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On Non-Linear Instabilities of Cellular Premixed Flames

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Abstract—A study of finite amplitude instabilities occurring in cellular premixed flames is undertaken, both in the case of 2D and 3D flames. We use two different models: the complete equations of isobaric flames with one global exothermic reaction and a modified Michelson-Sivashinsky equation (including gravity effects). Different instabilities are found that can be explained in terms of Eckhaus type instabilities of cellular patterns. An oscillatory instability also exists, which produces self turbulizing flames.

Key words: Darrieus-Landau instability, Michelson Sivashinsky equation, cellular patterns, Eckhaus instability, oscillatory instability.

1 INTRODUCTION

Plane premixed flames are only observed in the case of downwards propagating flames at very low flame velocities. With increasing flame speeds, it is well known since the work of Markstein (1951) that stable cellular flames, with hexagonal cells, can be obtained (see also Quinard et al (1984)). Increasing further the velocity, self turbulizing flames appear, on which cells continuously appear and merge.

The linear instability of plane flames has been theoretically explained in different papers (Pelcé and Clavin (1982), Matalon and Matkowsky (1982), Frankel and Sivashinsky (1983)). In these works, the mechanism responsible of this transition is the hydrodynamic Darrieus-Landau instability (see Darrieus (1938) and Landau (1944)): because of gas expansion, deflection of streamlines through the flame is produced. This has a destabilizing influence, giving rise, in the absence of other effects, to a positive growth rate proportional to the wave vector. Opposite effects, such as diffusion, which tends to stabilize short wavelengths, and gravity, which tends to stabilize large wavelengths in the case of downwards propagating flames, are also taken into account in the linear theory. The analytical predictions for the values of the growth rates have been roughly confirmed by recent numerical simulations of the equations of combustion with a simplified kinetics, in the isobaric approximation and without radiative heat losses (Fröhlich and Peyret (1991), Denet and Haldenwang (1992)). Another possible explanation for the instability exists, i.e. the thermal-diffusive instability, caused only by the destabilizing influence of species diffusion (Sivashinsky (1977a)). The aforementioned theoretical results (see e.g. Pelcé and Clavin (1982)) show that in the absence of heat losses, the thermal-diffusive instability does not seem to be a very plausible mechanism for the instability (i.e. the Markstein length remains positive), except perhaps in the case of lean hydrogen oxygen flames. However, it is possible to argue that a sufficient amount of radiative heat losses could be sufficient to induce a negative Markstein length, and thus produce a thermal-diffusive instability. Another argument against the Darrieus-Landau mechanism is the existence of cellular Bunsen burner flames both for downward and upward propagating flames, whereas flames should be cellular only for downward propagating flame in the Landau picture. However the Landau analysis is mainly concerned by (plane on average) flames propagating in tubes (the hydrodynamics is different from Bunsen burner flames), and in this case cells are observed only for downward propagating flames. Upward propagating flames are in this case curved flames, as predicted in the Landau picture. As can be

seen from the previous discussion, the situation is not yet completely clear, for a recent review see Sivashinsky (1990).

In the sequel, we shall be concerned with secondary instabilities of flames controlled by the Landau mechanism. Our purpose will be to find what is the stability domain of non linear cellular solutions and to explain the nature of the instabilities that develop. We shall be particularly interested in the spatial structure of perturbations that grow because of these secondary instabilities. It will be found that all the instabilities found do not saturate to a finite amplitude, but that instead the basic cellular structure is completely changed because of these instabilities.

From the point of view of numerical computations, simulations of the complete equations of combustion are very expensive, so that it seems difficult to obtain results in the non linear domain, particularly in the case of three dimensional flames. Fortunately, it has been shown by Sivashinsky (1977b) that it was possible to derive a model equation, the Michelson Sivashinsky (MS) equation, from the basic equations of the problem, in the case of low gas expansion. This equation, which contains the effects of gas expansion and diffusion, is much simpler and easy to compute numerically than the original equations. It has been further shown by Sivashinsky and Clavin (1987) that the MS equation is actually valid up to the second order in gas expansion. The MS equation has even been extended in a phenomenological manner by Joulin and Cambray (1992) to yield good quantitative agreement with for instance amplitudes of flames produced by the Darrieus Landau instability.

Numerical simulations (and analytical solutions in the case of a 1D front) of the MS equation have shown (see e.g. Thual *et al.* (1985)), that the solution was a curved flame (i.e. with only one cell in the computational domain). However, the MS equation is easily extended to include gravity effects and thus to produce cellular flames. Our purpose here will be to study numerically the nonlinear instabilities occurring in these flames, and to show that there is a qualitative agreement with experimental results.

It will be shown here that these non linear instabilities have a close relation to instabilities of other cellular patterns. These type of instabilities, often encountered in convection (e.g. Rayleigh Benard convection) have attracted a lot of attention: theoretical methods have been developed to deal with these problems, particularly in the weakly non linear approximation. We will make reference here to the method of amplitude equations valid close to threshold of cellular instability. Instabilities of compression dilatation type, and also an oscillatory instability will be found to occur in the case of cellular flames.

The paper is organized as follows. In Section 2 we describe the basic equations (i.e. both the MS equation and the complete isobaric equations). In Section 3 we recall briefly the basics of amplitude equations and Eckhaus instability. In Section 4 we present the numerical studies we have made in the case of 1D fronts. Section 5 is devoted to the presentation of a set of numerical results concerning the instabilities of 2D fronts.

2 BASIC EQUATIONS

We first recall the complete equations of premixed flames in the isobaric approximation and with the simplest chemistry: a single one-step chemical reaction is assumed. Non-dimensional quantities are obtained using a classical approach: the length scale is the flame thickness obtained from asymptotics, likewise the velocity unit is the asymptotic flame speed. The use of the normalized variables allows us to write the model, in a frame moving with the flame front, as follows:

$$\rho \frac{\partial T}{\partial t} + \rho (\vec{v} \cdot \vec{\nabla}) T = \Delta T + \Omega \quad \text{II.1.a}$$

$$\rho \frac{\partial C}{\partial t} + \rho (\vec{v} \cdot \vec{\nabla}) C = \frac{1}{Le} \Delta C - \Omega \quad \text{II.1.b}$$

with

$$\Omega = \frac{\beta^2}{2Le} C \frac{\rho}{1-\gamma} \exp\left(\frac{\beta(T-1)}{1+\gamma(T-1)}\right) \quad \text{II.2}$$

and

$$\frac{\partial(\rho)}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \quad \text{II.3.a}$$

$$\frac{\partial(\rho \vec{v})}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v} \vec{v}) = -\vec{\nabla} P + \rho \vec{F} + \mu \Delta \vec{v} + (\lambda + \mu/3) \vec{\nabla}(\vec{\nabla} \cdot \vec{v}) \quad \text{II.3.b}$$

$$\rho = \left(1 + \frac{\gamma}{1-\gamma} T\right)^{-1} \quad \text{II.4}$$

where T and C correspond to the reduced temperature of the gas mixture and concentration of a reactant (the other reactant being in excess). The boundary conditions on T and C are

$$T(x = -\infty, y) = 0, \quad T(x = +\infty, y) = 1$$

$$C(x = -\infty, y) = 1, \quad C(x = +\infty, y) = 0$$

all quantities being periodic in the y direction (the direction transverse to the flame).

$Le = D_{th}/D_{mol}$, $\beta = (E/RT_b^2)(T_b - T_u)$ and $\gamma = (T_b - T_u)/T_b$ are respectively the Lewis number of the reactant (ratio of thermal to molecular diffusivity), the reduced activation energy (or Zeldovich number) and the heat release parameter (T_b and T_u being respectively the temperature of burnt and unburnt gases, R is the constant of perfect gases). ρ , v and P are the density, velocity and pressure; F is an external specific force per unit volume (gravity for instance). μ and λ are respectively shear and bulk viscosities. Let us define U, the reduced flame speed which is an unknown of the problem. U is supposed to be parallel to the x-direction. F will also be parallel to this direction, positive and negative F corresponding respectively to downward and upward propagating flame (F being the inverse of the Froude number).

These equations have been described in Denet and Haldenwang (1992), as well as the numerical method we use here. Because of the cost of the numerical computations of such equations, we will limit ourselves to 2D simulations, corresponding to a 1D front separating fresh and burnt gases. Theoretical predictions for the growth rate of perturbations to the plane solution can be found in Pelcé and Clavin (1982). This analysis relies on an expansion in the wave vector k. Essentially, outside of a boundary layer around $k=0$ (corresponding to a growth rate $\sigma = 0$), the growth rates are approximately given by a second order polynomial in k. In order to use an explicit form for the growth rates in the Michelson-Sivashinsky equation, we will have to consider that (see Rakib and Sivashinsky (1987))

$$\sigma(k) = 0 \text{ for } k = 0$$

$$\sigma(k) = G + |k| - \nu k^2 \text{ for } k \neq 0 \quad \text{II.5}$$

where this expression is valid in suitably rescaled variables. G depends on gravity and is negative for flames propagating downwards, ν depends on diffusive effects, which are stabilizing: ν is always positive. It is possible to have a transition from stable plane flames to cellular flames by increasing G (with the complete equations, it would correspond to lowering the parameter F). We emphasize that the band of unstable k will always

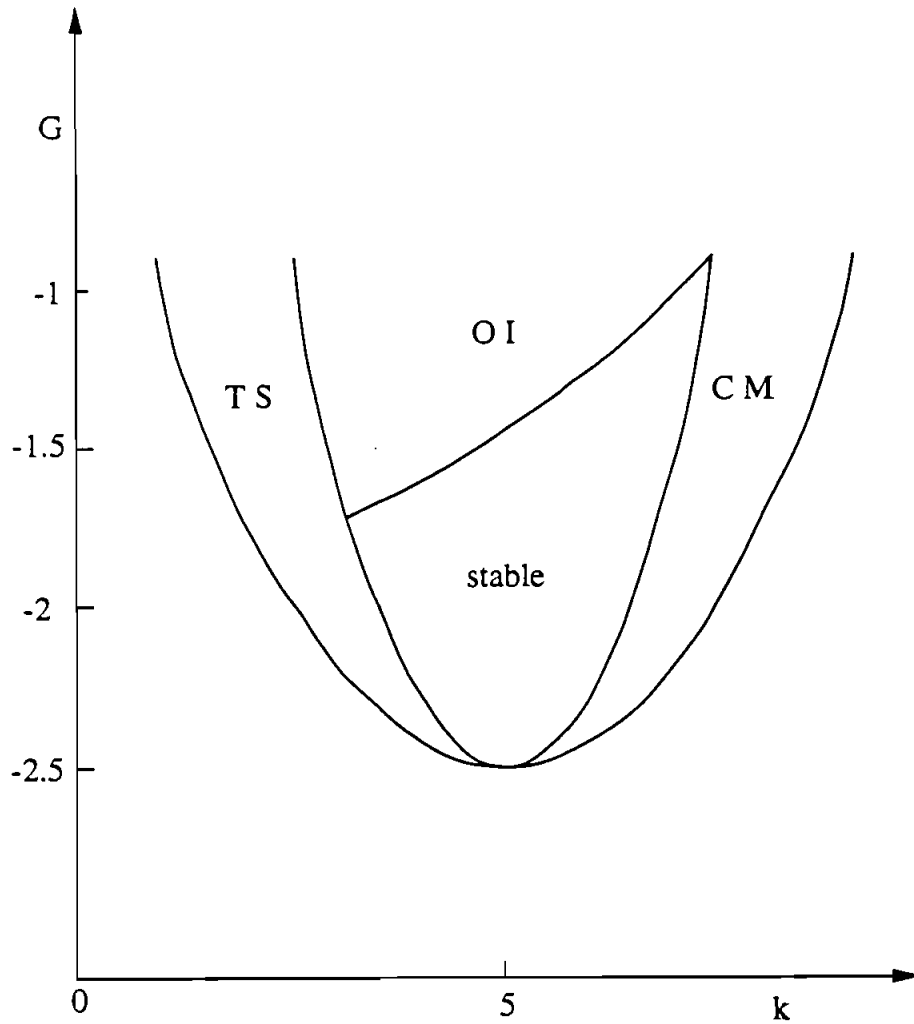


FIGURE 1 Sketch of the 1D stability domain (possible instabilities are: cell merging (CM), tip splitting (TS) oscillatory instability (OI))

be confined to low values of the wave number. This will be seen to have important consequences on the cellular flame's instabilities.

With the form II.5 of the dispersion relation, the modified Michelson Sivashinsky (MS) equation (with periodic boundary conditions) then reads (see Rakib and Sivashinsky (1987) to see an example of the same type of equation but in the case of a thermo-diffusive instability with gravity).

$$\alpha_t + \frac{1}{2} (\vec{\nabla} \alpha)^2 = I(\alpha)$$

where α represents the flame position, $I(\alpha)$ is an operator corresponding to the multiplication by $\sigma(k)$ in Fourier space. The MS equation, which can be solved easily in both one and two dimensions by Fourier pseudo spectral methods, describes the

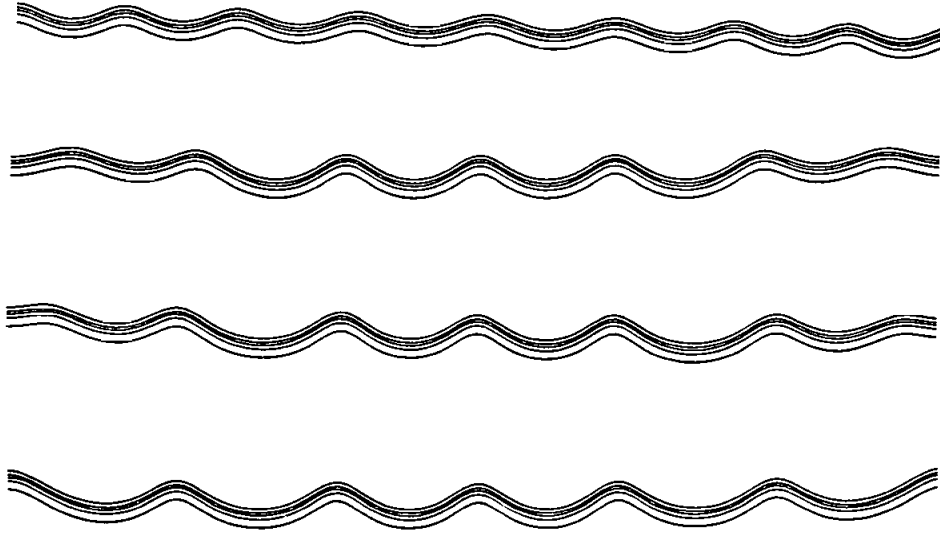


FIGURE 2 (a b c d) Cell merging for an isobaric flame (the physical parameters are $\beta=5$ $Le=1$ $\mu=0.7$ $F=0.3$ $k=0.22$). The flames are represented by their temperature lines for different times with a slight dilatation in the direction of the mean flame propagation. It can be seen that the number of cells reduces from eight cells (Fig. 2.a) to seven cells (Fig. 2.b) and finally six (Fig. 2.d).

behaviour of a flame submitted to the above linear instability; this instability saturates because of the non linear term, which is the lowest order pertinent term and results actually from a geometrical effect associated with the slope of the flame advancing with a constant normal velocity (see Sivashinsky (1977b)).

The difference between the MS equation used here and the usual one is that in the form II.5 of $\sigma(k)$, we have taken into account the G term, which is due to gravity effects. The solution of the usual MS equation consists of only one cell in the computational domain; here, by tuning the parameter G , we can also obtain what will interest ourselves in the sequel, a cellular flame close to the threshold of instability.

3 AMPLITUDE EQUATIONS AND ECKHAUS INSTABILITY

In this section, we will give a short overview of theoretical analysis of cellular structures near threshold. All results are fairly classical: the theory has been developed to deal with problems such as Rayleigh Benard convection, Taylor Couette instability ... See for instance Wesfreid and Zaleski (1984) for different examples of applications.

We are interested in a system undergoing a linear instability, when a control parameter (G in the case of the MS equation) is varied. We suppose that just above threshold, the band of unstable wave vectors is centered on a particular value q_0 corresponding to the maximum of the dispersion relation; around q_0 the dispersion relation $\sigma(k)$ (real) can be approximated by a parabola. For the moment, we restrict ourselves to 1D cellular structures. It is possible to express the solution $u(x,t)$ as $u(x,t)=A(X,T)\exp(iq_0x)+c.c$ (complex conjugate), where $A(X,T)$ is a complex amplitude function, slowly varying in

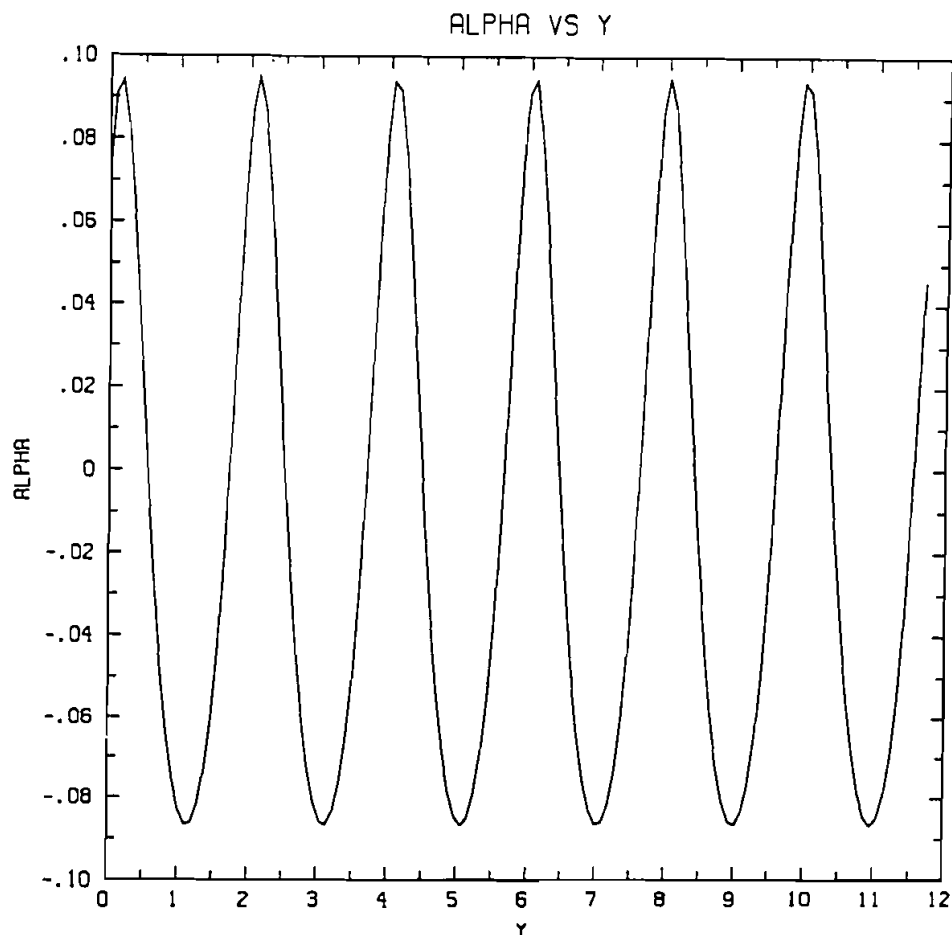


FIGURE 3 (a) Tip splitting instability with the 1D MS equation (physical parameters $G=-2$ $\nu=0.1$ $k=3.2$). Starting from a six cells solution (Fig. 3.a) a twelve cells solution is obtained in a short time (Fig. 3.b)

time and space. It can be shown that for every control parameter above threshold, in suitably rescaled units, $A(X,T)$ satisfies the following amplitude equation:

$$\frac{\partial A}{\partial T} = A + \frac{\partial^2 A}{\partial X^2} - |A|^2 A$$

If A is taken of the form $\exp(iQX)$ (where Q measures actually the difference between the wave vector and q_0), the linear terms of the amplitude equation will give a growth rate $\sigma=1-Q^2$, i.e. corresponding to the parabolic shape of $\sigma(k)$ near q_0 . The non linear term of the amplitude equation is the lowest order non linear term to be retained. Since the original equation is invariant by translation, the equation satisfied by A must be invariant under a change in phase. The non linear term satisfies this property. Also, without the spatial dependence, A is solution of a Landau type equation: the amplitude undergoes essentially a second order phase transition.

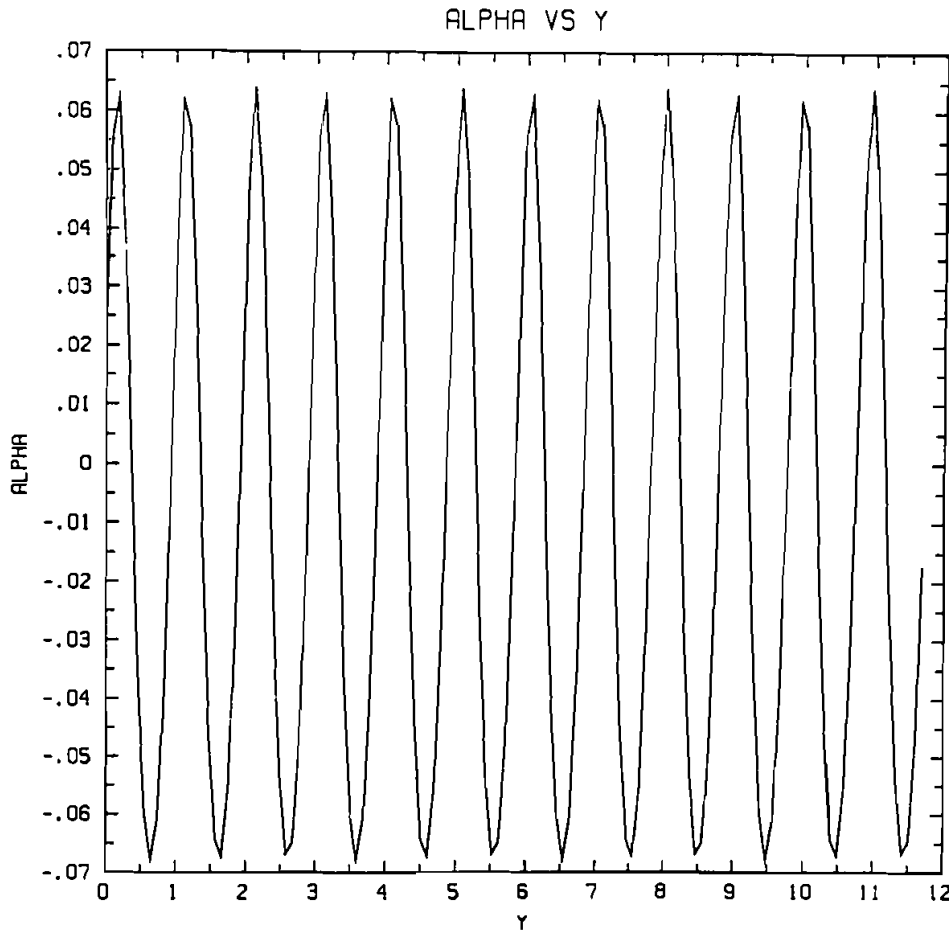


FIGURE 3 (b) Tip splitting instability with the 1D MS equation (physical parameters $G=-2$ $\nu=0.1$ $k=3.2$). Starting from a six cells solution (Fig. 3.a) a twelve cells solution is obtained in a short time (Fig. 3.b)

Let us now talk about the Eckhaus instability of 1D patterns. The control parameter will be denoted by ϵ , we will suppose that $\epsilon=0$ is the threshold. The band of unstable wave vector in the plane (ϵ, k) is limited by a parabola originating in $(0, q_0)$. In the previous units unstable wave vectors correspond (for every ϵ to $Q^2 \leq 1$ (of course the units chosen depend on the control parameter, in order to reduce in each case the dispersion relation to $\sigma=1-Q^2$). To each wave vector in this band is associated a non linear cellular solution:

$$A = (1 - Q^2)^{1/2} \exp(iQX) + c.c$$

It is possible to study the stability of this solution of the amplitude equation, submitted to the perturbation

$$w = w_1 \exp(i(Q + H)X) + w_2 \exp(i(Q - H)X)$$

To be stable the solution should be stable against all perturbations of this type, it is easily demonstrated that this is the case only for $Q^2 \leq 1/3$ (this defines a new parabola inside

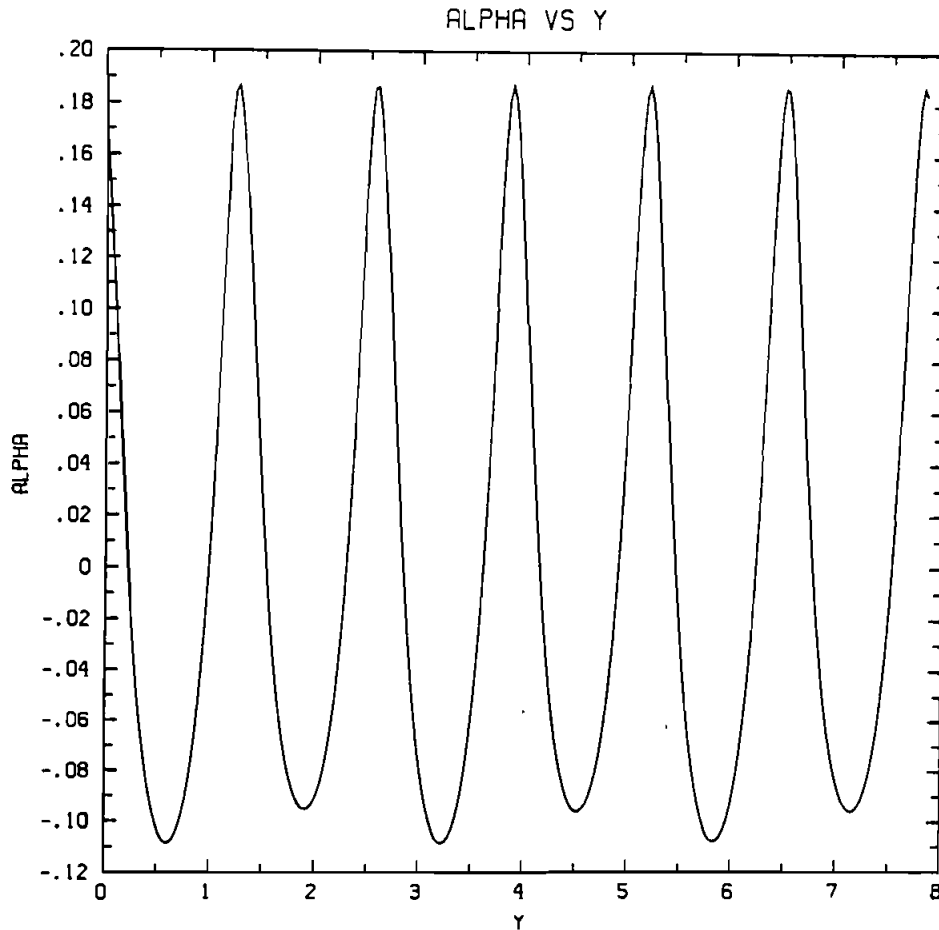


FIGURE 4 (a) Oscillatory instability (1D MS equation with parameters $G=-1.25$ $\nu=0.1$ $k=4.8$. A subharmonic perturbation of the basic cellular structure can be seen in Fig. 4.a; this perturbation oscillates in time (Fig. 4.b) grows and produces a self turbulizing flame (Fig. 4.c).

the linear instability parabola). Outside this new parabola, the basic solution is unstable to perturbations with small H : this is the Eckhaus instability.

Looking at the form of the perturbation, one can see that the Eckhaus instability is a compression dilatation instability of the basic cellular pattern. There will be compression at some place of the pattern and dilatation elsewhere until a new cell appears in the dilatation zone (if the original wave vector was too small) or two cells merge in the compression zone (if the wave vector was too great).

We will have to use these concepts about cellular patterns in the rest of the paper.

4 NUMERICAL SIMULATIONS OF 1D FRONTS

We now turn to simulations of 1D fronts. We will consider two models: the MS equation in 1D solved by standard Fourier pseudo spectral methods, and the complete equations of premixed flames in two dimensions (one dimension perpendicular to the flame). These models have been described in Section 2.

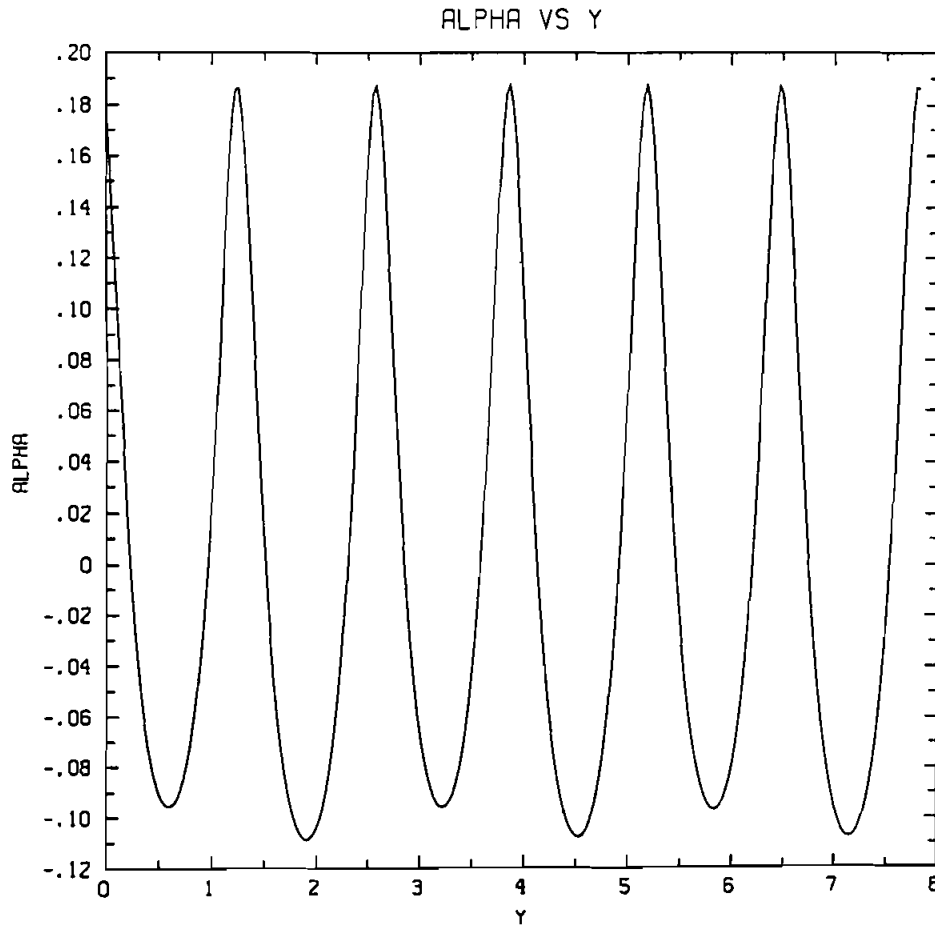


FIGURE 4 (b) Oscillatory instability (1D MS equation with parameters $G=-1.25$ $\nu=0.1$ $k=4.8$). A subharmonic perturbation of the basic cellular structure can be seen in Fig. 4.a; this perturbation oscillates in time (Fig. 4.b) grows and produces a self turbulizing flame (Fig. 4.c).

With both models, we will choose the physical parameters in order to get cellular flames slightly above threshold. We first obtain a stable cellular flame, then the width of the computational domain is suddenly changed and we observe for certain widths (and for the associated wavelengths) the development of some instabilities. We then deduce from these numerical experiments the qualitative shape of the domain of stability in the plane (control parameter, wave vector). This approach is of course reminiscent of the work of Busse and coworkers on the Rayleigh-Benard problem (see Busse (1978)). Here and in the rest of the paper, we don't pretend to find precisely the limits of stability; actually, since all the instabilities we are interested in are secondary instability (very slow), it would necessitate a huge CPU time to locate precisely these limits.

For 1D fronts, the limits are qualitatively plotted in Fig. 1. To be concrete we have taken here the control parameter to be G (of the MS equation), but close to threshold the figure has exactly the same aspect with the complete equations. The instabilities are also of the same type, which will give us some confidence about the results we will obtain

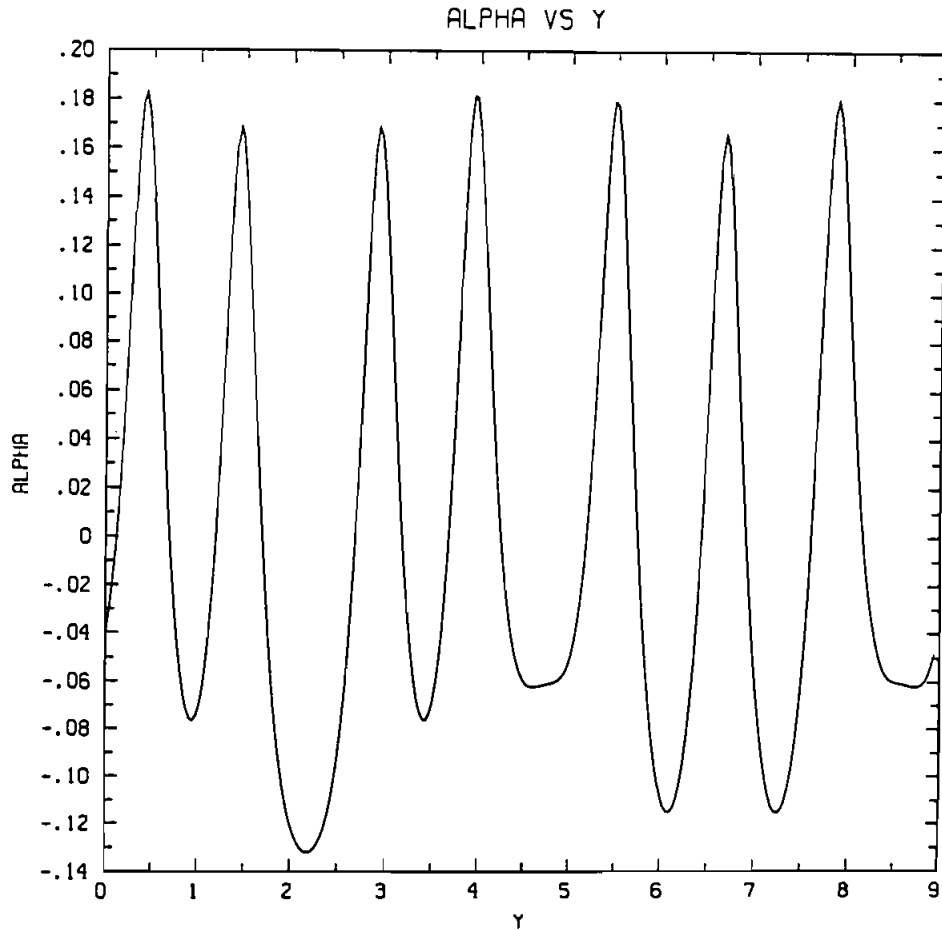


FIGURE 4 (c) Oscillatory instability (1D MS equation with parameters $G=-1.25$ $\nu=0.1$ $k=4.8$. A subharmonic perturbation of the basic cellular structure can be seen in Fig. 4.a; this perturbation oscillates in time (Fig. 4.b) grows and produces a self turbulizing flame (Fig. 4.c).

when integrating numerically the 2D MS equation. The major difference between both cases seems to be that it is a hundred times cheaper in CPU time to work with only the MS equation.

As can be seen in Fig. 1, the stability domain of the basic cellular flame takes the form of a parabola inside the linear instability curve. By increasing G , it is however possible to get an oscillatory instability. For even higher values of G , we observe a phenomenon not plotted on the figure, that we have not studied in detail: an effective reduction of the number of cells as G is increased; starting from eight cells in the domain at the instability threshold, it is possible to get progressively four, two cells. . . This phenomenon allows to recover continuously the behaviour of the pure MS equation without G term: i.e. a flame with only one cell in the whole computational domain. A study of this phenomenon would necessitate however a high precision, because the flames obtained seem very sensible to numerical noise.

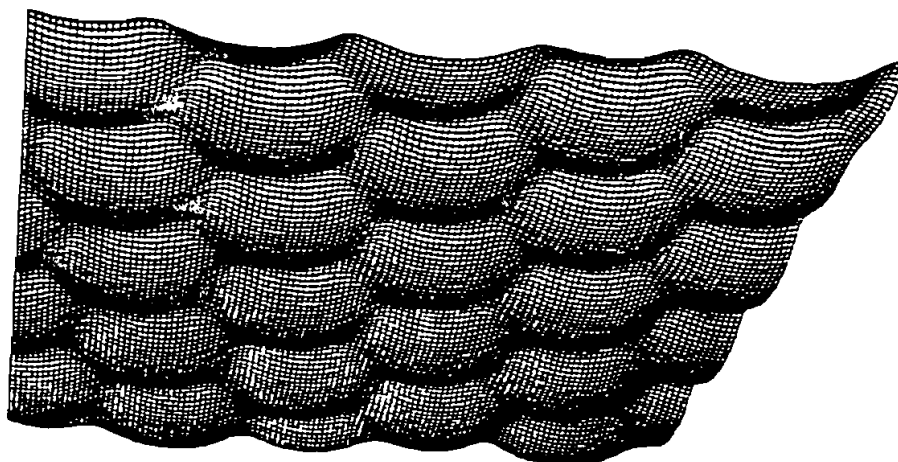


FIGURE 5 (a) Stationary solution of the 2D MS equation with physical parameters $G=2$ $\nu=0.1$ $k=5$. Fig. 5.a surface representation, Fig. 5.b lines of constant flame position.

Coming back to the behaviour near threshold, the parabolic shape of the stability limit reminds us of the Eckhaus parabola (see Section 3). It seems quite normal: a one dimensional cellular structure (here a flame) is usually described by an amplitude equation and exhibits an Eckhaus instability. But in the case of flames there is an important point to keep in mind: the linear (Darrieus-Landau) instability occurs at low values of k . Thus, it is possible to be very close to threshold and to have in the unstable band of wave vectors two values k_1 and k_2 such as $k_2=2 k_1$. This property is rather unusual, and it seems that we are here at the limit of validity of the amplitude equation approach. Nevertheless, the shape of the stability limit is of the same type as for the usual Eckhaus instability. As we will see, the instabilities (we will sometimes talk in the sequel of degenerate Eckhaus instabilities) are qualitatively different because of this property.

We now describe the behaviour on the high k side of this instability, on the basis of a simulation with the complete equations. The physical parameters chosen (see Section 2) are $\beta=5$ (this is a low value of β , chosen in order to be able to use an equidistant mesh), $Le=1$, $\mu=0.7$, $F=0.3$, $k=0.22$. These particular values of the parameters correspond to a flame slightly above threshold; we have numerically measured in this case the unstable band of wave vectors to be $[0.09,0.23]$ (a method for measuring growth rates is given in Denet and Haldenwang (1992)). The band of wave vectors inside the theoretical Eckhaus parabola should then be $[0.13,0.19]$ (see Section 3): The value $k=0.22$ is thus very close to the linear stability limit, but still linearly unstable, and is very likely to be unstable to an Eckhaus type instability. The results are plotted in Figure 2, where five temperature lines are plotted to locate the flame position. We start from a eight cells solution (Fig. 2.a). In order to accelerate the instability development, this solution is not purely sinusoidal, but actually there is already some compression at some place of the structure. In Fig. 2.b, we can see that two cells have merged in this compression zone, and we have now only seven cells. The amplitude of these cells then grows, because the mean wave vector of the structure corresponds now to a higher growth rate (Fig. 2.c). However, the wave vector is still relatively high and we can see in Fig. 2.d that another cell has disappeared and we have now only six cells solution.

All these phenomena are of course relatively close to a standard Eckhaus instability, but here, although there is obviously the same mechanism of cell reduction as in the high k

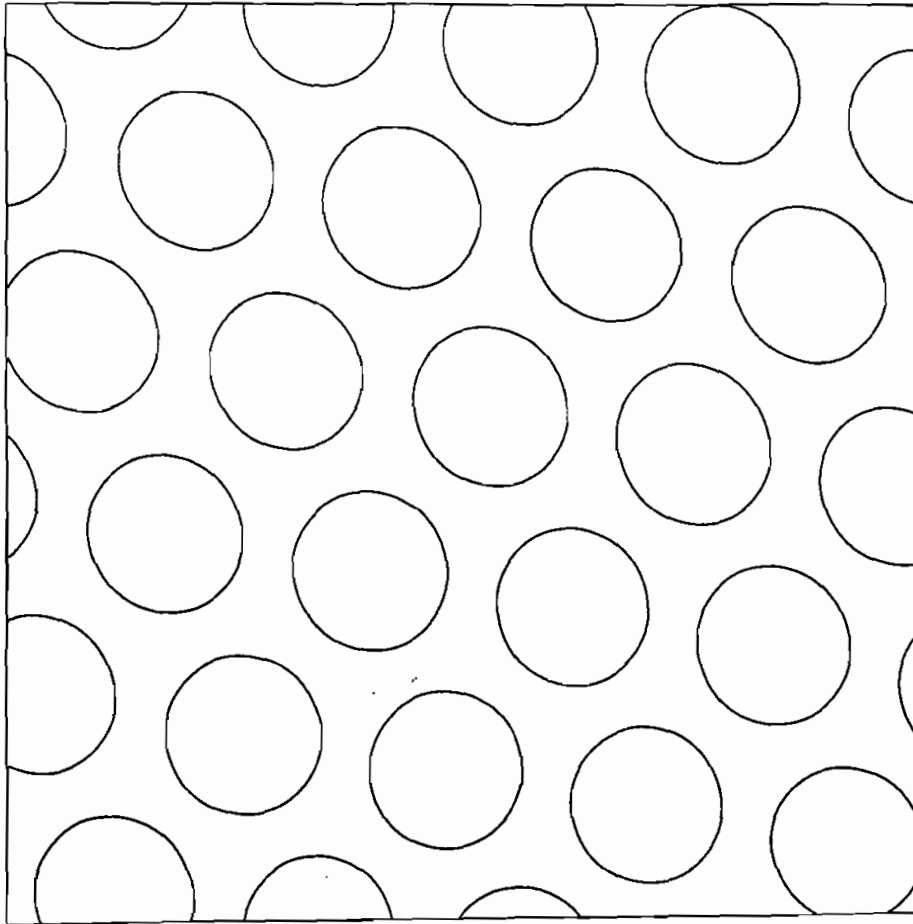


FIGURE 5 (b) Stationary solution of the 2D MS equation with physical parameters $G=-2$, $\nu=0.1$, $k=5$. Fig. 5.a surface representation, Fig. 5.b lines of constant flame position.

Eckhaus instability, it is important to note that the cells move only very slightly during the computation. In an usual Eckhaus instability, the cells first move to create compression at some place in the structure, then begin to merge. Here there is practically no phase of cell movements before cell merging. This conclusion is supported by simulations made with the MS equation with a more sinusoidal initial condition.

We now turn to the low k degenerate Eckhaus instability (see Fig. 1). We will use here the MS equation but similar phenomena occur with the complete isobaric equations. We use the parameters $G=-2$, $\nu=0.1$, $k=3.2$ (the band of linearly unstable wave vectors is $[2.76, 7.24]$, the band of wave vectors inside the theoretical Eckhaus parabola being $[3.70, 6.29]$ (see Section 3)). Here and in the rest of the paper, we will always use $\nu=0.1$ because we had some numerical experience with this value of the “viscosity” for $G=0$ and $k=1$, but actually we could have as well chosen the units such as $\nu=1$. We start from a six cells solution (Fig. 3.a). Because the initial k chosen lies outside the theoretical Eckhaus stable band, it is not a surprise that we observe a secondary instability. But the type of instability is surprising: a tip splitting mechanism occur; each cell breaks in

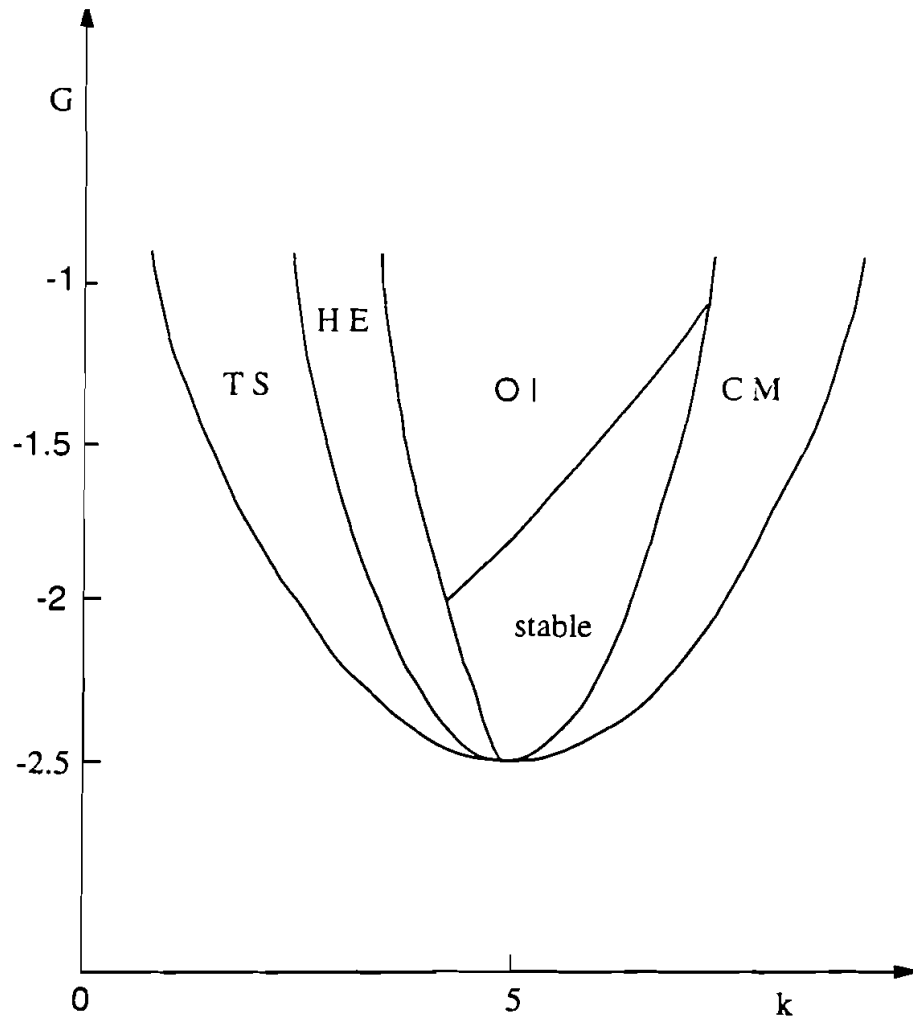


FIGURE 6 Sketch of the 2D stability domain (possible instabilities are cell merging (CM) tip splitting (TS) hexagonal Eckhaus (HE) oscillatory instability (OI)).

two parts, from the initial six cells we now get twelve cells (Fig. 3.b)! The twelve cells solution has now a value of k that is too high and still outside the Eckhaus stable band, but after some cell merging (not plotted) of the type previously described, a stable eight cell solution will appear.

To conclude on compression dilatation instabilities in 1D fronts, degenerate Eckhaus instabilities are observed approximately at the location foreseen by theoretical analysis. Nevertheless, they take unusual forms because the linear instability appears at very low k : cell merging and tip splitting mechanisms are observed on the high and the low k sides respectively. We note that cell destructions, without preliminary displacement, have been also observed in directional solidification (see Bechofer and Libchaber (1986)).

Let us now describe the oscillatory instability that occurs for sufficiently high G (see Fig. 1). For the moment, we have only observed this instability with the MS equation,

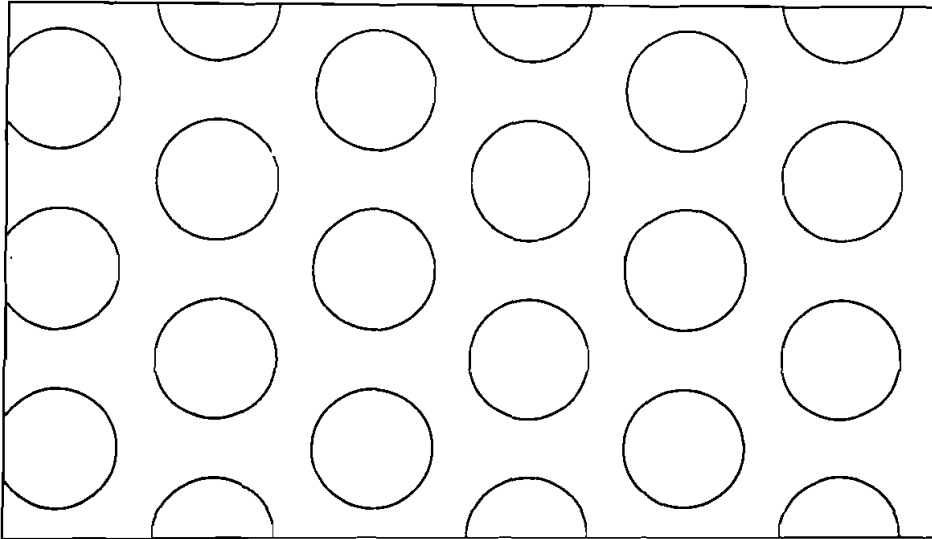


FIGURE 7 (a) 2D example of cell merging ((physical parameters $G=-2$ $\nu=0.1$ $k=6.8$). Evolution of the solution with time: the amplitudes of some cells decrease (Fig. 7.b); the cells merge (Fig. 7.c); the final stationary solution is obtained (Fig. 7.e).

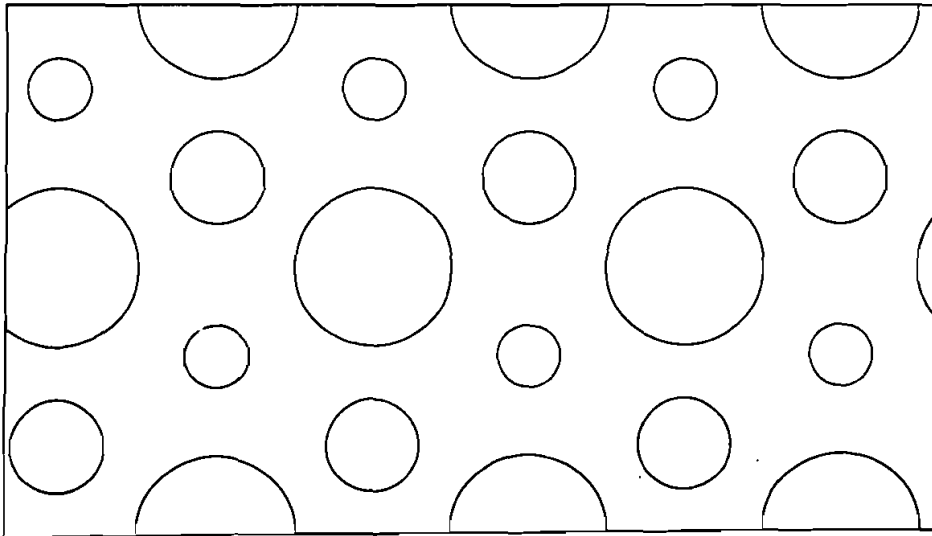


FIGURE 7 (b) 2D example of cell merging ((physical parameters $G=-2$ $\nu=0.1$ $k=6.8$). Evolution of the solution with time: the amplitudes of some cells decrease (Fig. 7.b); the cells merge (Fig. 7.c); the final stationary solution is obtained (Fig. 7.e).

and have not tried to find it with the complete isobaric equations, because this instability is relatively slow and much more difficult to locate in parameter space than the previous Eckhaus type instabilities. As a result, a numerical simulation of this effect with the

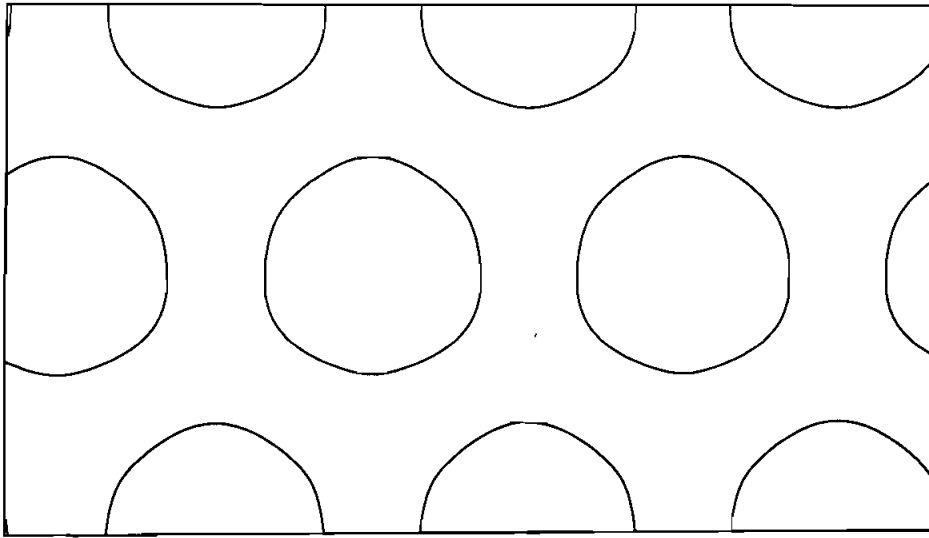


FIGURE 7 (c) 2D example of cell merging ((physical parameters $G=-2$ $\nu=0.1$ $k=6.8$). Evolution of the solution with time: the amplitudes of some cells decrease (Fig. 7.b); the cells merge (Fig. 7.c); the final stationary solution is obtained (Fig. 7.e).

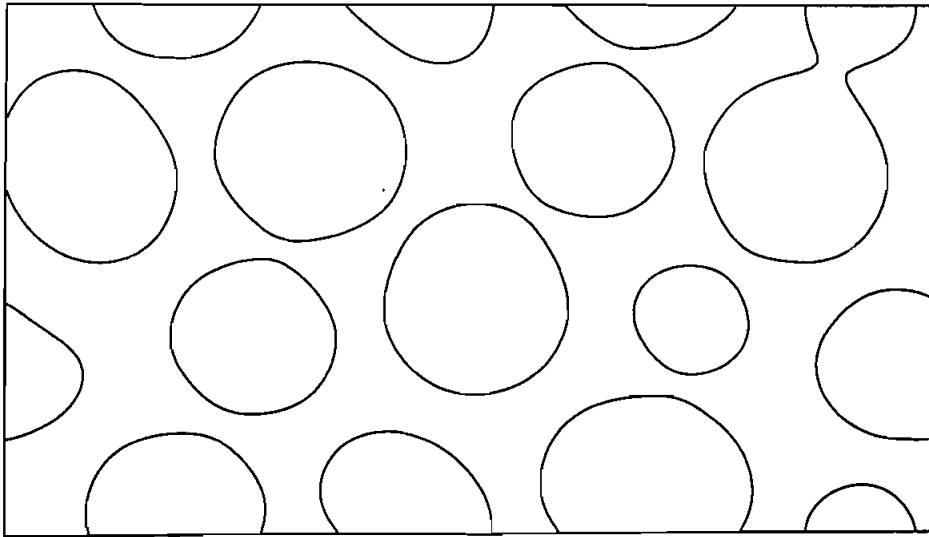


FIGURE 7 (d) 2D example of cell merging ((physical parameters $G=-2$ $\nu=0.1$ $k=6.8$). Evolution of the solution with time: the amplitudes of some cells decrease (Fig. 7.b); the cells merge (Fig. 7.c); the final stationary solution is obtained (Fig. 7.e).

complete equations would be extremely expensive. We take the parameters $G=-1.25$, $\nu=0.1$, $k=4.8$.

This instability is actually a subharmonic instability, i.e. a modulation of the cellular structure with a wavelength of twice the original one (i.e. a wave vector twice as small) develops (Fig. 4.a). This modulation oscillates in time (Fig. 4.b) and grows. After a

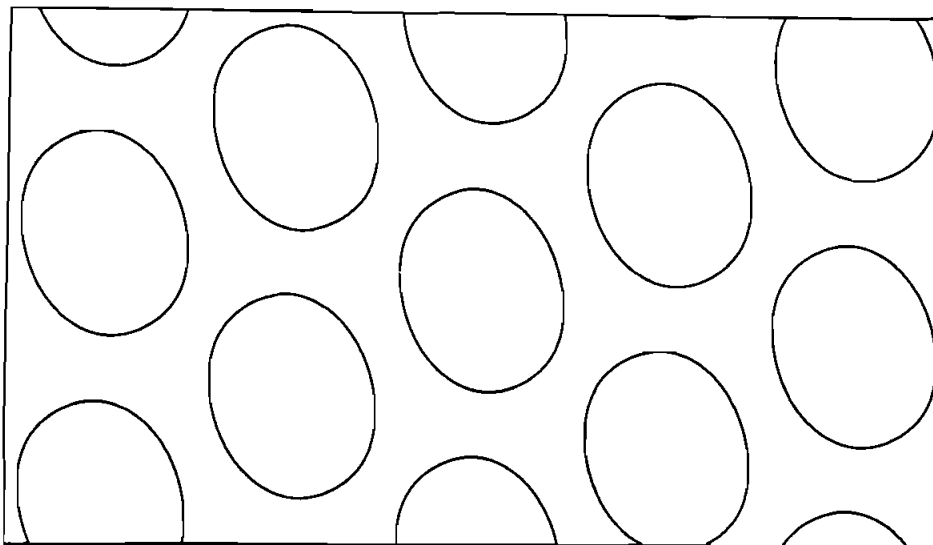


FIGURE 7 (e) 2D example of cell merging ((physical parameters $G=-2$ $\nu=0.1$ $k=6.8$). Evolution of the solution with time: the amplitudes of some cells decrease (Fig. 7.b); the cells merge (Fig. 7.c); the final stationary solution is obtained (Fig. 7.e).

short time the modulation completely destroys the basic cellular structure and a self turbulizing flame is obtained (Fig. 4.c). The integration has been continued for a long time: the solution remains always chaotic. We will see in the next section that similar phenomenons can be observed in the case of 2D fronts.

It might in principle be possible to build a theoretical model exhibiting this oscillatory (subharmonic) instability. One should truncate the problem by analytically retaining only the lowest order Fourier modes, thereby obtaining a model close to the Lorenz model (Lorenz (1963)) of convection. But it seems to be difficult to do such a truncation, because we have verified that the amplitude of the Fourier modes decreases in an extremely slow way. A pertinent model should then contain a high number of modes, and would not be very tractable.

5 NUMERICAL SIMULATIONS OF 2D FRONTS

To deal with the instabilities of 2D cellular fronts, we have confined ourselves to the 2D MS equation, because the possibility of an equivalent 3D simulation of the isobaric equations seems to be completely unrealistic for the moment. First of all, let us note that a numerical simulation of the 2D MS equation is actually rather easy to perform in the framework of Fourier spectral methods. The operator $I(\alpha)$ of the modified MS equation (see Section 2) continues to correspond to the multiplication by the growth rate $\sigma(k)$ in Fourier space, except that now k has to be interpreted as the modulus of a two dimensional wave vector.

We will begin by describing the stable cellular structures occurring in the 2D MS equation. As can be seen in Figure 5.a, and as was previously shown by Michelson and Sivashinsky (1982), the obtained cellular structures are actually hexagonal (the parameters are $G=-2$, $\nu=0.1$, $k=5$). The MS equation closely resembles the experimental results for laminar cellular flames (see Markstein (1951), Quinard *et al.* (1984)). Note however

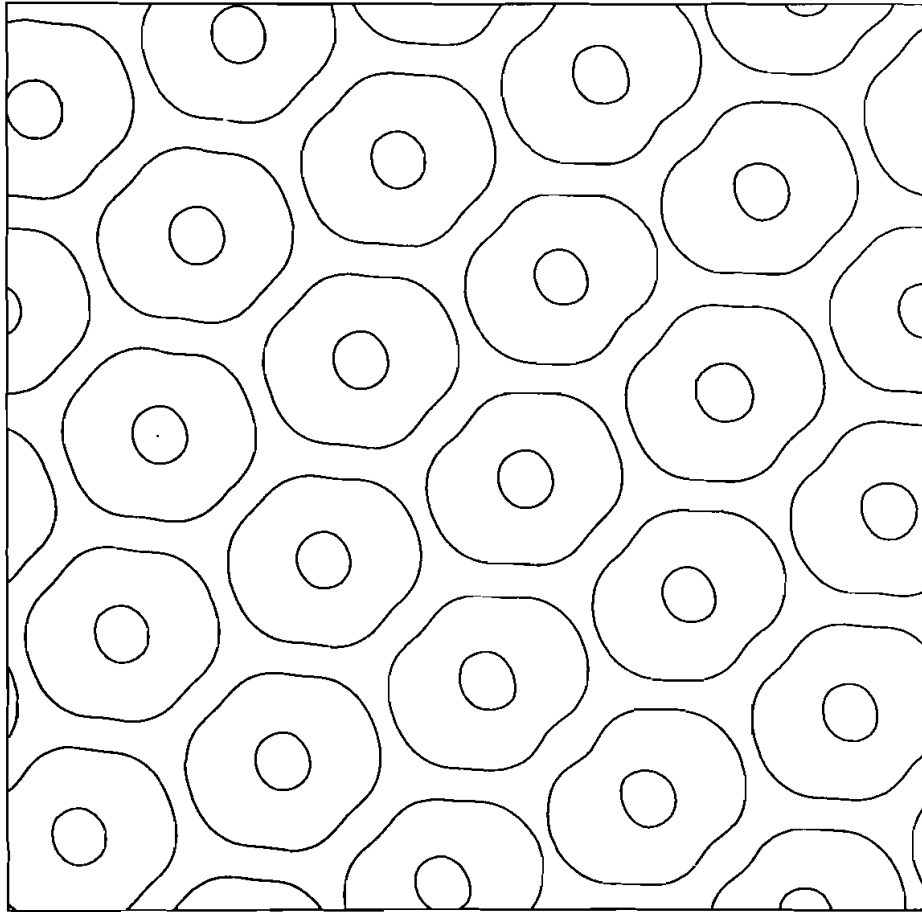


FIGURE 8 2D example of tip splitting (physical parameters $G=-2$ $\nu=0.1$ $k=3$). A new cell begins to grow inside each old cell.

that hexagonal structures can also be obtained in the framework of the thermal diffusive model (Shtilman and Sivashinsky (1990)). We won't be interested in these stationary solutions, only in non linear instabilities of this type of cellular flame.

In Fig. 5.a, the solution was plotted as a surface in a 3D space. This representation looks very like the experimental photographs, but is rather difficult to interpret when instabilities develop. We have chosen instead to plot the lines of constant flame position α , and, in order to get easy to read figures, to only plot one line of this type, i.e. the line corresponding to the mean of the minimum and the maximum of α . This value is of course not constant with time but depends on the form of the front obtained at a certain time step. This representation will permit us to give a good qualitative idea of the solution; as an example we have plotted in this manner in Figure 5.b the same stationary solution as in Fig. 5.a. As can be seen in this figure, the lines plotted look essentially circular.

Limits of stability are plotted in Figure 6. A parametric study was undertaken in order to discover whether the basic cellular solution was stable or not. Most of the instabilities

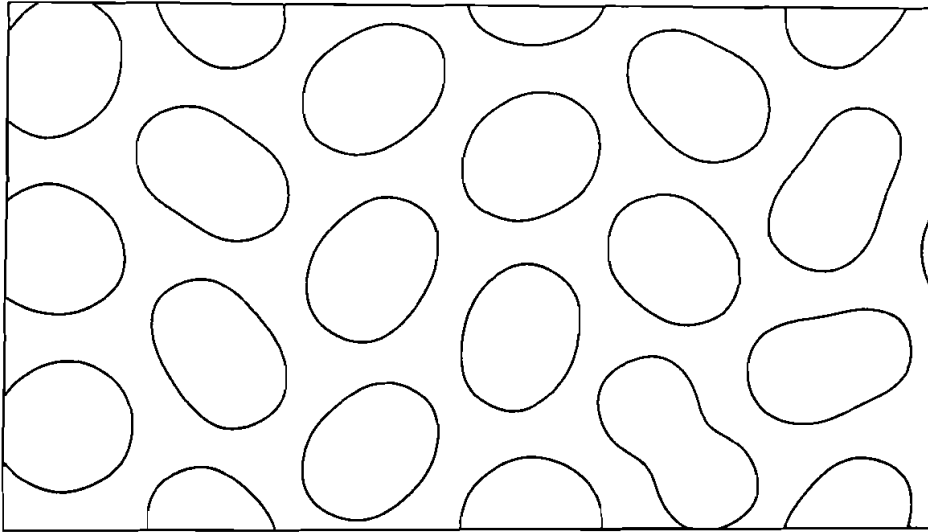


FIGURE 9 Hexagonal Eckhaus instability ($G=-2.25$ $\nu=0.1$ $k=4.5$). A dilated cell begins to break into two new cells.

encountered are the same as in the 1D case: we have 2D equivalents of tip splitting on the low k side, cell merging on the high k side, and oscillatory (subharmonic) instability for sufficiently high G . The only differences are that the oscillatory instability occurs for lower values of G , and a new instability exists close to the tip splitting, that we have called hexagonal Eckhaus.

Of course, the qualitative aspects of these different instabilities are not quite the same in two dimensions. In each case, we initiate the calculation with a stable hexagonal structure and study the instabilities that develop.

An example of cell merging is shown in Figure 7 (the parameters are $G=-2$, $\nu=0.1$, $k=6.8$). We start from a 18 cell solution (Fig. 7.a); after some time we obtain Fig. 7.b: some cells have an amplitude that decreases (the circles have a lower radius). On the contrary, it is easily seen in the same figure that the relative amplitude of other cells grow (the circles have a greater radius). Then the cells merge, and we obtain in Figure 7.c a six cell solution. This time the wavelength of the solution is too small to lie in the stability domain of Fig. 6, thus the cells break into new smaller cells and the disordered solution of Fig. 7.d is obtained. After some time this disordered solution becomes the ten cell regular stationary solution of Fig. 7.e. One can conclude from this simulation that, as in one dimension, the role of the Eckhaus type instabilities is to select a cellular solution, whose wave vector lies in the stable domain of Figure 6, in general completely different from the original solution. In the case of tip splitting and of hexagonal Eckhaus instabilities, we shall only show how the instability develops and not the subsequent evolution toward another cellular solution. What is really important is to determine the stable domain of Figure 6, every final solution produced by the Eckhaus instabilities has to lie in this domain.

In Figure 8, we show an example of tip splitting (the parameters are $G=-2$, $\nu=0.1$, $k=3$). As in 1D, a new cell appears in the middle of each initial cell, in the rounded part of the front that would be pointing towards fresh gases in a real flame. This mechanism is very fast, and one obtains a lot of small cells that are again unstable because the wave vector is now too great; after a complicated evolution of the type shown in Figure 7, a cellular solution with a wave vector in the stable range of Fig. 6 would be obtained.

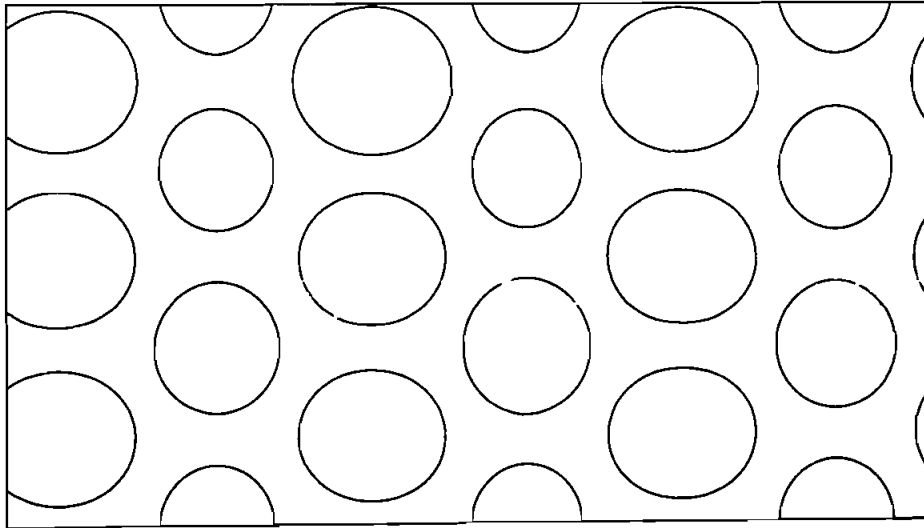


FIGURE 10 (a) Development of the oscillatory instability in 2D ($G=-1.6$ $\nu=0.1$ $k=5.3$).

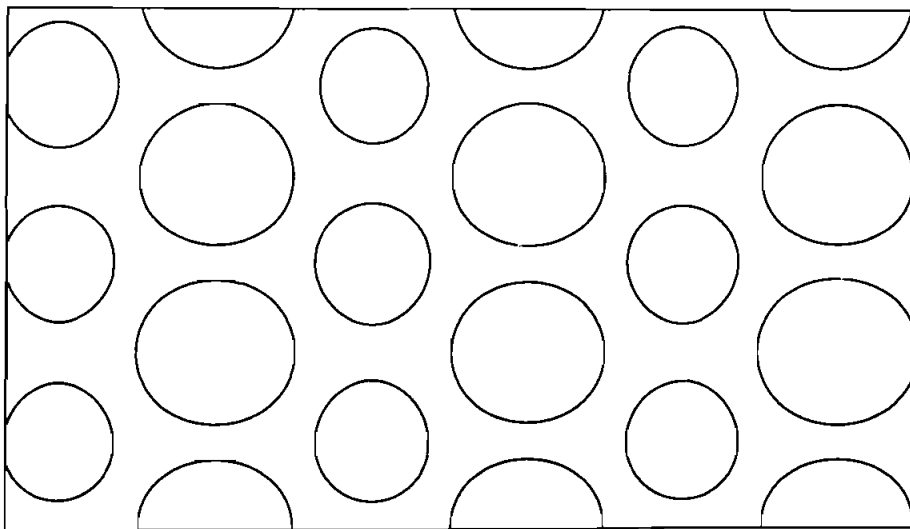


FIGURE 10 (b) Development of the oscillatory instability in 2D ($G=-1.6$ $\nu=0.1$ $k=5.3$).

In Figure 9, a case of hexagonal Eckhaus instability is presented ($G=-2.25$, $\nu=0.1$, $k=4.5$). As in standard Eckhaus instability, the structure gets compressed in some places, dilated in others, but above all the structure is twisted. Then a new cell forms at a place where dilatation occurred (Fig. 9 is plotted at the moment where an old cell breaks into two new cells), and as usual a solution with a wave vector in the stable range would appear after a long transient.

A theoretical description of instabilities of hexagonal structures close to threshold can be found in Caroli, Caroli and Roulet (1984). Essentially an hexagonal structure can be considered as the superposition of three systems of rolls, with angles $2\pi/3$ between each

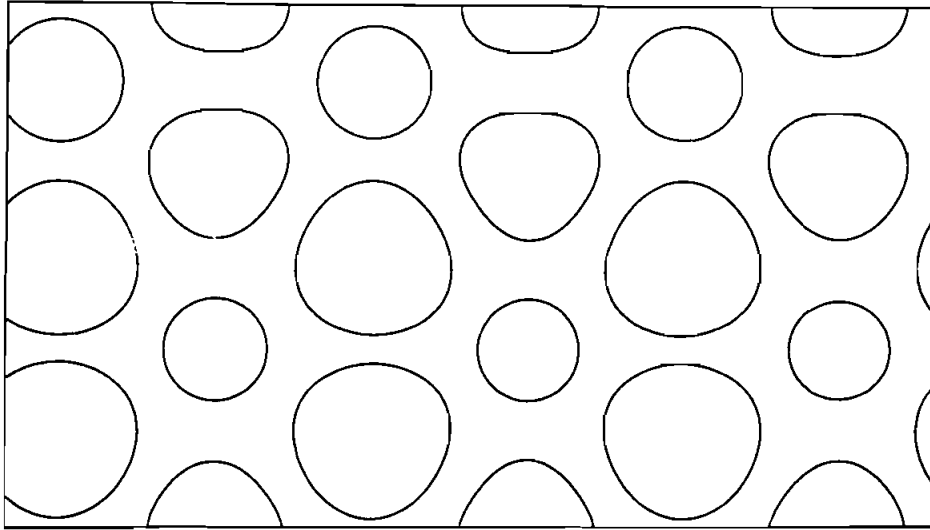


FIGURE 10 (c) Development of the oscillatory instability in 2D ($G=-1.6$ $\nu=0.1$ $k=5.3$).

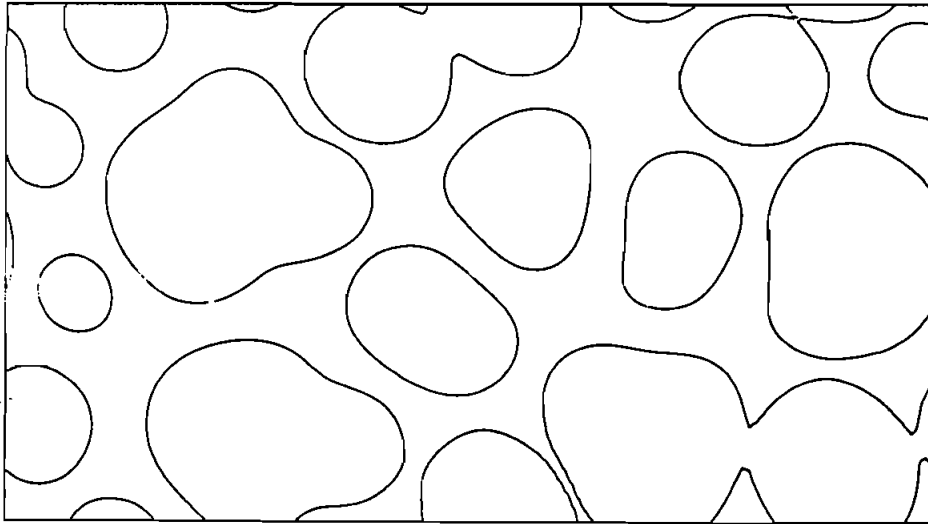


FIGURE 11 Example of 2D self turbulizing flame obtained for $G=-1.5$ $\nu=0.1$ $k=5.3$.

others. Each system of roll has its own amplitude, and the structure can be properly described by using three amplitude equations coupled. The coupling not only involves cubic terms, but also quadratic terms. Instabilities of Eckhaus type, as we have observed here, can be found, but apparently only in the case where quadratic terms are sufficiently low. The observations we have made seem compatible with these results, even though we are here in a case of low wavevectors, which, as we have previously seen, modifies the qualitative aspects of the different instabilities.

In Figure 10 ($G=-1.6$, $\nu=0.1$, $k=5.3$), we show the development of an oscillatory

instability. As in the 1D case, the instability produces spatial subharmonics of the basic cellular structure (both small and big cells can be seen when moving in the x direction). The oscillatory character of the instability becomes evident when an inversion of these small and big cells appear. As in the 1D case, these oscillations never saturate to a finite amplitude, on the contrary, the basic cellular structure gets destroyed by the oscillations, leading to a self turbulizing flame. An example of such a flame is plotted in Fig. 11, which corresponds to the slightly different parameters $G=-1.5$, $\nu=0.1$, $k=5.3$. This type of self turbulizing cellular solution was first obtained in the pioneering paper of Michelson and Sivashinsky (1982). The model used was the same, with the same periodic boundary conditions, however it was considered in connection with the problem of expanding spherical flames. Our results confirm the existence of these self turbulizing solutions and make clear that the origin of these solutions is the subharmonic instability previously described. Also the results are interpreted for the case of a downward propagating flame submitted to a hydrodynamic instability. As previously noted, the 2D instability exists however for lower values of G than the 1D one. It seems that the domain of parameters where the instability grows is modified a little with different boundary conditions (i.e. for instance a larger computational domain with a structure of the same wave vector). But an oscillatory instability exists in any case.

6 CONCLUSION

In this paper we have been interested in secondary instabilities of cellular premixed flames propagating downwards. We have been able to show that, even relatively close to threshold, a lot of instabilities can be found. Some instabilities have the role of changing the number of cells in the domain when the wave vector gets too high or too low, they produce cellular stationary solutions completely different from the original one. The oscillatory (subharmonic) instability occurs at higher values of the control parameter G (i.e. corresponds to faster flames). It has the important property of producing self turbulizing flames quite similar to those observed in experiments. These results are obtained in a model based on the Landau picture where the basic cellular structure is caused by an hydrodynamic instability. Our study is limited to the case of periodic boundary conditions; however it seems that the hydrodynamic instability could be sufficient to explain experimental results on (plane on average) cellular downward propagating flames in tubes.

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