Internship Proposal

Internal shear layers and zonal flows in rotating and stratified fluids

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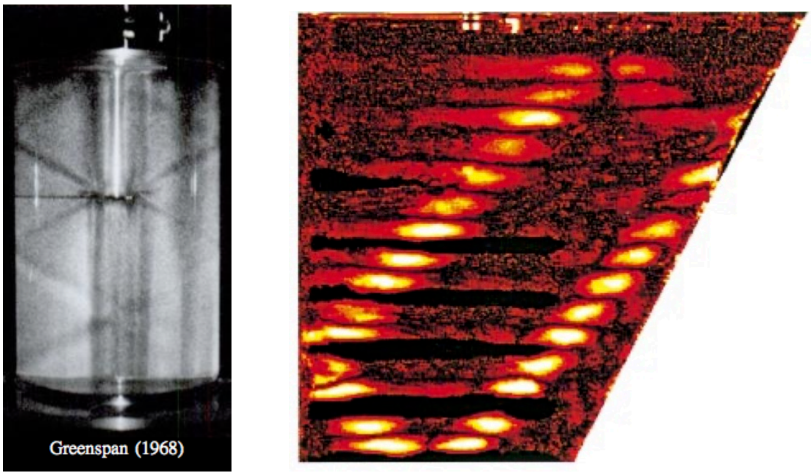
Type of project : Numerical and theoretical

Stratified and/or rotating fluids support waves that can transport and dissipate energy away from their sources. These waves are suspected to play an important role in atmospherical sciences, oceanography, and in the dynamics of stars and planets. In the atmosphere, they transport momentum from the convective regions to the high altitude regions where they break and create mean flows. In the ocean, they are mostly generated by tides and winds and are expected to provide the missing contribution to the global energy budget of the ocean. In planets and stars, they are excited by gravitational effects or convection and could play a role in the generation of zonal flows [1] and in dissipative processes [2].

In a fluid, rotating with the rotation rate Ω around the axis Oz and stably stratified with a constant buoyancy frequency N along the same axis, an harmonic forcing excites waves when its frequency ω lies within the inertia-gravity interval $\min(N,2\Omega) < \omega < \max(N,2\Omega)$. These waves propagate along cones (in 3D) or planes (in 2D) with a fixed angle α with respect to the horizontal plane given by $\sin^2\alpha = (\omega^2 - 4\Omega^2\})/(N^2 - 4\Omega^2)$.

The cone (or the plane) tangent to the oscillating object or to a local topographical feature corresponds to a “critical” surface across which the wave field changes of nature. The singularity of the wave field across these surfaces if smoothed by viscosity gives rise to thin internal shear layers which possess some generic features [3,4]. These internal shear layers are for instance visible when oscillating a small object as they form the familiar St Andrews cross pattern (see figure 1a).



(a) (b)

Figure 1: *(a) St Andrews cross pattern obtained by oscillating an object in a rotating fluid [5]; (b) Internal wave attractor in a confined stratified fluid [6].*

In a closed domain, the waves reflect on boundaries while keeping the same propagation

angle. Depending on the geometry and the propagation angle, they may contract and focus on a specific region of space forming an attractor. In the presence of weak viscosity, these attractors also manifest as thin internal shear layers. They have been experimentally evidenced for a stratified fluid in 2D container (see figure 1b).
For a rotating fluid in a spherical shell geometry, both critical surfaces and attractors can be present at the same time. Various wave patterns can then be obtained when the frequency and the nature of the harmonic forcing are changed.
A spectral numerical code has been developed to analyse these structures in this geometry for very small viscosity [7]. So far, it has been used to obtain the linear response. Examples of linear wave patterns are illustrated in figure 2.

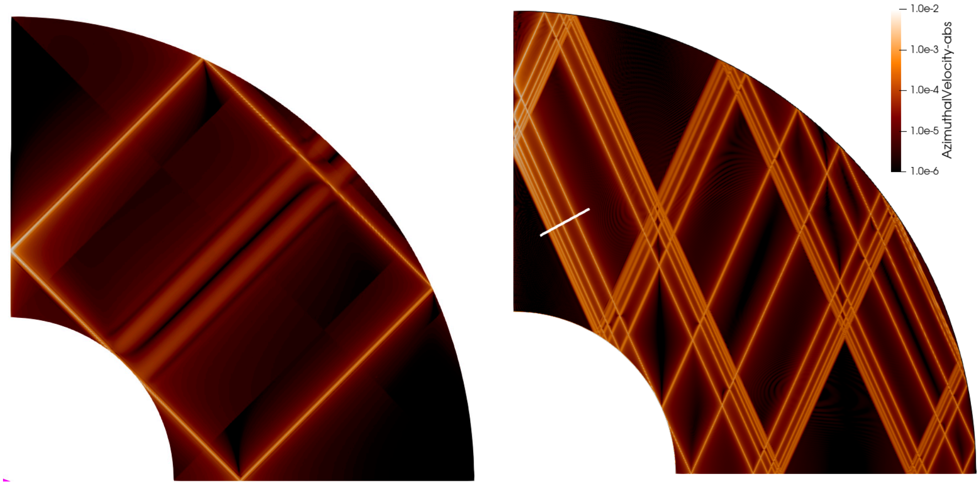


***Figure 2**: Examples of linear wave patterns obtained by librating the inner core for a rotating fluid in a spherical shell (left: ω/Ω=1.41; right: ω/Ω=0.82) (© J. He).*

During the internship, this code will be adapted to determine the first nonlinear corrections. We shall focus on the mean flow correction which is expected to have a specific cylindrical structure. The mean flow correction will be analysed for harmonic responses of increasing complexity. The objective will be to try to understand its structure using recent asymptotic results obtained for small viscosity [8].

Opportunities to continue the work in a PhD Project will be proposed.

Références :

[1] TILGNER, A. 2007 Zonal wind driven by inertial modes. *Phys. Rev. Lett.* **99,** 194501.

[2] OGILVIE, G. I. & LIN, D. N. C. 2004 Tidal dissipation in rotating giant planets. *Astrophys. J.* **610**, 477–509.

[3] MOORE, D. W. & SAFFMAN, P. G. 1969 The structure of free vertical shear layers in a rotating and the motion produced by a slowly rising body. *Phil. Trans. R. Soc.* A **264**, 597-634.

[4] LE DIZES, S & LE BARS, M. 2017 Internal shear layers from librating objects. *J. Fluid Mech*. 826 653-675.

[5] GREENSPAN, H. P. 1968 The Theory of Rotating Fluids. Cambridge University Press.

[6] MAAS, L. R. M., BENIELLI, D., SOMMERIA, J. & LAM, F.-P. A. 1997 Observation of an internal wave attractor in a confined, stably stratified fluid. *Nature* **388**, 557–561.

[7] HE, J., FAVIER, B., RIEUTORD, M. & LE DIZES, S. 2022 Internal shear layers in librating spherical shells: the case of periodic characteristic paths. *J. Fluid Mech.* **939**, A3.

[8] LE DIZES, S. 2020 Reflection of oscillating internal shear layers: nonlinear corrections. *J. Fluid. Mech*. 899, A21.

The candidate should possess basic knowledge of fluid mechanics, a strong interest in fundamental research and a solid mathematical background. Applications should be sent to stephane.ledizes@univ-amu.fr and benjamin.favier@univ-amu.fr