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Long- and short-wave instabilities in helical vortices

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Abstract. We review two instability mechanisms that may be active in wind turbine wakes and contribute to their downstream evolution, by considering simplified configurations of one or several spatially uniform helical vortices. One category of instabilities involves displacement perturbations of the vortices, with wavelengths that are large compared to the size of their cores; they can be analysed using a filament approach. Previous theoretical results, confirmed by our recent experiments, show that the predicted instability modes are related to the pairing phenomenon found in periodic arrays of vortices. A second group of instabilities involves internal perturbations of the vortex cores, with wavelengths scaling on the core size. They result from deformations of the cores due to curvature, torsion or the strain induced by neighbouring helix loops. Our experiments show that the non-linear evolution of the short-wave instabilities, combined with the pairing mechanism, leads to a rapid destruction of the helical wake vortices.

1. Introduction

The near and intermediate wake behind a horizontal-axis wind turbine – or any rotor, in general – consists of a system of interlaced helical vortices generated at the rotor blade tips. This initially well-organized vortex system evolves into a turbulent far wake over a downstream distance of several rotor diameters. Although much of this evolution is dictated by atmospheric conditions, intrinsic vortex instabilities may also play a role in this transition.

We here consider an idealized model of a rotor wake, which consists of either one or several (N) interlaced infinite helical vortices, and possibly a straight central “hub” vortex as depicted in figure 1. A single helical filament is characterised by its circulation Γ and core radius a , as well as the helix radius R and pitch h , which leads to three non-dimensional parameters: the Reynolds number $Re = \Gamma/\nu$ (ν : kinematic viscosity) and two length ratios, *e.g.*, a/h and h/R . For multiple helices, h is the distance between neighbouring vortices, and N is an additional parameter. In this paper, we consider configurations with small core size (a/h), small pitch (h/R), and Reynolds numbers sufficiently large ($Re \gtrsim 10^4$) that viscous effects can be neglected.

The stability of helical vortex systems has been analysed theoretically by a number of authors. These studies can be divided into two groups, according to the type of vortex interaction considered for the stability analysis. One mechanism involves perturbations displacing the vortices locally as a whole, *i.e.* without change of their core structure. The vortices can then be approximated by filaments, whose shapes evolve under the effect of their mutually- and self-induced velocities. A second mechanism is linked to the amplification of short-wave perturbations inside the vortex cores. The

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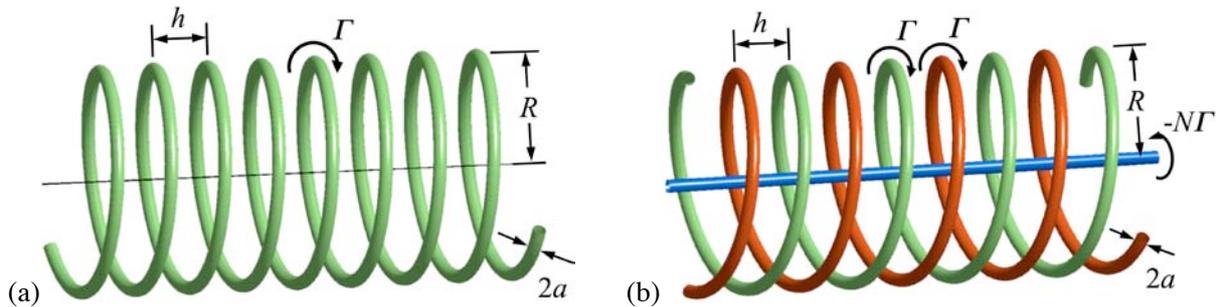


Figure 1. Schematics of (a) a single helical vortex and (b) two interlaced helical vortices with a central “hub” vortex, including the relevant flow parameters.

growth of these modes is due to the modification of the core structure by an external strain field, induced, *e.g.*, by a second nearby vortex, or by the curvature and/or torsion of the vortex.

In the following, we recall the main conclusions, from past studies and on-going work, concerning the two instability mechanisms in helical vortices, and show examples of our recent experimental observations illustrating these phenomena.

2. Long-wave instabilities

The stability of a single helical vortex filament with respect to displacement perturbations was studied by Widnall [1] and subsequently by Fukumoto & Miyazaki [2] (a much earlier study by Levy & Forsdyke [3] of the same configuration was shown to be erroneous). Gupta & Loewy [4] and Okulov & Sørensen [5] have extended the analysis to systems of more than one helix, in the latter case also including a straight central vortex.

For small h/R and a/h , various unstable modes are found, consisting of periodic deformations (displacements) of the vortices at different wavelengths in all of these configurations. The most unstable deformations are characterised by an out-of-phase displacement of successive helix turns, leading to a pairing of these loops. The corresponding growth rates (σ) are close to the value $\sigma = (\pi/2)(\Gamma/2h^2)$ for the classical pairing instability of an infinite row of point vortices in two dimensions [6]. If one defines a perturbation wavenumber k as the number of wavelengths in one helix turn, then out-of-phase displacements are obtained for a single helical vortex when k is an odd multiple of $1/2$. Two such cases are shown schematically in figures 2(a) and 2(b), where they are compared to dye visualisations of a rotor wake. The corresponding experiments were carried out in a water channel, using a single-bladed rotor of diameter 160 mm. Details about the experimental set-up and flow conditions can be found in [7] and [8]. The helical wake vortex was perturbed by a controlled modulation of the rotation rate of the rotor, in order to trigger the same displacement perturbations as considered in the theoretical stability analyses. Whereas the theory predicts a spatially uniform temporal growth in an infinite helical vortex, the corresponding perturbations evolve spatially (in the downstream direction) in the rotor wake. The white triangles in figure 2 indicate the approximate location where these perturbations have the same amplitude as in the corresponding schematic from theory; the qualitative agreement is very good. The growth rate of the unstable perturbations was determined from video recordings of dye visualizations such as in figure 2 [8]. The values are very close to the theoretical prediction for the experimental flow configuration (figure 3). At late times, the non-linear growth of the unstable displacement perturbations results in a complicated three-dimensional vortex structure, which can be seen to the right of the triangles in figure 2. Surprisingly, no significant merging or breakdown of the vortices into small-scale structures is seen in the observed field of view, which covers a downstream distance of about 3.5 rotor diameters.

Whereas for a single helix, pairing can only occur locally (with respect to the azimuthal direction of the cylindrical coordinate system associated with the helix), *e.g.* at the top in figure 2(a), multiple

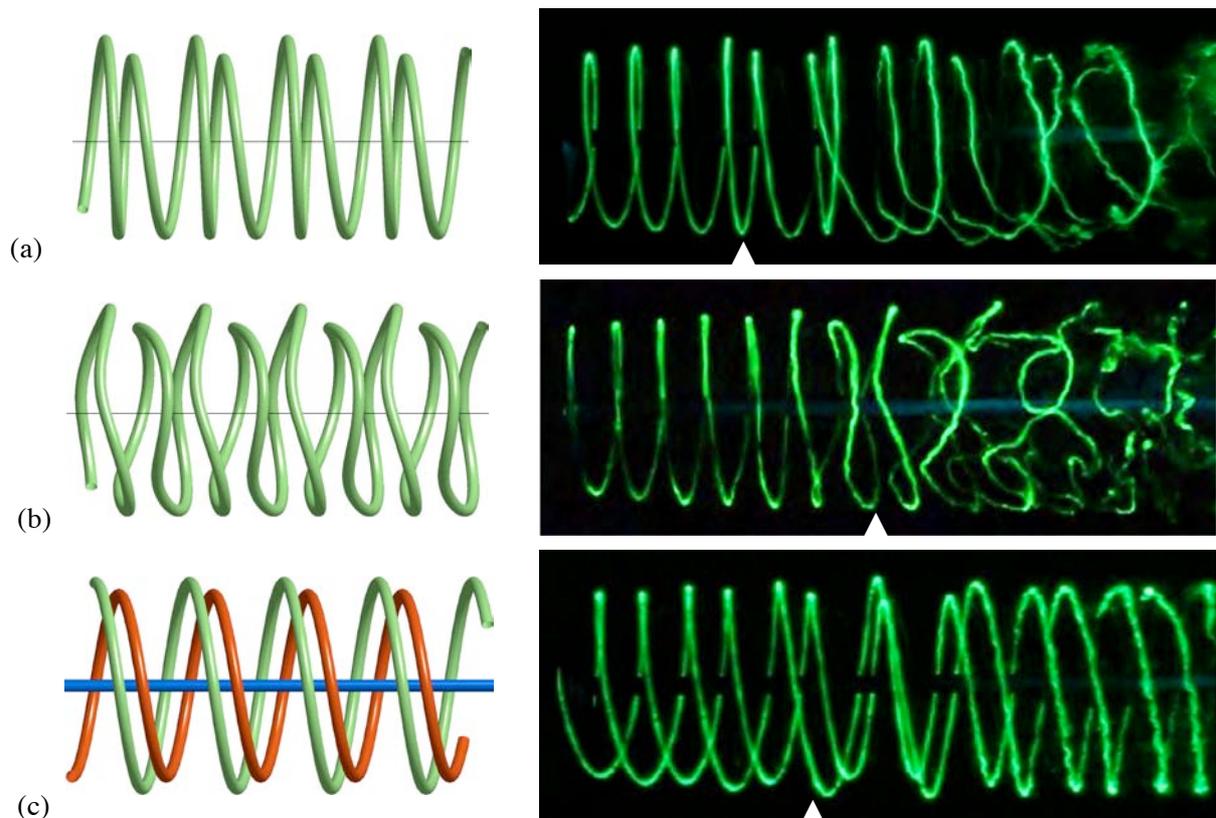


Figure 2. Examples of unstable displacement perturbations. Single helix with (a) $k = 1/2$ and (b) $k = 5/2$; (c) two helices with uniform radial perturbations. Left column: theoretical predictions by (a,b) Widnall [1] and (c) Okulov & Sørensen [5]. Right column: experimental visualisations of one- and two-bladed rotor wakes in a water channel [8]. The rotor is on the left; flow is from left to right. The white triangles show the location where the perturbation amplitude is approximately equal to the one in the schematic on the left.

helices allow for a global (azimuthally uniform) pairing between different helices. This is illustrated in figure 2(c), again comparing the theoretical prediction [5] with the corresponding experimental visualization of a two-bladed rotor wake [8]. As for the single helix, the qualitative and quantitative agreement is good: the initial perturbation structure is the same in theory and experiment (one helix contracts, the other one expands), even if for the latter its amplitude increases in the downstream direction; and the measured growth rate is very close to the relevant theoretical prediction [5] (figure 3b). The late-time behaviour is again quite surprising: after a swapping of neighbouring helix loops (just to the right of the triangle in figure 2c), the vortices rearrange themselves to form again a double-helix structure, similar to the one of the initial state near the rotor. As before, no breakdown is observed; the vortices remain concentrated and intact.

Systems of two or more interlaced helical vortices, such as the ones generated by multi-bladed rotors, may exhibit a combination of unstable modes, involving local and uniform pairing (see, *e.g.*, the simulations by Ivanell *et al.* [9] and the experiments by Felli *et al.* [10]). The resulting deformations may have quite a complicated structure, but as long as the associated wavelengths are large compared to the vortex core size, they can in general always be linked to some type of pairing mechanism, due to the mutual induction between the different parts of the wake vortices.

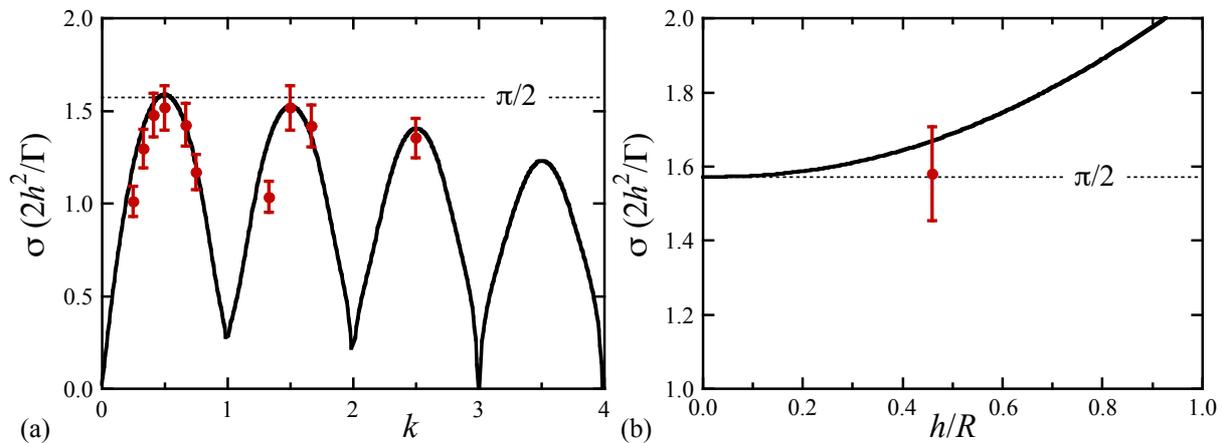


Figure 3. Growth rates of the pairing instability: comparison between experiment (symbols, $Re = O(10^4)$) and theory (lines, inviscid). (a) Local pairing of a single helix: experiment with $a/R = 0.058$ and $h/R = 0.55$; theoretical result for the same parameters obtained from Gupta & Loewy [4]. (b) Uniform pairing of a double helix: experiment with $a/R = 0.054$ and $h/R = 0.46$; theoretical result according to Okulov & Sørensen [5] for the same core size and perturbation mode.

3. Short-wave instabilities

Short-wavelength instabilities develop inside the vortex cores, and the wavelengths of the corresponding perturbations are typically of the order of the core diameter. They are associated with a mechanism of resonant amplification of vortex waves (Kelvin modes) through an interaction with the background flow. Kelvin modes are vortex perturbations with a velocity field of the form $\mathbf{u}(r) \exp(i\omega t + im\theta + ik_z z)$ in the local cylindrical coordinates (r, θ, z) of the vortex. ω is a (complex) frequency, and m and k_z are the azimuthal and axial wavenumbers. The Kelvin modes of axisymmetric (isolated) straight vortices are usually neutral or slightly damped. The short-wavelength instability mechanism has been described first in the context of columnar vortices subject to an external strain field [11], which can be induced by other nearby vortices or boundaries. It is now termed “elliptic instability”, owing to the elliptic streamlines (see figure 4c) of such strained vortices [13]. In this case, two perturbation waves of the vortex are resonantly coupled with each other through the background strain field. The conditions of resonance are such that, when there is no axial flow within the vortex, the most unstable perturbation involves two helical stationary waves, of azimuthal wavenumbers $m = -1$ and $m = 1$, and deforms the vortex core in a sinusoidal way (figure 5a). In the presence of an axial core flow, two different waves, with $m = 0$ and $m = 2$, may become the most unstable resonant configuration [17], producing a double-helix perturbation of the core (figure 5b).

A similar framework can be used to analyse the short-wavelength instability of helical vortices, when the core size a is small compared to both the pitch h and the helix radius R . In that case, the helical vortex is similar to a columnar vortex deformed at order a/h by curvature effects and at order $(a/h)^2$ by torsion and strain [19]. The presence of the strain field at the second order indicates that the elliptic instability is also expected to be present in helical vortices.

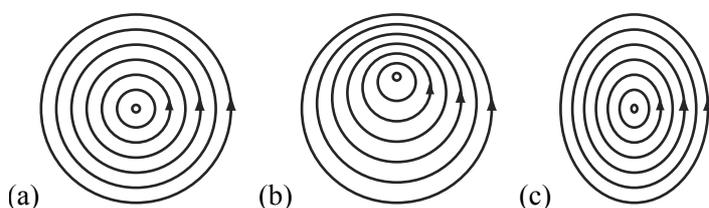


Figure 4. Schematic vortex core streamline patterns for axisymmetric flow (a), and flow with deformations of azimuthal wave-number $m = 1$ (b) and $m = 2$ (c).

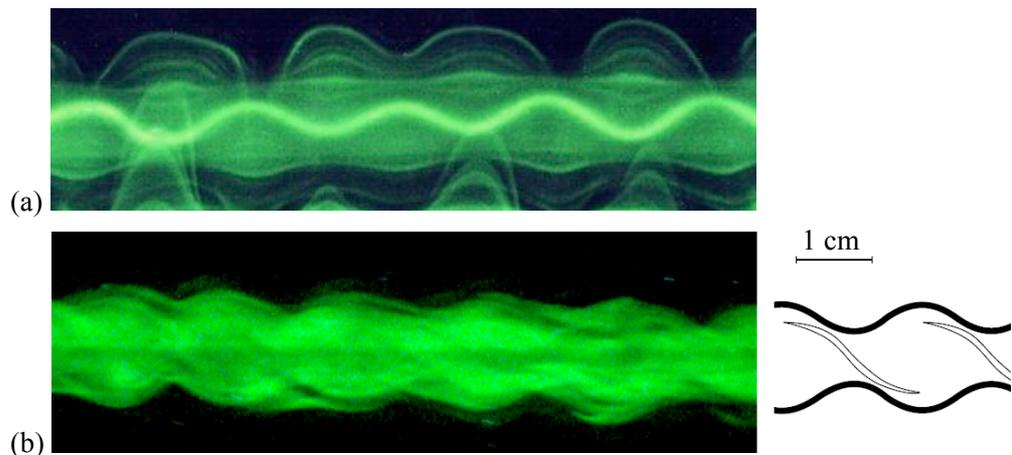


Figure 5. Experimental dye visualizations (side views) of the elliptic instability of strained vortices: (a) without and (b) with axial flow in the core (from [15] and [16], respectively).

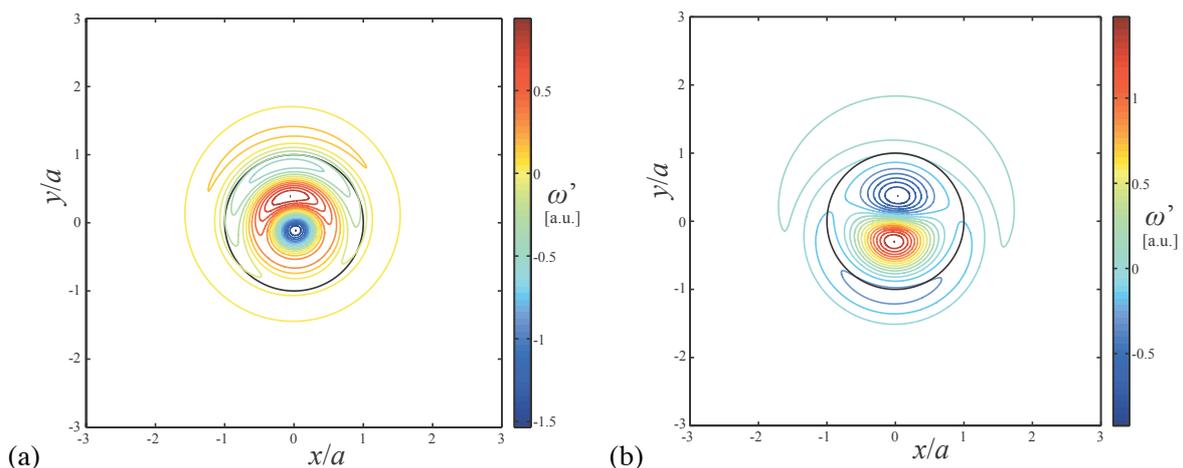


Figure 6. Most unstable perturbation modes for the short-wave curvature instability of a helical vortex with the same parameters as measured in the experiments: $a/R = 0.058$ and $h/R = 0.55$. Shown is the axial component of the perturbation vorticity in a plane perpendicular to the local vortex axis. (a) Helix with weak axial core flow: $2\pi a V_a / \Gamma = 0.2$; (b) helix with strong axial flow: $2\pi a V_a / \Gamma = 0.5$, representative of the experimental case.

The deformation associated with curvature is stronger and has a different azimuthal structure ($m = 1$) than a strain field ($m = 2$) – see figure 4; other resonances between vortex waves are thus *a priori* possible. For a helical vortex with a constant-vorticity core (Rankine vortex), Hattori & Fukumoto [20] demonstrated that these new resonances are indeed associated with instability. In the present work, we show that curvature is also a source of instability when the vortex has a more realistic Gaussian vorticity profile, and that the presence of flow along the vortex axis inside the core can have a large influence on the mode structure and growth rate. We find that the waves involved in the resonance are different from those obtained for the Rankine vortex. In the absence of axial core flow, or with only weak axial velocities, they exhibit a more intricate structure, as shown, *e.g.*, in figure 6(a). For a larger axial flow, the most unstable resonant waves have a simpler structure (figure 6b), which resembles the elliptic instability mode for vortices without axial flow. The axial

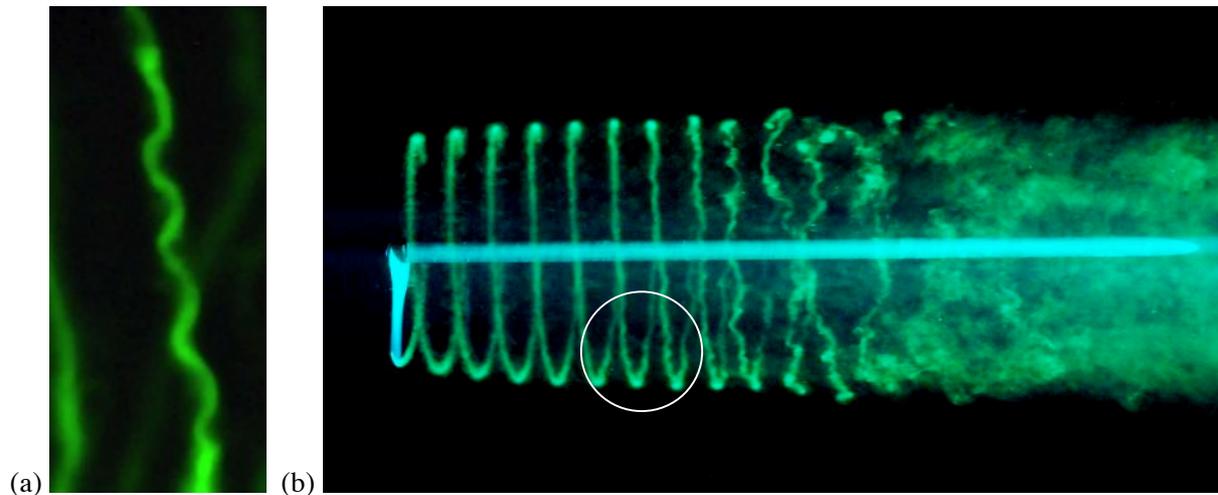


Figure 7. Examples of short-wave instability in a rotor wake (dye visualizations in water). (a) Close-up of a short-wave perturbation behind a two-bladed rotor. (b) Evolution of a low-pitch helical wake of a one-bladed rotor. The simultaneous presence of short waves and pairing leads to a rapid breakdown of the vortices.

wavelength λ of this curvature instability mode is, however, approximately twice as large. For the parameters of the experiments ($a/R = 0.058$, $h/R = 0.55$, $2\pi a V_a / \Gamma = 0.5$), we obtain a growth rate $\sigma \approx 0.15$ ($\Gamma/2h^2$) and a wavelength $\lambda \approx 5.7a$. The elliptic instability properties of the current flow can be estimated using the results obtained by Lacaze *et al.* [17] for straight vortices. In the absence of axial core flow, the most unstable mode involves a sinusoidal displacement of the vortex centre line (figure 5a), with a perturbation vorticity similar to the one shown in figure 6(b). This mode has a wavelength $\lambda \approx 2.8a$ and a growth rate $\sigma \approx 0.65$ ($\Gamma/2h^2$). For vortices with the amount of axial flow found in the experiments, the unstable mode involves the double-helix perturbation shown in figure 5(b), with an axial wavelength $\lambda \approx 3.3a$ and a similar growth rate as for the case without axial flow. It is not yet clear, however, how much these predictions are affected by the curvature and torsion of the helical vortex.

Our recent water channel experiments on rotor wakes [7] have shown evidence of short-wave core instability in the helical wake vortices. The observed perturbations appear mostly to have the typical sinusoidal shape of the elliptic instability mode (figure 7a), even if it cannot be ruled out that it is actually a helical perturbation (looking like a sine wave when seen from the side), which would indicate a curvature instability mode. Further careful observations and measurements are needed to identify the precise mechanism leading to these short-wave perturbations.

A noteworthy observation concerns the late stages of the short-wave instability. Figure 7(b) shows that its interaction with the long-wave deformation modes produces a rapid and violent disruption of the system of concentrated vortices, which is very different from the late stages of the pairing instability acting alone, as shown in figure 2. This behaviour is similar to the one observed previously during the interaction of straight vortices [15]. It could be related to the non-linear evolution of the elliptic instability [21].

4. Conclusions

Systems of one or several helical vortices, such as the ones generated in the wake of a rotor, are unstable with respect to different instability mechanisms. Previous stability analyses have predicted various unstable displacement modes, whose wavelengths are large compared to the vortex core size, and which can be related to the pairing phenomenon of periodic vortex arrays. Experimental

observations confirm the existence of these modes, with a good agreement concerning the spatial structure and the growth rates.

A different type of instability involves the growth of short-wave perturbations inside the vortex cores. This instability, which has previously been studied theoretically and experimentally for straight vortices, is also predicted for helical vortex systems, and some first experimental evidence is presented. Combined with the long-wave pairing, it leads to a rapid transition to turbulent flow in the wake of a rotor.

The additional presence of curvature and torsion may induce new short-wave instability modes. Further theoretical and experimental work concerning this yet unobserved curvature instability is currently underway.

Acknowledgments

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