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Intrinsic Instabilities of Curved Premixed Flames.

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Abstract. – A study of instabilities occurring in downward-propagating curved premixed flames is undertaken, on the basis of a Michelson-Sivashinsky equation including gravity effects. Contrary to the qualitative idea of Zel'dovich *et al.* on flame stability (*Combust. Sci. Tech.*, 24 (1980)), intrinsic instabilities exist in this model even in the low-gravity case.

In the last few years there has been a great interest in the problem of stability of curved fronts, *e.g.* Saffman-Taylor finger, free needle crystals in solidification fronts and curved premixed flames (see [1]). Here we will be interested in the last problem. The typical transverse dimension of a curved flame is much larger than the most amplified wavelength of the corresponding plane front. This has led to explain the «anomalous stability of curved flames» by a heuristic argument of Zel'dovich *et al.* [2].

These authors consider the development of a perturbation of the curved flame: the perturbation is amplified because of the linear instability; because of the tangential velocity the perturbation is stretched and at the same time advected away from the tip. The conclusion is that although the flame is supposed to be linearly stable, a small amount of noise is sufficient to produce nonlinear instabilities.

Another approach has been initiated by Sivashinsky [3]: in a limit of low gas expansion and large wavelength, a nonlinear partial differential equation has been derived, the Michelson-Sivashinsky (MS) equation. This equation describes the behaviour of a flame submitted to a hydrodynamic Landau instability [4]. It has been shown [5] that the solution to this equation consists in only one cell in the computational domain, *i.e.* is a curved flame. This solution is stationary for every size of the domain, although with increasing size it becomes more and more sensible to noise. It seems that these results are compatible with Zel'dovich's qualitative description.

In the MS equation, two effects are retained, the destabilizing gas expansion, and the stabilizing diffusive effects. Usually the gravity effects, which stabilize plane downward-propagating flames, are considered negligible for sufficiently fast flames. However, it is not difficult to include such effects in the MS equation. In the sequel, we will describe a numerical study concerning stability of curved flames with low gravity on this modified MS equation.

Let us first recall the linear-stability analysis of the plane flame front. Theoretical predictions for the growth rate of perturbations can be found in [6]. This analysis relies on an expansion in the wave vector k. Essentially, outside a boundary layer around $k = 0$

(corresponding to a growth rate $\sigma = 0$), the growth rates are approximately given by a second-order polynomial in k. In order to use an explicit form for the growth rates in the Michelson-Sivashinsky equation, we will consider that

$$\sigma(k) = 0 \quad \text{for} \quad k = 0\,,$$

$$\sigma(k) = G + |k| - \nu k^2 \quad \text{for} \quad k \neq 0\,,$$

where this expression is valid in rescaled variables. G is proportional to gravity, inversely proportional to the laminar-flame speed squared and is negative for flames propagating downwards; ν depends on diffusive effects, which are stabilizing: ν is always positive. In the simulations, we will take a constant value $\nu = 0.1$ (we could have as well chosen the units such as $\nu = 1$) and vary G and the width of the computational domain.

With the previous form of the dispersion relation, the modified Michelson-Sivashinsky (MS) equation (with periodic boundary conditions, as in [5]) is obtained by simply adding to a partial differential equation corresponding to the above linear instability the lowest-order pertinent nonlinear term, which results from a geometrical effect associated with the slope of a flame advancing with a constant normal velocity (see [3] for a complete derivation). The equation reads

$$\alpha_t + \frac{1}{2}(\nabla\alpha)^2 = I(\alpha)\,,$$

where α represents the flame position, $I(\alpha)$ is an operator corresponding to the multiplication by $\sigma(k)$ in Fourier space. The MS equation, which can be solved easily in one dimension by Fourier pseudospectral methods, describes the behaviour of a flame submitted to the previous linear instability; this instability saturates because of the nonlinear term. We will limit ourselves to 1D flame fronts corresponding to 2D flames.

The difference between the MS equation used here and the usual one is that in the previous form of $\sigma(k)$ we have taken into account the G term, which is due to gravity effects. The MS equation with this term was first introduced in [7] in the case of 2D flame fronts. Simulations of cellular flames were presented in this paper, showing stable flames close to the linear-instability threshold and unstable (self-turbulizing) flames for a higher value of G. This transition was explained in [8] in terms of a subcritical Hopf bifurcation. The last paper also mentioned a more or less regular transition from cellular flames to curved flames with increasing G.

Our motivation in this work is to reconnect the zero-G stable curved flame to cellular solutions with lower values of G (*i.e.* slower flames), which are well known to be highly oscillatory (self-turbulizing) for downward-propagating flames. It has been possible to show in [8] that this type of solutions can be obtained with the MS equation with a sufficient amount of gravity even in one dimension. The question naturally arises, what is the mechanism that will permit us to obtain an oscillatory behaviour by starting from stable curved flames and lowering G?

The domain of stability of the MS equation with gravity is plotted in fig. 1 in the plane (G, k), where k is the wave vector corresponding to the total width of the domain. It has been shown by Joulin [9] (see also [5]) that among other possible solutions to the zero-G MS equation, the stable one was an n-pole solution for a value of νk in the interval $[1/(2n+1), 1/(2n-1)]$ $(n \geqslant 1)$. The stability limit takes the same type of form in all intervals of the type that we have studied, except the first one, corresponding to $n = 1$, where the solutions seem to be stable for all values of G, if k is not close to the left end of the interval.

Consider an interval of the previous form, with $n \neq 1$. At each end of the interval, the

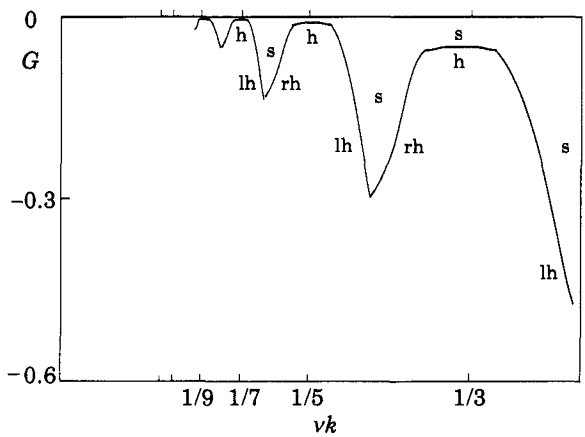

Fig. 1. – Sketch of the stability domain of curved flames in the plane $(\nu k, G)$ ($\nu = 0.1$) (possible bifurcations are: homoclinic bifurcation (h), left Hopf bifurcation (lh), right Hopf bifurcation (rh)); the stable domain is denoted by s.

solution becomes unstable because of a homoclinic bifurcation occurring at values of G very close to zero. In the middle of the interval, on the contrary, the solution becomes unstable because of two different Hopf bifurcations, appearing at lower values of G. These bifurcations will be hereafter referred to as right and left Hopf bifurcations, depending on their position in the interval. Obviously the solution becomes more easily unstable at the ends of the intervals, because the corresponding solution with zero gravity is itself less stable for these values of k. In each case, the instabilities occur for values of G not very far from zero, and much higher than the critical value of G for the appearance of cellular flames ($G = -2.5$ for $\nu = 0.1$). Furthermore, for smaller k, the same kind of structure is again found, but closer to $G = 0$.

Let us now take examples of these various bifurcations. We will plot the velocity increase compared to the plane front velocity (sometimes simply called velocity and defined as the spatial mean value of $(1/2)(\partial\alpha/\partial y)^2$) *vs.* time. In fig. 2, an example of relaxation oscillations

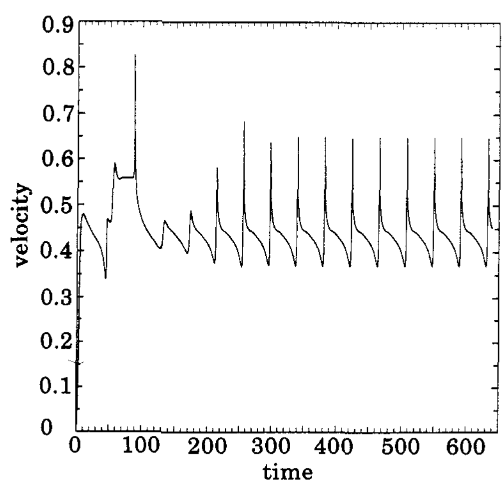

Fig. 2. – Homoclinic bifurcation leading to nearly periodic relaxation oscillations: velocity increase *vs.* time (physical parameters $G = -0.03$, $\nu = 0.1$, $k = 1.1$).

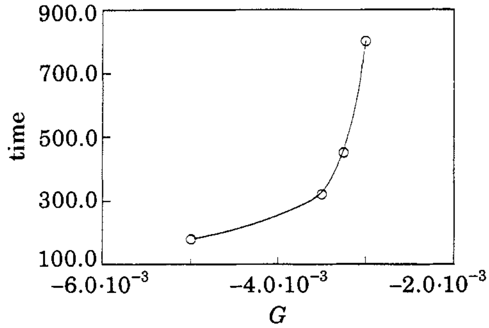

Fig. 3. – Divergence of the time necessary to get the first velocity maximum when the homoclinic-bifurcation line is approached from below ($\nu = 0.1$, $k = 1.35$, G varies).

obtained above the homoclinic bifurcation is shown. In this figure different phases can be seen in the simulation. First, the velocity increases in order to get close to the value of the stationary solution (with a value of G close to zero), *i.e.* close to 0.5 (see [9] for a derivation of the value of velocity *vs.* k for the zero-G MS equation, always close to 0.5 for k not too high). This stationary solution is unstable and the velocity begins to decrease. In the figure, this phenomenon is relatively fast, but is slower and slower as one gets closer to the bifurcation. In the case of the figure, a tip splitting perturbation forms during the decrease in velocity (in some cases this is only a lateral perturbation of the Zel'dovich type); this perturbation is shifted on one side and merges with the cusp, which corresponds to the high velocity maximum well above 0.5. This transient phase is followed by the installation of nearly periodic relaxation oscillations. The maxima of these relaxation oscillations correspond to merging of lateral perturbations with the cusp.

This bifurcation is indeed of a homoclinic type because, as seen in fig. 3, the time taken to see the first high maximum of velocity above 0.5 diverges close to the bifurcation. We have chosen to plot this time because it is of the same order of magnitude as the period of the final oscillations and does not necessitate a huge CPU time to be measured. On the contrary the amplitude of the oscillations is more or less constant when the transition is approached. We

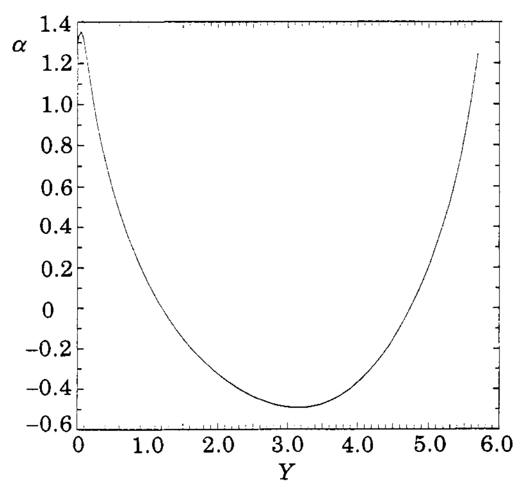

Fig. 4. – Lateral perturbations (note the dissymmetrical aspect of the front) occurring during the nearly periodic relaxation oscillations seen in fig. 2 (parameters $G = -0.03$, $\nu = 0.1$, $k = 1.1$).

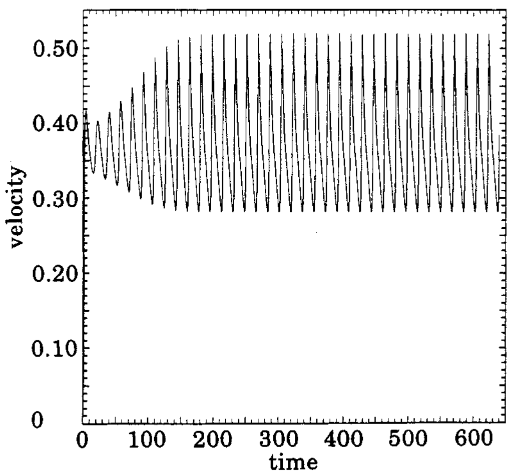

Fig. 5.

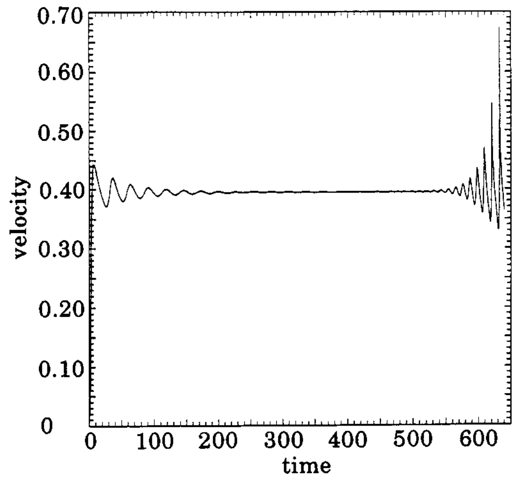

Fig. 6.

Fig. 5. – Oscillations caused by the left Hopf bifurcation: velocity increase *vs.* time (parameters $G = -0.12$, $\nu = 0.1$, $k = 1.6$).

Fig. 6. – Oscillations caused by the right Hopf bifurcation: velocity increase *vs.* time (parameters $G = -0.06$, $\nu = 0.1$, $k = 1.2$). The damped oscillations seen at the beginning of the simulation are visible because we are close to the left Hopf bifurcation line.

have seen examples of periodic relaxation oscillations, but also of more chaotic cases. From the point of view of the spatial structure, these oscillations of velocity are produced when lateral perturbations of the Zel'dovich type, but intrinsic (fig. 4) are amplified and merge with the cusp of the curved front. At this moment the flame surface and thus the flame velocity are maximum. Merging of perturbations with the cusp causes continuous translation of the basic curved solution toward the right or the left, as in the case of a curved flame (with periodic boundary conditions) submitted to noise. This is compatible with [10], where it has been shown that homoclinic bifurcations of partial differential equations generally produce travelling-wave solutions. This translation of the basic solution proceeds always in the same direction, but we have seen cases where it reverses itself during the simulation, probably because of noise. It is to be noted that slowly travelling-wave solutions to the MS equation with a gravity term (breathing solutions) were obtained analytically in [11]. But the expansion used in this paper is not valid for values of G close to zero, so it is difficult to know whether our results are connected with these solutions.

We will now describe the two Hopf bifurcations occurring in this problem. In fig. 5 the amplification of the oscillation produced by the Hopf bifurcation appearing on the left side of each interval is shown. Contrary to the previous homoclinic bifurcations, no translation of the basic solution is generated by this left Hopf bifurcation, but in this case only the amplitude of the curved flame oscillates, and the solution is always symmetric. This instability saturates and a periodic finite amplitude oscillation is obtained. When parameters are varied, it is possible to have a more or less continuous evolution from oscillations generated by Hopf bifurcations to relaxation oscillations generated by the homoclinic bifurcations.

Damped oscillations caused by this left Hopf bifurcation are visible far away from the stability limit; actually they are visible even in a domain unstable because of the second (right) Hopf bifurcation. This has the unexpected consequence of leading to fig. 6: damped left Hopf oscillations occur first, then the solution seems stationary, and after a very long time right Hopf oscillations become visible. At the same time, the solution begins to translate periodically to the right and to the left, which indicates the presence of lateral perturbations moving to the cusp of the front.

The results we obtain are rather surprising: by adding a small gravity effect, we completely destabilize the stationary solutions to the zero-G Michelson-Sivashinsky equation. This is not in agreement with the usual view of the anomalous stability of curved fronts. Furthermore, it is difficult to interpret these instabilities in terms of Zel'dovich's WKB analysis: actually the gravity effect is a stabilizing factor for the development of Zel'dovich-type perturbations! On the contrary, for positive G (corresponding to upward-propagating flames), we have found that the curved flames are always stable (at least for G low enough), although Zel'dovich's perturbations should be more amplified in this case. The discrepancy can be explained simply: Zel'dovich's mechanism is a mechanism to produce nonlinear instabilities of curved flames, whereas in this study we have found various linear instabilities.

The fact that all the instabilities we have found appear at values of G close to zero is very surprising. It is difficult to know whether this result is quantitatively reliable. On the one hand the experiments on curved downward-propagating flames are rare and not very precise. The solution to the MS equation is also different from an experiment: it is only 1D, the periodic boundary conditions used in the numerics are different from those in a tube with boundary layers close to the wall, and the MS equation is only the leading order of a development in powers of a gas expansion parameter. It is thus possible for the stability limit of real flames to be displaced to lower values of G. However, we recall that instabilities have to happen with decreasing G in order to recover oscillatory behaviours characteristic of self-turbulizing flames.

Finally, the analogy between the nearly periodic perturbations of curved premixed flames found in this paper and the nearly periodic sidebranching seen in growing dendritic crystals is striking. At present, works on stability of curved solidification fronts seem to show that these fronts are stable; one has then to rely on noise in order to explain sidebranching [1]. However, such an explanation is not completely convincing, particularly the regularity of the patterns obtained is difficult to reconcile with perturbations produced by a random noise. It is possible to suggest as an alternative that, as in our study of curved flames, a small neglected effect could be sufficient to generate intrinsic oscillations of the growing crystal.

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