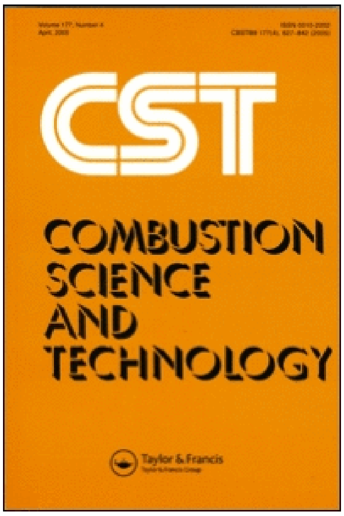

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Laminar Premixed Flame Dynamics: A Comparison of Model and Complete Equations

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Laminar Premixed Flame Dynamics: A Comparison of Model and Complete Equations

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ABSTRACT—Instabilities occurring in downwards propagating flames (in plane geometries) are studied in two different models: the complete equations of isobaric flames with one global exothermic reaction and a model equation: a modified version of Michelson–Sivashinsky equation (including gravity effects). Results obtained in both models are compared, showing a good qualitative agreement for a gravity not too small.

Key Words: Darrieus–Landau instability, Michelson Sivashinsky equation, downwards propagating flames.

1 INTRODUCTION

In this work, we will be interested in downwards propagating and zero gravity premixed flames in tubes, in a plane geometry. These flames represent a classical problem of flame stability. It is well-known that for very low flame velocities, or equivalently for a large value of the gravity (the relevant parameter is the Froude number $(u_l)^2/gd$, where u_l is the flame speed, d the flame thickness and g the gravity), the flame obtained is plane. With increasing flame velocities, stable cellular flames, are obtained.

The linear instability of plane flames has been theoretically explained in different papers (Pelcé and Clavin (1982), Matalon and Matkowsky (1982), Frankel and Sivashinsky (1983)). In these works, the mechanism responsible for this transition is the hydrodynamical Darrieus–Landau instability (see Darrieus (1938) and Landau (1944)): because of gas expansion, deflection of streamlines through the flame is produced. This has a destabilizing influence, giving rise, in the absence of other effects, to a positive growth rate proportional to the wave vector. In these studies, diffusive effects tend to stabilize the plane flame.

Another possible explanation for the instability exists. i.e., the thermal-diffusive instability, that occurs at very low Lewis number of the limitant species and is caused only by the destabilizing influence of species diffusion (Sivashinsky (1977a)). There is some controversy between the two explanations, and the aforementioned theoretical results (see e.g., Pelcé and Clavin (1982)) show that the thermal-diffusive instability, although possible, does not seem to be a very plausible mechanism for the instability in usual conditions (i.e., the Markstein length remains positive), except in cases of very low Lewis numbers such as lean hydrogen oxygen flames. However, it is possible to argue that some effects neglected in the theoretical analysis, for instance a sufficient amount of radiative heat losses, could favour he thermal-diffusive mechanism and induce a negative Markstein length. It is also quite possible that in geometries others than the plane one (i.e., Bunsen burner flames or spherically expanding flames), the appearance of cells could be controlled by the thermal-diffusive mechanism. The situation is not yet completely clear, however it seems that the hydrodynamical mechanism plays an important role in many cases.

We will be interested here only in flames where this mechanism is dominant. In this case, it has been shown by Sivashinsky (1977b) that it is possible to derive a model equation, the Michelson Sivashinsky (MS) equation, from the basic equations of the problem, in the case of low gas expansion. This equation, which contains the effects of gas expansion and diffusion, is much simpler and easy to compute numerically than the original equations. It has been further shown by Sivashinsky and Clavin (1987) that the MS equation is actually valid up to the second order in gas expansion. The MS equation has even been extended in a phenomenological manner by Joulin and Cambray (1992) to yield good quantitative agreement with for instance amplitudes of flames produced by the Darrieus Landau instability.

Strictly speaking, the MS equation describes only zero gravity flames. However, it is easily extended to include gravity effects. This modified equation has been used in some papers in order to exhibit complicated secondary instabilities (i.e., instabilities occurring after the initial Landau instability): see Denet (1993a, 1993b). However a validation has still to be made in order to get convinced that these secondary instabilities appear in real situations, and are sufficiently robust to resist to variations of physical parameters. In this work, we will test the results obtained with the modified MS equation by comparing them to the results obtained with complete hydrodynamical equations, with the simplification of only an overall one-step Arrhenius reaction.

The paper is organized as follows. In Section 2 we describe the basic equations (i.e., both the MS equation and the complete hydrodynamical equations). In Section 3 we shall study the secondary instabilities of the cellular flames caused by the Landau instability for a relatively important gravity. In Section 4 we shall show a mechanism of reduction of the number of cells, which occurs when gravity is reduced. In Section 5 we shall examine the case of zero gravity flames, where analytical predictions exist in the case of the MS equation.

2 BASIC EQUATIONS

We first describe the complete equations of premixed flames in the isobaric approximation and with the simplest chemistry: a single one-step chemical reaction is assumed. Non-dimensional quantities are obtained using as units of length scale the flame thickness obtained from asymptotics, and as velocity unit the asymptotic flame speed. The use of normalized variables allows us to write the model, in a frame moving with the flame front, as follows:

$$\rho\frac{\partial T}{\partial t}+\rho(\overline{v}.\overline{\nabla})\,T=\Delta T+\Omega \tag{2.1a}$$

$$\rho\frac{\partial C}{\partial t}+\rho(\overline{v}.\overline{\nabla})\,C=\frac{1}{Le}\Delta C-\Omega \tag{2.1b}$$

with

$$\Omega = \frac{\beta^2}{2\,Le} C \frac{\rho}{1-\gamma} \exp\left(\frac{\beta(T-1)}{1+\gamma(T-1)}\right) \tag{2.2}$$

and

$$\frac{\partial(\rho)}{\partial t} + \vec{\nabla}.(\rho\vec{v}) = 0 \tag{2.3a}$$

$$\frac{\partial(\rho\vec{v})}{\partial t} + \vec{\nabla}.(\rho\vec{v}\,\vec{v}) = -\vec{\nabla}P + \rho\vec{F} + \mu\Delta\vec{v} + (\lambda + \mu/3)\vec{\nabla}(\vec{\nabla}.\vec{v}) \tag{2.3b}$$

$$\rho = \left(1 + \frac{\gamma}{1-\gamma} T\right)^{-1} \tag{2.4}$$

Where T and C correspond to the reduced temperature of the gas mixture and concentration of a reactant. (The other reactant being in excess). The boundary conditions on T and C are

$$T(x = -\infty, y) = 0, \quad T(x = +\infty, y) = 1$$

$$C(x = -\infty, y) = 1, \quad C(x = +\infty, y) = 0$$

all quantities being periodic in they y direction (the direction transverse to the flame).

$Le = D_{\text{th}}/D_{\text{mol}}$, $\beta = (E/RT_b^2)(T_b - T_u)/T_b$ and $\gamma = (T_b - T_u)/T_b$ are respectively the Lewis number of the reactant (ratio of thermal to molecular diffusivity), the reduced activation energy (or Zeldovich number) and the heat release parameter (T_b and T_u being respectively the temperature of burnt and unburnt gases, R is the constant of perfect gases). ρ, v and P are the density, velocity and pressure; F is an external specific force per unit volume (gravity for instance). μ and λ are respectively shear and bulk viscosities. Let us define U, the reduced flame speed which is an unknown of the problem. U is supposed to be parallel to the x-direction. F will also be parallel to this direction, positive and negative F corresponding respectively to downward and upward propagating flame (F being the inverse of the Froude number $(u_l)^2/gd$, where u_l is the flame speed, d the flame thickness and g the gravity).

We will limit ourselves to 2D simulations, corresponding to a 1D front separating fresh and burnt gases. Theoretical predictions for the growth rate of perturbations to the plane solution can be found in Pelcé and Clavin (1982). This analysis relies on an expansion in the vector k. Essentially, outside of a boundary layer around $k = 0$ (corresponding to a growth rate $\sigma = 0$), the growth rates can be developed in a Taylor series in the wave vector and are approximately given by a second order polynomial in k. In order to use an explicit form for the growth rates in the Michelson–Sivashinsky equation, we will consider that (see Denet (1993a))

$$\sigma(k) = 0 \text{ for } k = 0$$

$$\sigma(k) = G + |k| - \nu k^2 \text{ for } k \neq 0 \tag{2.5}$$

where this expression is valid in suitably rescaled variables. G depends on gravity and is negative for flames propagating downwards, ν depends on diffusive effects, which are stabilizing: ν is always positive; we will keep this parameter to 0.1 in all the calculations presented in the sequel.

With the form 2.5 of the dispersion relation, the modified Michelson Sivashinsky (MS) equation (with periodic boundary conditions) then reads

$$\alpha_t + \frac{1}{2}(\vec{\nabla}\alpha)^2 = I(\alpha)$$

where α represents the flame position, $I(\alpha)$ is an operator corresponding to the multiplication by $\sigma(k)$ in Fourier space. The MS equation is solved by Fourier pseudo spectral methods.

The difference between the MS equation used here and the usual one is that in the form 2.5 of $\sigma(k)$, we have taken into account the G term, which is due to gravity effects. We will perform 1D simulations (one transverse dimension) of the MS equation, which are equivalent to 2D simulations with the complete equations, (one transverse and one longitudinal dimension), the longitudinal dimension being parallel to the average flame speed, whereas the transverse one is of course perpendicular to this velocity. In the sequel, we shall vary the parameter G (or equivalently the parameter F for the complete equations) and the transverse size of the computational domain. From an experimental point of view, tuning the parameter G at constant wave vector would correspond to varying the flame speed at constant expansion ratio (which gives essentially the coefficient of the k term of the dispersion relation) and constant Markstein length (which controls the coefficient of the k^2 term). A modification of the expansion ratio or of the Markstein length, which can easily occur when dilution and equivalence ratio are varied, would correspond to an effective wave vector modification.

3 SECONDARY INSTABILITIES OF CELLULAR FLAMES

When G is varied in the MS equation, a lot of widely different sorts of flames can be obtained. For G sufficiently negative, (or F sufficiently positive: remember that the signs of these two parameters have been taken different) the plane flame is obtained. For G slightly higher the plane flame becomes unstable to the Landau instability and a cellular flame is observed. In Denet (1993a) a stability domain of this cellular flame in the plane (G, k) was defined for the MS equation, both in the 1D and 2D case.

In the case of 1D fronts, which concerns us here, the limits of stability of the cellular flame are qualitatively plotted in Figure 1. As can be seen in Figure 1, for G close to the threshold of linear Landau instability, the stability domain of the cellular flame takes the form of a parabola inside the linear instability curve. On the high k side of the instability, we have an instability that has been called cell-merging, on the low k side a tip splitting instability. For higher values of G, an oscillatory instability exists.

The cell merging and tip splitting have the role of changing the wavelength of the cellular structure: if the wavelength is too small, cells merge, whereas if the wavelength is too great, each cell of the cellular structure breaks into two new cells: this is the tip splitting case. In the case of the complete hydrodynamical equations, the cell merging instability was already exhibited in Denet (1993a). We will show here the tip splitting instability. We take as an initial solution a flame with two large cells with physical parameters $F = 0.11$ and $k = 0.17$, which correspond to a wave vector outside the

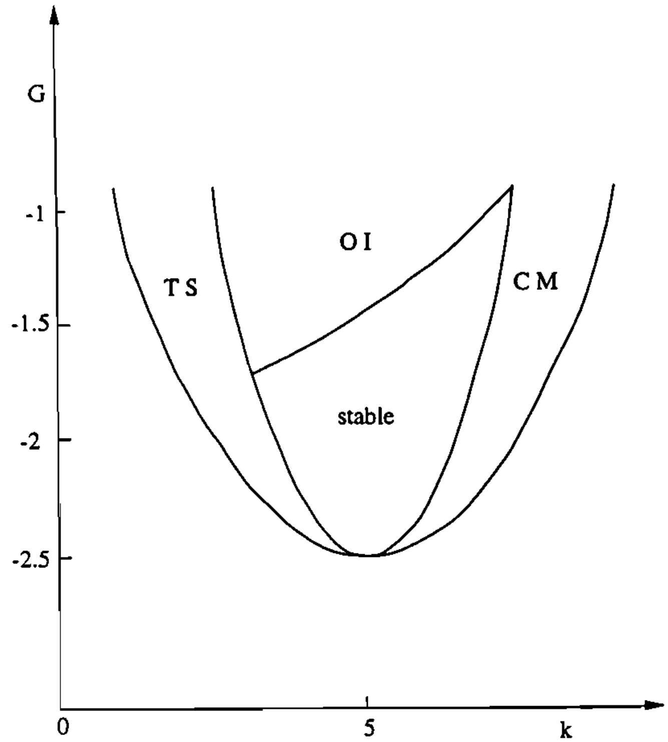

FIGURE 1 Sketch of the 1D stability domain (possible instabilities are: cell merging (CM), tip splitting (TS) oscillatory instability (OI)).

stability zone of the cellular solution. This flame is shown in Figure 2a, where five equidistant contour lines of the temperature field have been plotted (note that the scale in the longitudinal direction is not the same as in the transverse direction). Then we let the solution evolve in time; this evolution is quite fast and new cells begin to grow in the

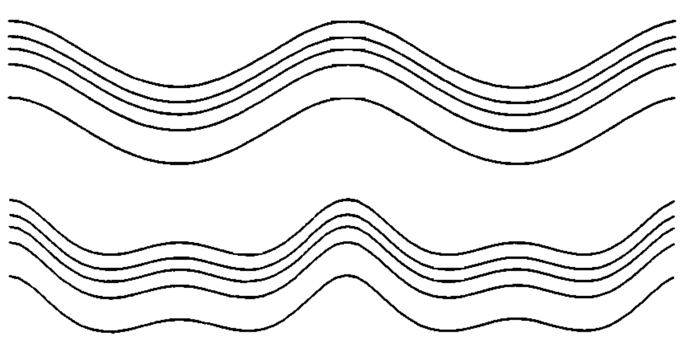

FIGURE 2(a, b) Tip splitting for an isobaric flame (the physical parameters are $\beta = 5$, $Le = 1$, $\mu = 0.7$, $F = 0.11$, $k = 0.17$ length 250 in the x direction). The flames are represented by their temperature lines for different times with a dilatation in the direction of the mean flame propagation. It can be seen that the number of cells increases from two cells (Figure 2a) to four cells (Figure 2b).

zone of the cells pointing towards fresh gases. Thus a flame with four cells is obtained, as can be seen in Figure 2b. This behaviour is exactly the same as the one that occurs in the MS equation.

But the most interesting instability of the cellular flame is the so-called oscillatory instability. This effect occurs for values of G sufficiently high (see Figure 1), or equivalently F sufficiently low, i.e., relatively far from threshold of linear Landau instability. It has been shown in Denet (1993a) that in the case of the two dimensional MS equation, which corresponds actually to a 3D flame, this instability produces self-turbulizing flames, i.e., chaotic flames with cells continuously growing and merging on the flame front. Thus this instability could be important in explaining this kind of flames which are observed in experiments (see the historical work of Markstein (1951)).

In the case of the MS equation, a one dimensional version of this instability exists, except that the solution obtained are less chaotic than in the 2D case. We will show here that this oscillatory instability can be observed in the case of the complete hydrodynamical equations. We start with a solution with four cells, and after waiting for a sufficiently long time, a subharmonic perturbation of this cellular structure appears, which can be seen in Figure 3a: the second and fourth cells have a lower amplitude than the first and the third. This subharmonic perturbation presents an oscillatory character, and it is easily seen in Figure 3b, taken at a later time, that the second and fourth cells have now a larger amplitude. Furthermore, this subharmonic perturbation does not only oscillate in time, but also grows (compare the amplitudes in Figure 3a and 3b).

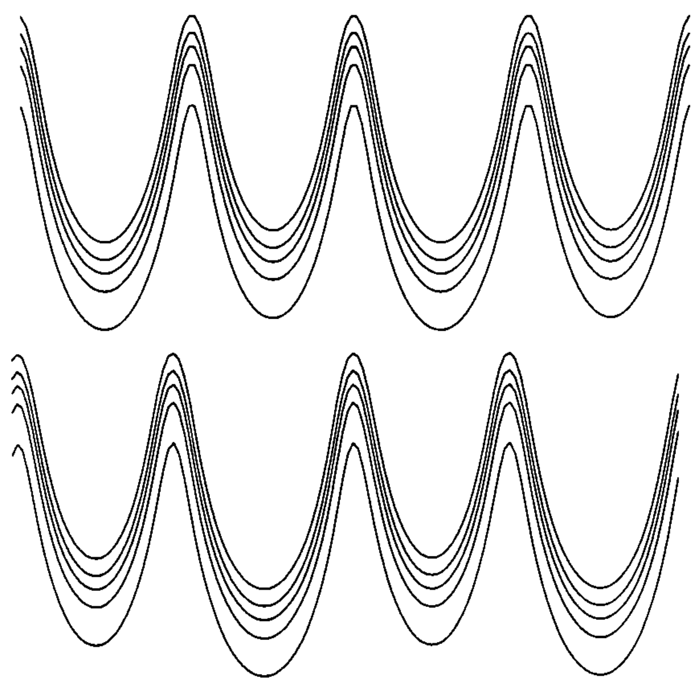

FIGURE 3(a, b) Oscillatory instability with the complete equations (physical parameters $\beta = 5$, $Le = 1$, $\mu = 0.7$, $F = 0.17$, $k = 0.17$ length 250). Subharmonic perturbation (Figure 3a) this perturbation oscillates in time (Figure 3b)

As in the case of the 1D MS equation, this instability is subcritical, and the amplitude of the subharmonic perturbation reaches a large value. However, as in the 1D MS case, this large amplitude solution decays towards a four cells solution similar to the original one, except for a translation. This solution is not stable, and is also subject to the same oscillatory instability, and the process just described occurs again. But recall that in the case of the two dimensional MS equation, there is much more disorder in the spatial structure of the solutions obtained and that a solution similar to the original one has never been obtained in this case.

Thus all the phenomenons observed in the MS equation in the case of cellular flames (i.e., cell merging, tip splitting and oscillatory instability) are again encountered with the complete equations, showing that these instabilities are robust. Actually, it has even been shown by Misbah and Valance (1993) that the Kuramoto Sivashinsky equation with the addition of gravity yields similar instabilities. It must be recalled that in the combustion context, the Kuramoto Sivashinsky equation describes a premixed flame submitted to a thermal diffusive instability (see Sivashinsky (1977b) for an analytical derivation of this equation in the case of the thermal-diffusive model). So a thermal diffusive instability damped by gravity would give similar results to those we have obtained in this paper in the case of secondary instabilities of cellular flames. We emphasize that these instabilities depend essentially on gravity and are thoroughly different from the complex instabilities encountered in the zero gravity Kuramoto Sivashinsky equation. This similarity in the non linear behaviour close to threshold of hydrodynamical and thermal diffusive instability can be explained by considering that the main difference between these two instabilities is the growth rate at very low wave number, proportional to k in the hydrodynamical case, and to k^2 in the thermal diffusive case. However, if one adds gravity, the growth rates are lowered (in the downwards propagating case) by a constant factor G; the low wavevectors growth rates near threshold become very negative and play a minor role in the dynamics. The important wave numbers are now the remaining unstable wave numbers, which are relatively similar in the two cases, i.e., a more or less parabolic shape in the neighbourhood of the wave vector with maximum growth rate. These two instabilities will be distinguished in the case of gravity less important, or equivalently higher flame speed. It will be the purpose of the forthcoming sections to study this behaviour in the hydrodynamical case.

4 CELLS NUMBER REDUCTION MECHANISM

The previous section studied the case of cellular flames with an important value of gravity. In the case of the zero gravity Michelson Sivashinsky equation, the solutions without gravity take the form of curved flames, with only one cell in the computational domain. In this section, we shall show the evolution of flames from cellular flames to curved flames with the MS and complete equations.

We shall show that a more or less regular reduction of the cells number occur. We start with a domain width where, close to Landau instability threshold, exist four cells, and we shall vary the gravity parameter (i.e., G for the MS equation and F fort the complete equations) to show that it is possible to get progressively three, two and only one cells.

The solutions we shall obtain with in general not be stationary. As it is not obvious to give a precise definition of the cells number of a solution which is not regular and periodic, we shall in the sequel call cells number the number of cusps pointing towards burnt gases which exist on the flame front, in agreement with an idea of Joulin and Cambray (private communication) on flames submitted to external noise. Others definitions of cells number could be possible, but as far as only the qualitative character of the cells number reduction mechanism is involved, we don't believe that the results will be changed.

In the previous section, we have already seen a case with the complete equations and four cells, corresponding to physical parameters $F = 0.17$ and were vector $k = 0.0425$ (corresponding to the total width and not to only the wavelength of the original pattern). We shall keep constant the size of the domain and reduce F, to see the number of cells of solutions.

We show in Figure 4 the equivalent case of a solution of the MS equation with four cells, exactly in the same case of the development of oscillatory (subharmonic) instability. The physical parameters are here $G = -1.15$ and $k = 1.25$ (corresponding to total width). Units are of course different from the complete equations, but the important point is the qualitative picture of evolution.

In Figure 5 we show a solution of the MS equation with three cells with parameters $G = -0.7$ and $k = 1.25$. An equivalent solution with three cells and the complete equations is given in Figure 6 for $F = 0.11$ and $k = 0.0425$.

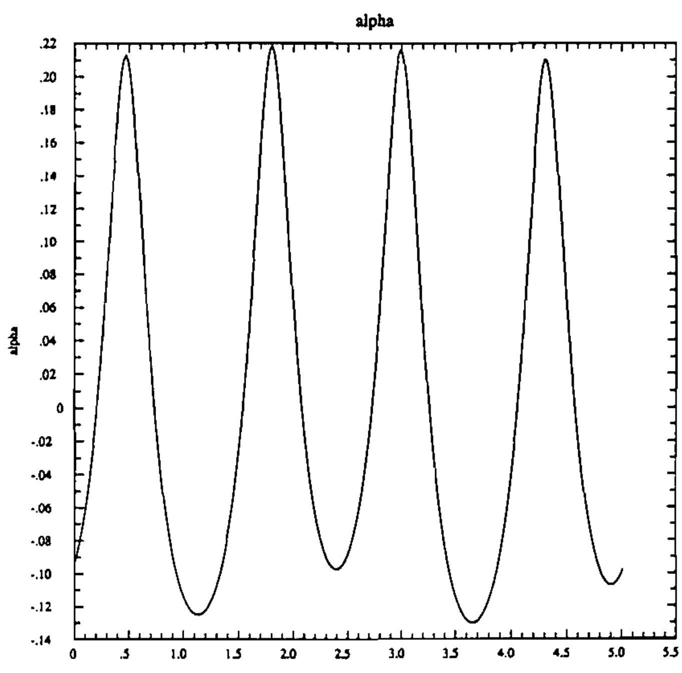

FIGURE 4 Oscillatory instability in the 1D MS equation with parameters $G = -1.15$, $\nu = 0.1$, $k = 1.25$, (corresponding to total width).

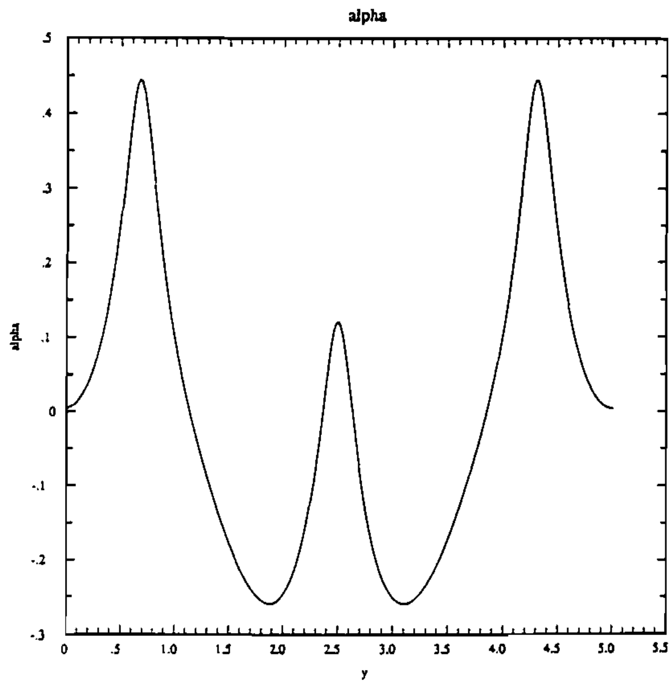

FIGURE 5 Three cells solution with the MS equation (physical parameters $G = -0.7$, $\nu = 0.1$, $k = 1.25$).

FIGURE 6 Three cells solution with the complete equation (physical parameters $\beta = 5$, $Le = 1$, $\mu = 0.7$, $F = 0.11$, $k = 0.0425$ length 250).

Reducing gravity further, we obtain two cells solutions: Figure 7, $G = -0.5$ $k = 1.25$ for the MS equation, Figure 8, $F = 0.07$, $k = 0.0425$ for the complete equations. We note that in the case presented here for the MS equation, the two cells solution is a stationary one. For some precise parameter values, it is possible to get such strange stationary solutions for the MS equation if one integrates for a sufficiently long time (the number of cells is fixed much before the final state is obtained). With the complete equations, we have not succeeded in producing stationary solutions. It is not impossible that such solutions could be found, but it would be very expensive. Furthermore, these solutions are not robust at all in the MS equation case and we have some doubt about their physical relevance in realistic situations, i.e., 3D flames with external noise.

Finally we have the case of zero G flames, in which we shall be interested in the next section, to show that there is some discrepancy between the two sets of equations we

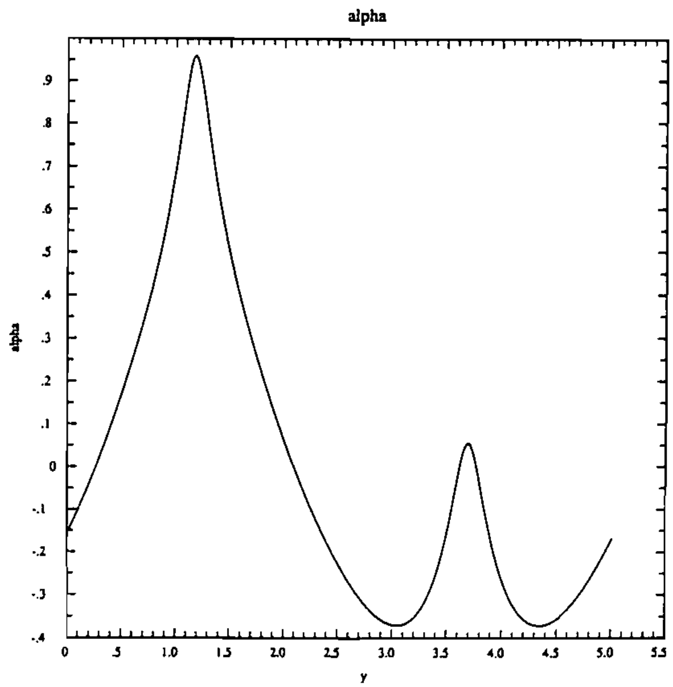

FIGURE 7 Two cells solution witht the MS equation (physical parameters $G = -0.5$, $\nu = 0.1$, $k = 1.25$).

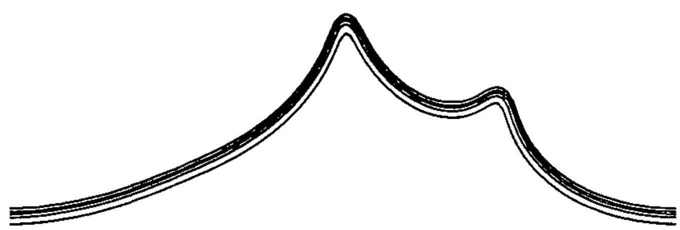

FIGURE 8 Two cells solution with the complete equations (physical parameters $\beta = 5$, $Le = 1$, $\mu = 0.7$, $F = 0.07$, $k = 0.0425$ length 250).

use. Let us simply recall for the moment that in the MS equation, the result is always a one cell stable solution, see for instance the solution with $G = 0$, $k = 1.25$ in Figure 9.

We emphasize that this cells number reduction, which should be easily seen in experiments, is important because it is a characteristic of the hydrodynamic flame instability. In the thermo-diffusive case, when gravity is reduced, the cells number stay more or less constant, however, cells become more and more chaotic. As we have seen in the previous section, secondary instabilities of cellular flames are relatively similar whatever the nature of initial instability, i.e., hydrodynamic or thermo-diffusive. So, in order to distinguish both cases, it is necessary to work farther from cellular flames. As it is difficult to work with zero gravity (microgravity experiments exist, but are expensive

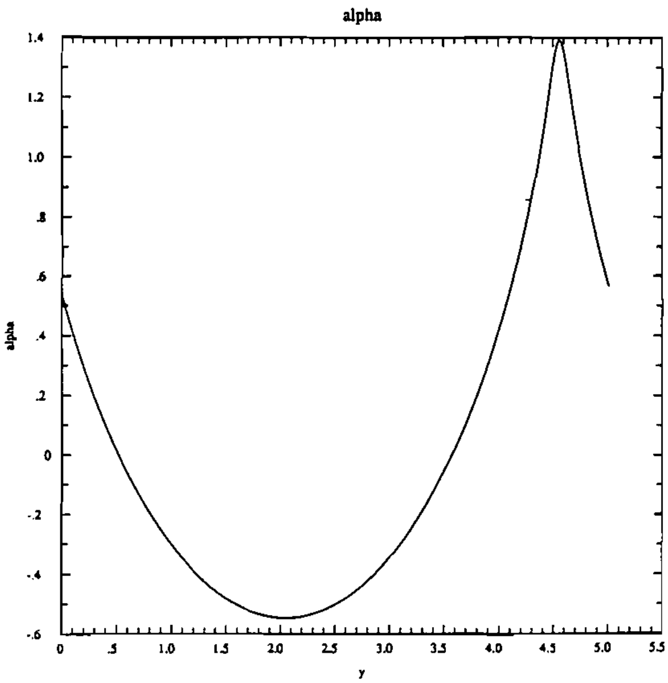

FIGURE 9 One cell solution with the MS equation (physical parameters $G = 0$, $\nu = 0.1$, $k = 1.25$).

and difficult to make), it will be in general only possible to work at higher flame speeds, i.e., smaller F, in the domain where cells number reduction appears. So perhaps the best experimental test concerning the nature of the basic linear instability is to perform the experiments in the domain of number of cells reduction. Some experiments have been made in a reduced gravity environment in Dunsky (1992), and in the cases studied, flames with a low number of big cells, like the ones obtained numerically in this section, are produced. This work supports the hydrodynamic hypothesis for the nature of the basic linear instability, however more complete studies are needed before deciding if the qualitative evolution presented here is relevant to experiments and in which cases it is.

The next section will be concerned with behaviour of flames without gravity, to compare the results with the hydrodynamical equations to the theoretical predictions made in the case of the Michelson Sivashinsky equation.

5 FLAMES AT ZERO GRAVITY

In the zero gravity case, a remarkable property of the Michelson Sivashinsky equation is that exact solutions have been analytically obtained. This is a rather exceptional case among non linear partial differential equations. The method used to get these solutions is a pole decomposition method. It was introduced by Lee and Chen (1982) in a class of partial differential equations in the context of plasma physics. Later, Thual, Frisch and Henon (1985) understood that the MS equation belonged to this class and studied the

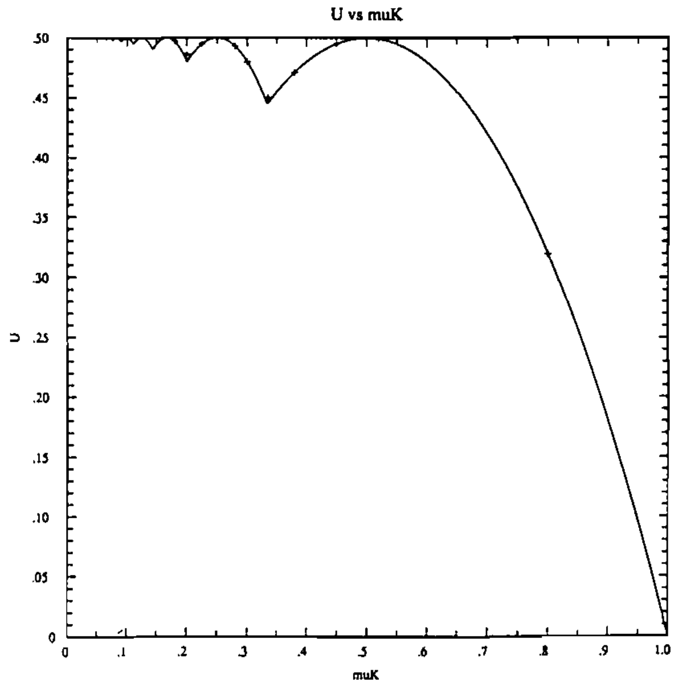

FIGURE 10 Increase in velocity vs. wave vector for the MS equation.

repartition of poles of solutions. Joulin used this method in different cases, with slightly different equations, for instance burner flames or interaction of flame with noise in the incident flow. However, we shall be concerned here with his result (Joulin (1987)) that gives precise values of increase in flame velocity depending of wave number. This curve is shown in Figure 10: the increase in velocity (equivalent to the spatial mean value of half the front slope squared) lies on different parabola (i.e., the number of poles is different) in each interval of the parameter νk of the form $[1/(2n+1), 1/(2n-1)]$. The precise values of the velocity increase inside each interval is

$$U = 2n\nu k(1 - n\nu k),$$

where n is the number of poles of the solution, k is the wavevector, and ν the coefficient of the k^2 term of the dispersion relation in the MS equation.

In Figure 10, crosses are points measured when solving numerically the MS equation: these points are actually on the theoretical curve. Another important feature of the MS equation is that the curved flames obtained are always stable, whatever the wave number. This property, that has been called "anomalous stability of curved flames", was explained in a heuristic manner in a paper of Zeldovich *et al.* (1980). In this section, we shall test these two basic results: the velocity vs. wave number and the stability of solutions, in the case of the complete equations.

In the previous sections, the predictions made with the MS equation were qualitatively similar to results with hydrodynamical equations. However, it was shown in Denet

(1993b) that the precise predictions of the zero G Michelson Sivashinsky equation were not so robust, i.e., that a small perturbation could change the stability of solutions. In this paper, the perturbation was simply the introduction of a very small gravity term in the equation: so, a very small term was sufficient to make the solutions of the MS equation unstable, in contradiction with Zeldovich's qualitative analysis. This observation can be compared to the known fact that in the zero G MS equation case, a very small noise is sufficient to produce important perturbations of the flame front form (see Cambray and Joulin (1992)). Microgravity experimental observations of Dunsky (1992) also support this idea of high sensitivity of zero G flames to small effects such as noise. In this paper, oblique flames are observed, with smaller cells that form and propagate along the oblique front. So actually, the zero G solutions seem very sensitive to different effects.

An argument which can be given concerning this sensitivity is that the success of the pole decomposition method in the zero G case is an exceptional property. The addition of a very small term in the equation is sufficient to prevent this method from working. More important than that, no one apparently knows for the moment how to construct a perturbation theory describing the behaviour of a slightly perturbed zero G MS equation. It seems that such a perturbation theory would be highly singular.

Now it is possible to discuss the relevance of the MS equation to the complete hydrodynamical equation. Let us remember that this model equation is obtained as the leading order of an expansion in powers of the gas expansion parameter $\gamma = (\rho_u - \rho_b)/\rho_b$, and that actually γ is not even a small number ($\gamma \sim 0.8$ in usual situations). So it could be that the difference between the hydrodynamical equations and the Michelson Sivashinsky equation is important for low gravities, because of the sensitivity of the solution, contrary to other cases studied in the previous sections of this paper.

We now examine the results of the simulations of the complete equations. In Figure 11, the numerical flame velocities are given vs. wave number, for two different values of length in the x direction, i.e., 50 flame thicknesses with a uniform mesh and 250 with use of a mapping to enlarge the domain (see Denet and Haldenwang 1992). The flame velocity is here defined as the opposite of the velocity of the incident flow needed to keep the flame at a constant position in the computational domain. For high values of the wave vector, we have stationary solutions and there is a good qualitative agreement with Joulin's results, i.e., the velocity lies on a parabola. However, only points on the first parabola have been obtained, because for lower wave vectors, the solutions become unstable, in contradiction with the theoretical predictions. For a longer domain, the flame is stable for lower wave vectors, however, the maximum of the velocity curve is also displaced, so that in this case, the flame becomes unstable close to the maximum.

An example of time evolution of flame velocity for a sufficiently low wavevector and in the case of the length of 250 is given in Figure 12. Relaxation oscillations are observed, so that it is not possible to define a mean velocity in a reliable way and continue the curve in Figure 11. These oscillations are in no way due to a numerical instability (a smaller time step gives similar effects), the only slight numerical problem that can be observed in Figure 12 concerns the slight undershoot observed before each peak. This undershoot is caused by the way flame speed is measured, i.e., the velocity of the incident flow is modified at each time step in order to keep the flame position at

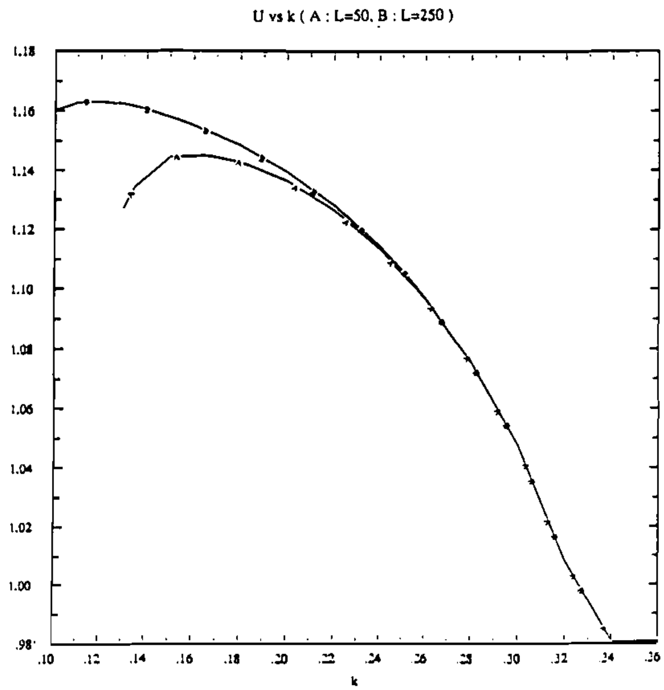

FIGURE 11 Velocity vs. wave vector for the complete equations (total length in the x direction: 50 (curve A) 250 (curve B)).

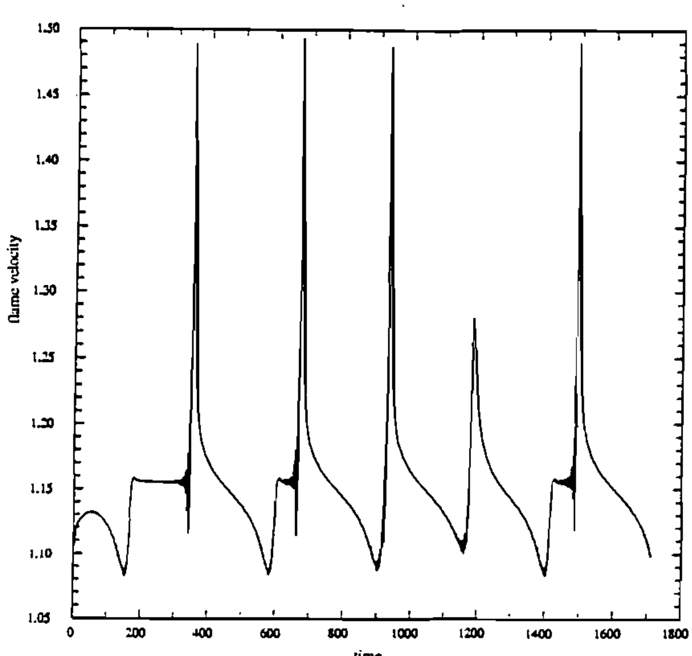

FIGURE 12 Time evolution of flame velocity for the complete equations (length 250) (parameters $F = 0$, $k = 0.08$).

a constant value. This method does not seem to work extremely well when there is a sudden change in velocity, and this explains the undeshoot. However the peaks can also be observed with other definitions of flame speed, for instance based on flame surface.

Solutions at different times of the simulation of Figure 12 are presented in Figure 13 a and b. In Figure 13a, a solution with one cell of large amplitude is observed, whereas in Figure 13b, a perturbation of the basic solution occurs, which reduces the overall amplitude.

In Figure 14, we give an example of an upward propagating flame ($k=0.08$ as in Figure 12 and $F=-0.01$). In this case, it can be seen that the oscillations produced at the beginning of the simulation are damped, and the curved flame is stable.

It has been suggested that instabilities of zero gravity flames could be due to finite size effects in the x direction (Joulin private communication). This idea is based on a study of flames close to a burner, where the extension in the longitudinal direction is

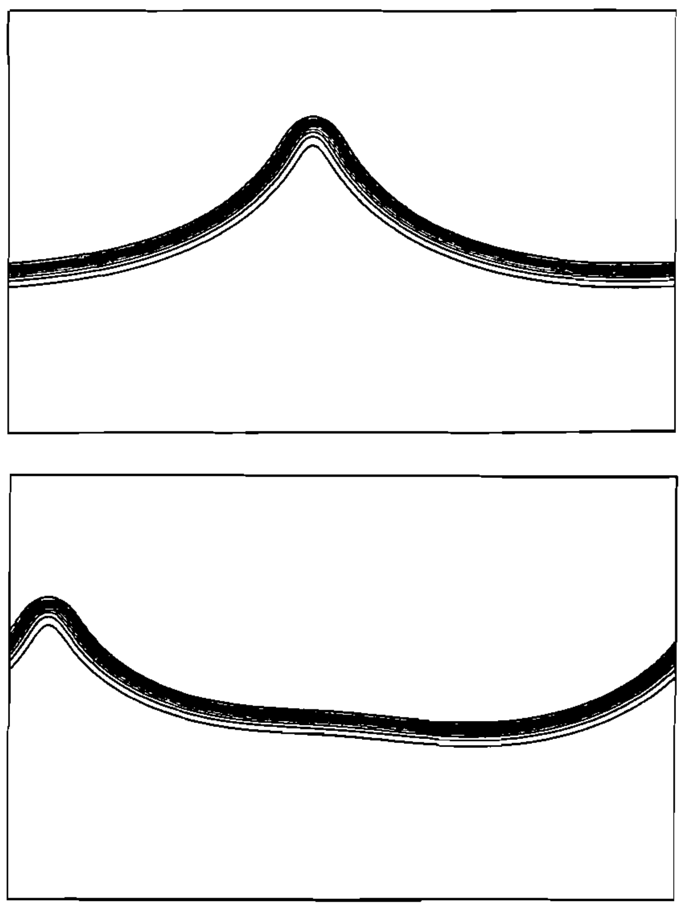

FIGURE 13(a, b) Solutions at different times corresponding to Figure 12 (length 250) (parameters) $F=0$, $k=0.08$).

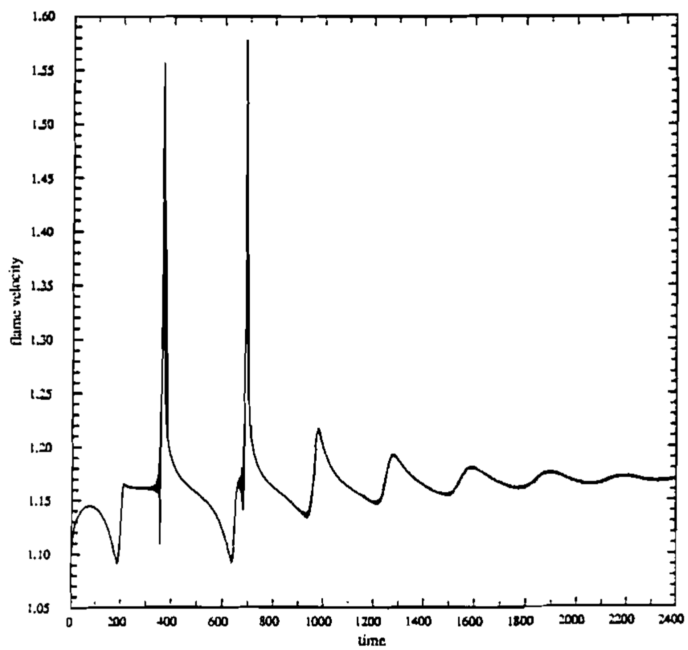

FIGURE 14 Time evolution of flame velocity for the complete equations (length 250) (parameters $F = -0.01$, $k = 0.08$).

limited, so that the growth rates of the Landau instability are slightly modified. A numerical simulation uses of course a domain of finite extent, however, here the mapping we have used gives us a relatively large domain (250 in units of flame thickness), so that we expect the finite size effects to be small. Furthermore, a comparison of simulations of length 50 and 250 shows that the curved solution becomes unstable in both cases, although not at the same value of wave vector.

It seems that for sufficiently large domains, zero gravity flames are unstable, although the oscillations should not be always very important and easy to see. Microgravity experiments have been performed, for instance in parabolic flights or drop towers... Interesting works of Dunsky (1992) and Strehlow *et al.* (1986) *et al.* have been published. In these works, oscillating zero gravity cases have been observed in a lot of cases, see for instance a figure in Strehlow *et al.* (1986), where a case of oscillating zero *G* methane flame, with more than one cell, is presented. However in other cases, flames seem stable. In the same paper of Strehlow *et al.*, it is reported that zero *G* propane flames do not give evidence for oscillations. This experiment doesn't rule out the possibility of small oscillations, very difficult to observe on the front form. On another hand, it could also be agrued that the oscillations observed are due primarily to noise in the incident flow. But it seems difficult to create new cells with only a low level noise, so we think that Strehlow's experiments favour the hypothesis that zero gravity flames can be unstable.

6 CONCLUSION

In this paper, we have shown various hydrodynamic instabilities of downwards propagating flames, for different values of gravity. For high gravities, cellular flames are unstable to an oscillating instability, which produces chaotic cellular flames. With decreasing values of gravity, the number of cells of these chaotic flames is progressively reduced. Finally, without gravity, we obtain the surprising results that flames are still unstable for sufficiently low wave vectors. In general, there is a good qualitative agreement between results of the model Michelson Sivashinsky equation and results with complete equations and simplified kinetics. However, in some cases, the Michelson Sivashinsky equation gives solutions which are too easily stable, particularly for zero gravity. It is perhaps reasonable to conclude that all results obtained with this model equation should not be accepted with total confidence, particularly if these results are very sensitive to changes in physical parameters. However the overall picture given by this equation is the good one if the basic instability of hydrodynamic. If this condition is fulfilled, it is probably sufficient in a number of cases to integrate numerically the model equation rather than the complete hydrodynamical equations, which result in very time-consuming computations. So this paper largely confirms the usefulness of model equations in combustion stability studies.

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