\bar{T}_N = 6$, $\varepsilon = 0.1$, $\gamma = 1.2$, $q = 0.25$, $2\pi/L = 0.5$. Top: Linear growth rate versus n, $k = 2\pi n/L$. Bottom: 2D solutions of (2) and (4) at three different times. The pattern left by the cusps for one period of a pulsating cell has the same diamond shape as in Fig. 2.

with $2g(\tau, \underline{\eta}) \equiv (\partial\alpha/\partial\tau)^2 - |\underline{\nabla}\alpha|^2$. In two-dimensional geometry and for simple waves traveling on the shock fronts, one has at the leading order $\partial\alpha/\partial\tau = \pm\partial\alpha/\partial\eta$ and the nonlinear term of pressure source vanishes in the burned gases, $g = 0$. The pressure field is then obtained for any ξ from the Euler equations by using the expression of U resulting from an integration with respect to ξ of (1). At the leading order, the solution of P written in Fourier space is $\tilde{P}(\tau, \xi, k) = \varepsilon\tilde{E}(\tau, k)\exp(-\varepsilon k\xi)/k$ where $E(\tau, \underline{\eta}) \equiv N(\alpha) - \partial[(\partial\alpha/\partial\eta)^2]/\partial\tau$ and where a boundedness condition at $\xi \rightarrow +\infty$ has been used. The final solution is obtained by requiring that the boundary conditions at the shock ($\xi = 0$) are fulfilled and by noticing that they do not contain terms of order $\varepsilon|\nabla\alpha|^2$. This leads to the compatibility condition $E = 0$ yielding

$$N(\alpha) = \partial[(\partial\alpha/\partial\eta)^2]/\partial\tau. \tag{4}$$

The same nonlinear term appears also in [12] for a wave approximated by a discontinuity unaffected by its internal structure and in conditions of sound radiation. But this approximation cannot describe gaseous detonations whose inner structure is unstable. Formation of cusps is easily understood from (2) and (4). When the integral terms in (2) are disregarded, the evolution on a slow time scale, $t \equiv \varepsilon^2\tau$, is given by an equation of the form $\varepsilon^2\partial\alpha/\partial t + (\partial\alpha/\partial\eta)^2/2 \approx -\varepsilon\sqrt{q}\,L(\alpha)$ with the same nonlinear mechanism of wave breaking as in the Burgers equation but with a different linear damping term to overcome the wave breaking at small wavelengths: the diffusion term $-k^2\tilde{\alpha}$ is replaced by $-k\tilde{\alpha}$. This leads to formation of cusps (abrupt change of $\partial\alpha/\partial\eta$) that are stiffer than in the solution of the Burgers equation. According to the wave equation (2), these cusps travel in transverse directions at nearly the sound speed in the burned gas. They are fed by a nonlinear transfer $N(\alpha)$ from large wavelengths where the linear instability due to the integral terms in (2) develops.

Numerical solutions of (2) and (4) in two-dimensional geometry with boundary conditions periodic in η exhibit periodic solutions in τ. The period of the pulsating cells is of order unity and the pattern left behind the cusps looks similar to those observed in experiments [2]; see Figs. 1 and 2. In small boxes the relaxation time is of order unity (same order as the period of oscillation). In large boxes with many linearly unstable modes, the final periodic solution is obtained after a much longer transient time and presents a cell size between the most amplified wavelength and the largest marginal wavelength; see Fig. 1. The case presented corresponds to a box width $L = 4\pi$ with 13 unstable modes; the selected wavelength corresponds to a wave number $n = 6$, $k_n = 2\pi n/L$, while the band of linearly unstable modes ranges from $n = 4$ to $n = 16$. A nonlinear selection mechanism is identified when starting the computation using a sinusoidal perturbation of small amplitude with 10 wavelengths plus a much smaller level of noise. After a few periods of oscillations, well ordered

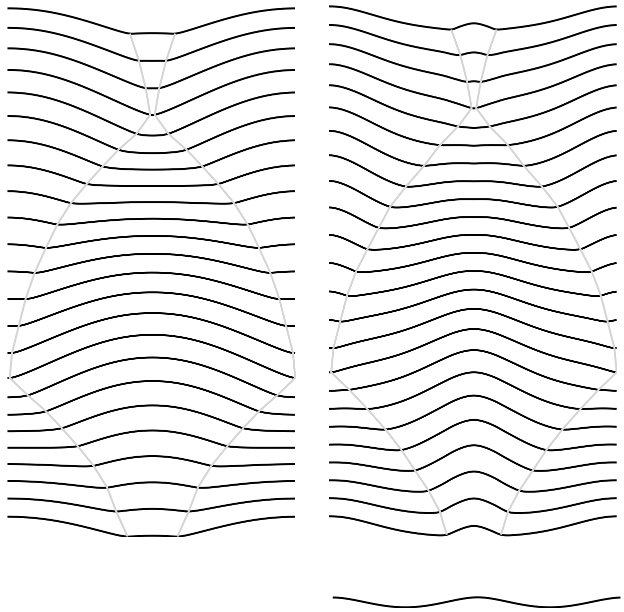

FIG. 2. Two-dimensional fronts (solid lines) propagating from bottom to top with the kinetics of Fig. 1, $E/R\bar{T}_N = 6$, and with $\varepsilon = 0.25$, $\gamma = 1.2$, $q = 0.2$, $2\pi/L = 3$; the gray lines are the trajectories of the cusps. Left: Solution of (2) and (4) for one period. Right: $\bar\alpha + \alpha'$ for one period; $\bar\alpha$ is shown at the bottom.

and developed pulsating cells with a small size, $n = 10$, are first observed followed by a chaotic regime in which the number of cells decreases down to the final stable state of 6 pulsating cells. The nondimensional relaxation time of this process is $\tau \approx 100$; see Fig. 1. This cell number reduction is also observed in direct numerical simulations [5].

However, the solutions in Fig. 1 with pulsating cells and crossover of cusps, are not simple waves as assumed in the derivation of (4). We next show that the size of the cells is accurately predicted by (4) but both the shape of the detonation front and the flow require a more elaborate treatment. Because of second order effects, neither the fluctuations of the flow nor the fluctuations of the wrinkled front average to zero but rather a net steady flow and a net steady wrinkled front are generated. These steady solutions must be taken into account at the leading order of the nonlinear analysis. Mean streaming flows have been known for many years in acoustics (acoustic streaming) [13]. The situation is somehow different here; the mean streaming flow is produced by the oscillatory rotational flows resulting from the unstable front. The nonlinear source term $g(\tau, \underline{\eta})$ is now nonzero and reaches its maximum during the crossover of two cusps, yielding pressure bursts of order $\varepsilon^2|\delta\alpha|^2$ that propagate in the burned gas with the rotational part of the flow. For a periodic solution, the time average $\bar g$ is also different from zero, $g = \bar g(\underline{\eta}) + g'(\tau, \underline{\eta})$, $\bar g(\underline{\eta}) \neq 0$, $\bar{g'} = 0$, generating a mean flow, $(\bar U(\xi, \underline{\eta}), \underline{\bar V}(\xi, \underline{\eta}), \bar P(\xi, \underline{\eta}))$, $U = \bar U + U'$, $\underline{V} = \underline{\bar V} + \underline{V'}$, $P = \bar P + P'$, $\tilde{\bar P}(\xi, k) \approx \tilde{\bar g}(k)[1 - \exp(-\varepsilon k \xi)]$. The shock conditions $\bar U(\xi = 0, \underline{\eta})$ and $\underline{\bar V}(\xi = 0, \underline{\eta})$ then require the existence of a nonzero mean of the wrinkled front, $\alpha(\tau, \underline{\eta}) = a\tau + \bar\alpha(\underline{\eta}) + \alpha'(\tau, \underline{\eta})$ with $\tilde{\bar\alpha}(k) \approx -\tilde{\bar g}(k)/\varepsilon k$ and $\tilde{\bar g}(k = 0) = 0$; see Fig. 2. The unsteady pressure term $\tilde P'(\tau, \xi, k)$ is computed in a similar way as before, yielding $\tilde P'(\tau, \xi, k) = \varepsilon \tilde E'(\tau, k)\exp(-\varepsilon k \xi)/k$, and the shock condition implies that $E'(\tau, \underline{\eta}) = 0$. This compatibility condition leads to the same equation as (2) for α' but now with $N(\alpha') = (1/2)\partial(\dot\alpha'^2_\tau + |\underline{\nabla}\alpha'|^2)/\partial\tau - \nabla^2 \int^\tau g'(\tau')\,d\tau'$ where the time average of the integral is zero by definition. This term involves the Gaussian curvature of the front. A more convenient form is obtained by using the leading order of the wave equation, $\partial^2\alpha'/\partial\tau^2 - \nabla^2\alpha' \approx 0$, to give

$$N(\alpha') = \partial(\dot\alpha'^2_\tau + |\underline{\nabla}\alpha'|^2)/\partial\tau - \nabla^2 \int^\tau [\dot\alpha'^2_{\tau'} - \overline{\dot\alpha'^2_{\tau'}}]\,d\tau',$$

with a zero time average of the integral. At the second order approximation, the quantity $z \equiv \alpha' + \int^\tau[(\partial\alpha'/\partial\tau')^2 - \overline{(\partial\alpha'/\partial\tau')^2}]\,d\tau'$ is a solution of (2) and (4), $N(z) = \partial|\underline{\nabla} z|^2/\partial\tau$, and the time dependent wrinkles of the front α' are obtained at the leading order from $z(\tau, \underline{\eta})$ by using $\partial\alpha'/\partial\tau \approx \partial z/\partial\tau - [(\partial z/\partial\tau)^2 - \overline{(\partial z/\partial\tau)^2}]$, valid for $|\partial z/\partial\tau| \ll 1/4$. Typical solutions $\bar\alpha(\underline{\eta})$ and $\bar\alpha(\underline{\eta}) + \alpha'(\tau, \underline{\eta})$ are shown in Fig. 2 by comparison with $z(\tau, \underline{\eta})$.

[1] Y. Denisov and Y. Troshin, Dokl. Akad. Nauk. SSSR **125**, 110 (1959).

[2] W. Fickett and W. Davies, *Detonation* (University of California Press, Berkeley, 1979); F. A. Williams, *Combustion Theory* (The Benjamin/Cummings Publishing Company Inc., Menlo Park, 1985).

[3] J. Lee, Adv. Heat Transf. **29**, 59 (1997).

[4] V. Gamezo, J. Wheeler, A. Khokhlov, and E. Oran, Astrophys. J. **512**, 827 (1999).

[5] V. Gamezo, D. Desbordes, and E. Oran, Combust. Flame **116**, 154 (1998); A. Bourlioux and A. Majda, Combust. Flame **90**, 211 (1992).

[6] A. A. Borissov, *Dynamic Structure of Detonation in Gaseous and Dispersed Media* (Kluwer, Dordrecht, 1991); J. Yao and D. S. Stewart, J. Fluid Mech. **309**, 225 (1996).

[7] J. J. Erpenbeck, Phys. Fluids **7**, 684 (1964).

[8] P. Clavin and L. He, C.R. Acad. Sci. Paris Série IIb **329**, 463 (2001); P. Clavin and B. Denet, C.R. Acad. Sci. Paris Série IIb **329**, 489 (2001).

[9] S. P. D'Yakov, Zh. Eksp. Teor. Fiz. **27**, 288 (1954); V. M. Kontorovich, Zh. Eksp. Teor. Fiz. **33**, 1525 (1957).

[10] P. Clavin, L. He, and F. A. Williams, Phys. Fluids **9**, 3764 (1997).

[11] P. Clavin and L. He, Phys. Rev. E **53**, 4778 (1996); J. Fluid Mech. **306**, 353 (1996).

[12] A. Majda and R. Rosales, SIAM J. Appl. Math. **43**, 1310 (1983).

[13] J. Lighthill, *Waves in Fluids* (Cambridge University Press, Cambridge, 1978).