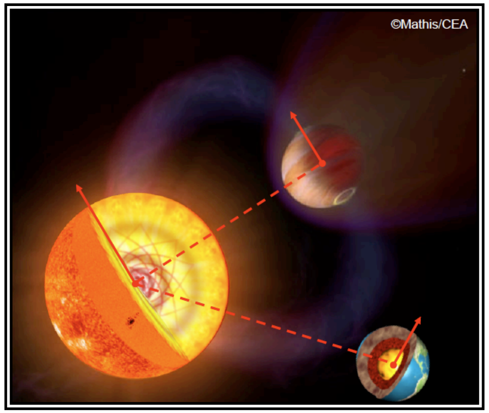
Internal waves in stellar interiors



S. Mathis

CEA/DSM/IRFU/SAp; Laboratoire AIM, CEA/DSM - CNRS - Université Paris Diderot

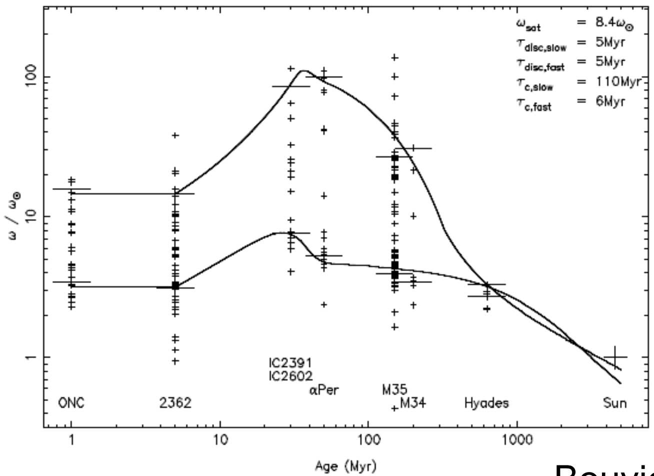
Laboratory Dynamics of Stars and their Environment

Waves and Instabilities in Geophysical and Astrophysical Flows;

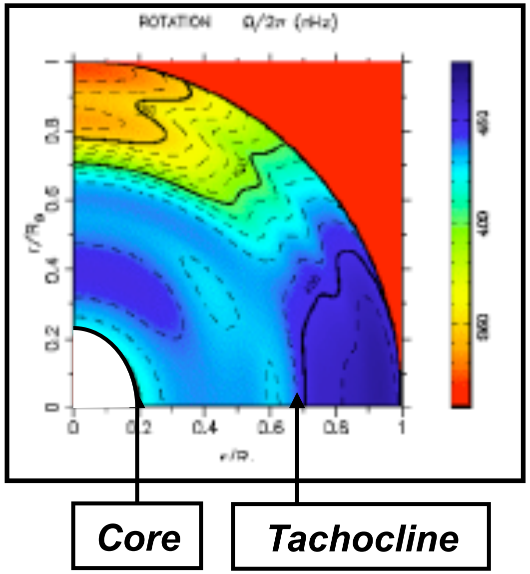
February 3rd - 8th 2013, Les Houches

MagnetoHydroDynamics of stellar interiors

Angular momentum history & differential rotation



Bouvier 2008

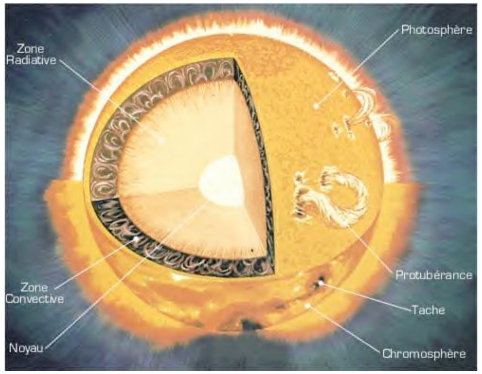


⊙: *Schou et al. 1998; Garcia et al. 2007*

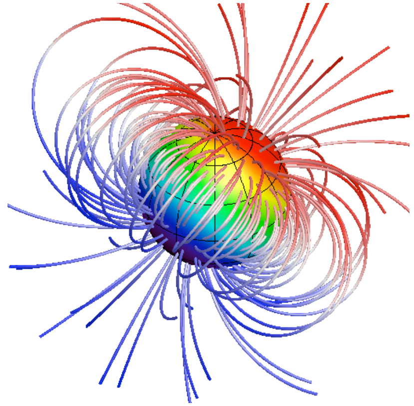
Beck et al. 2012; Deheuvels et al. 2012

Complex various magnetisms

Our Sun and solar-type stars

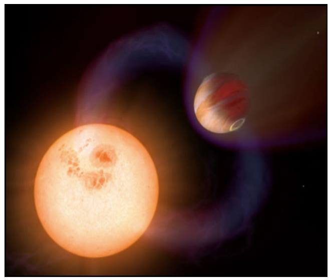


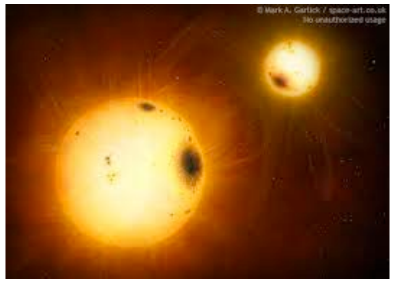
Intermediate & massive main sequence stars



Kochukhov 2010

Various companions





Coherent picture of stars, of their environment and of their evolution

→ need to obtain a precise physical picture of angular momentum exchanges

The major actor: the differential rotation

Rotation →
- large scale **circulations**
- **turbulence**
- **magnetic field (mean torque, instabilities, dynamos)**
- **modes & waves (transport)**
- **convection**
- **stellar winds (magnetic, radiative)** and **external torques (accretion, tides)**

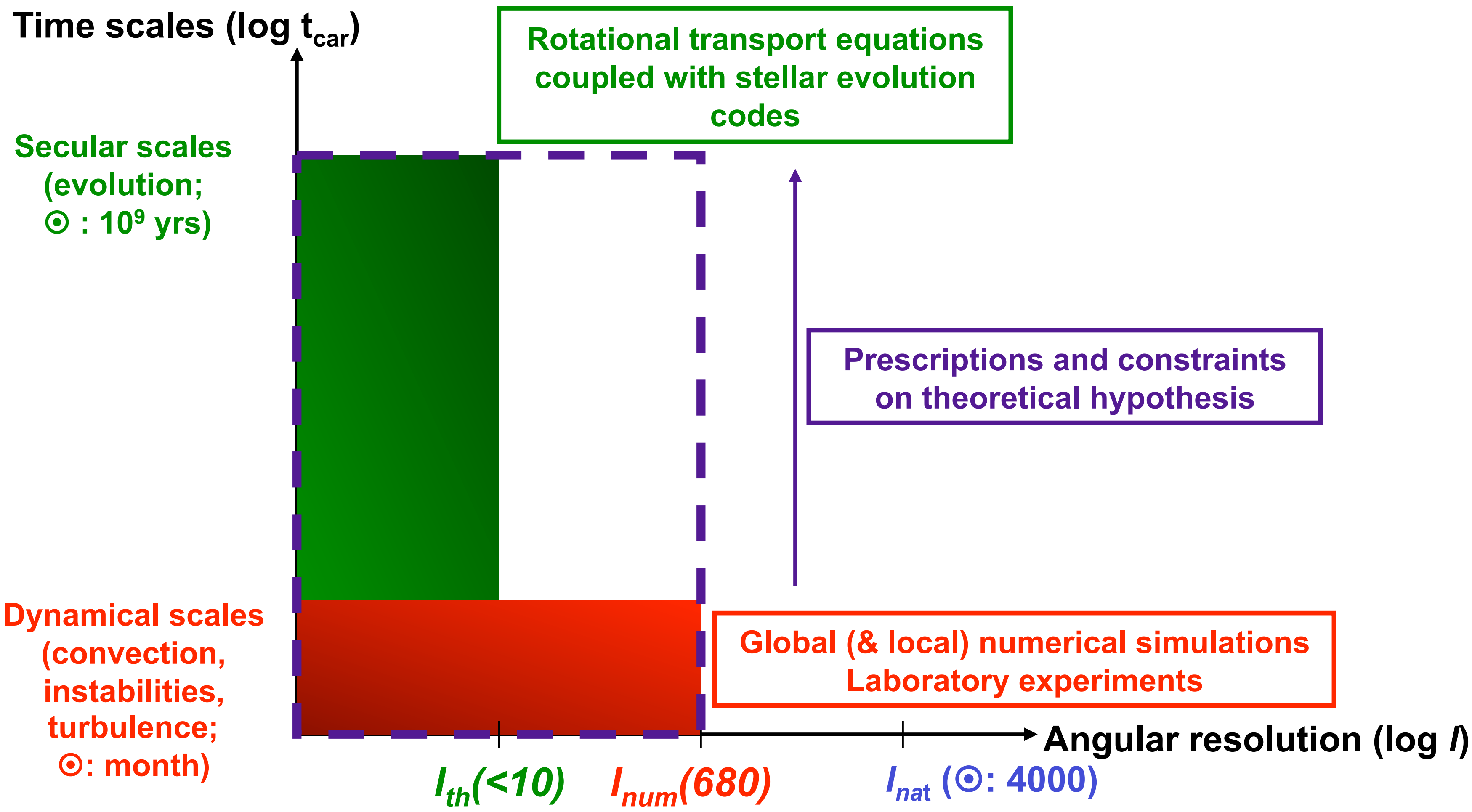
Those processes transport
- Angular Momentum → $\Omega(r,\theta)$
- Matter → modify chemical composition & nucleosynthesis

→ **Major impact on the internal dynamics,**

the evolution, and the environment of stars

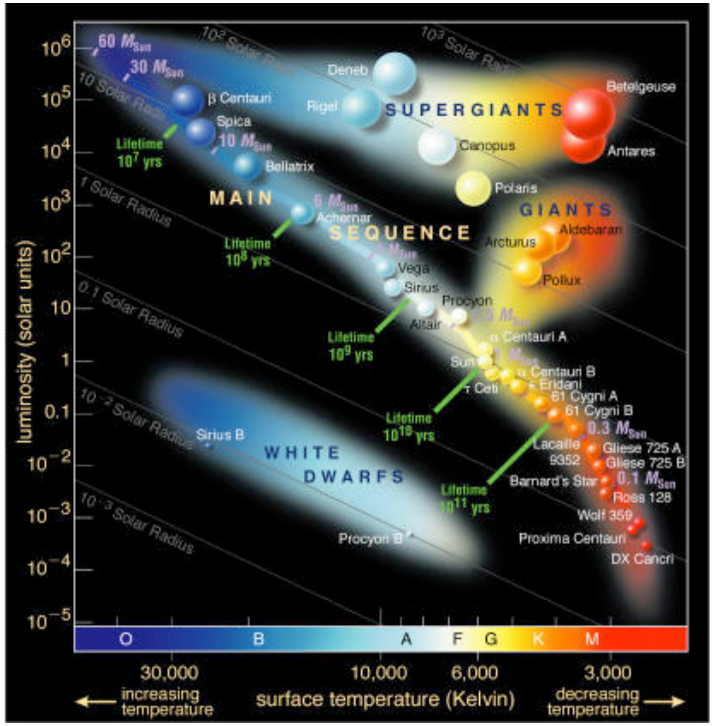
A multi-scales problem in time and space

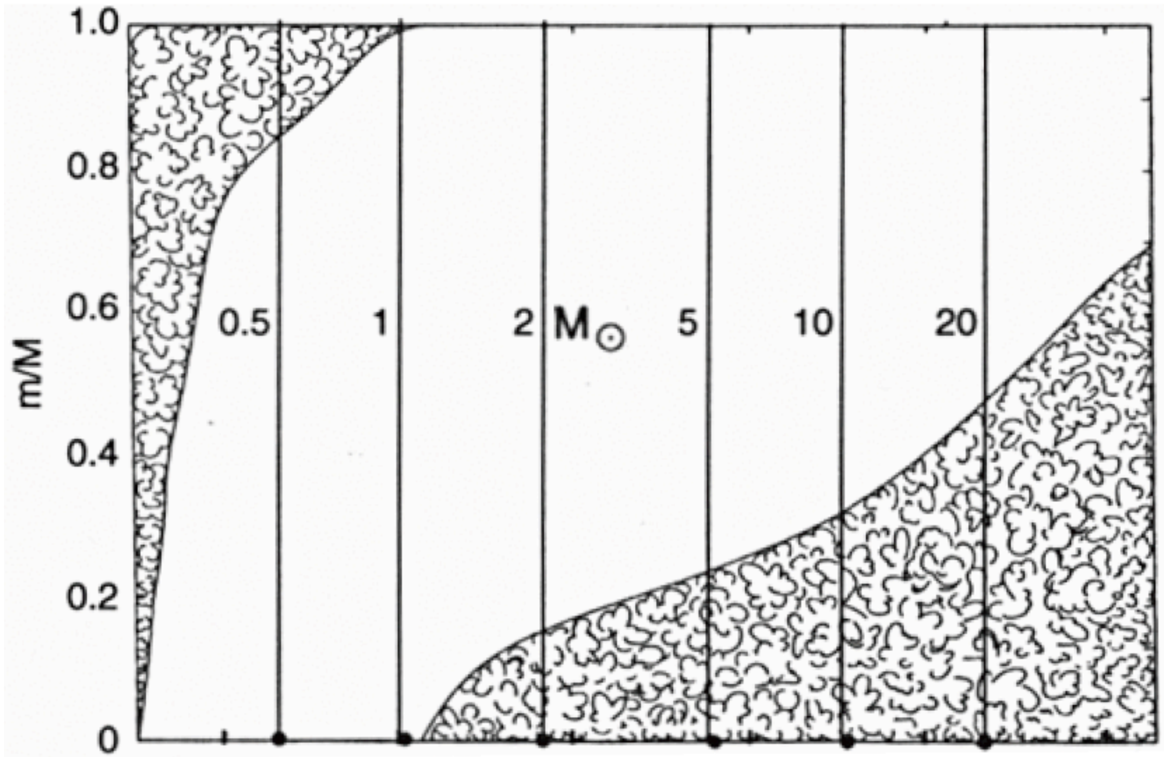
Decressin et al. 2009



Convection vs. radiation

Kippenhahn & Weigert 1997



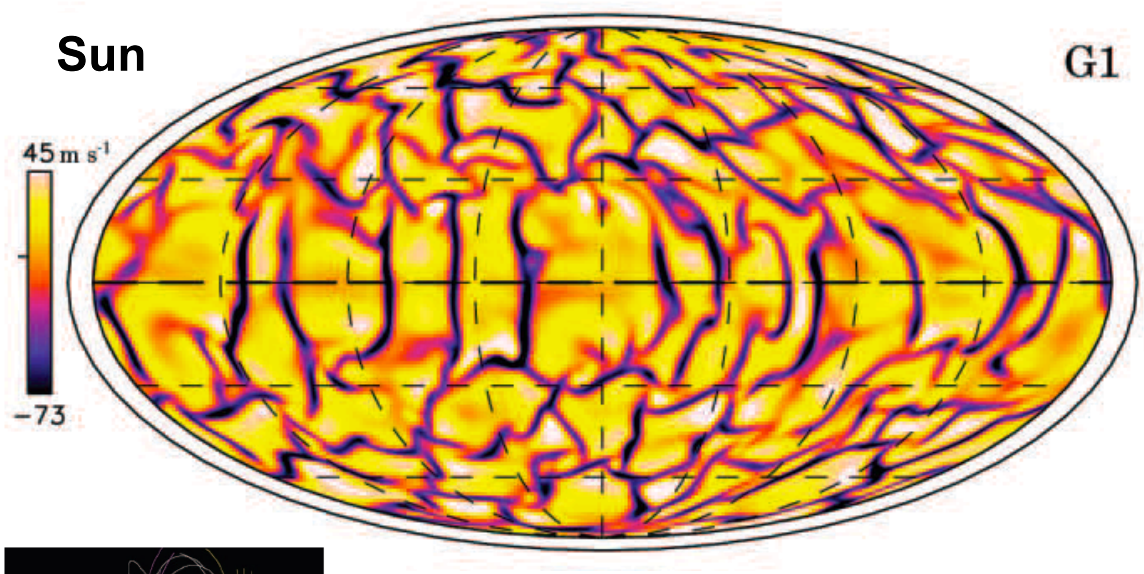


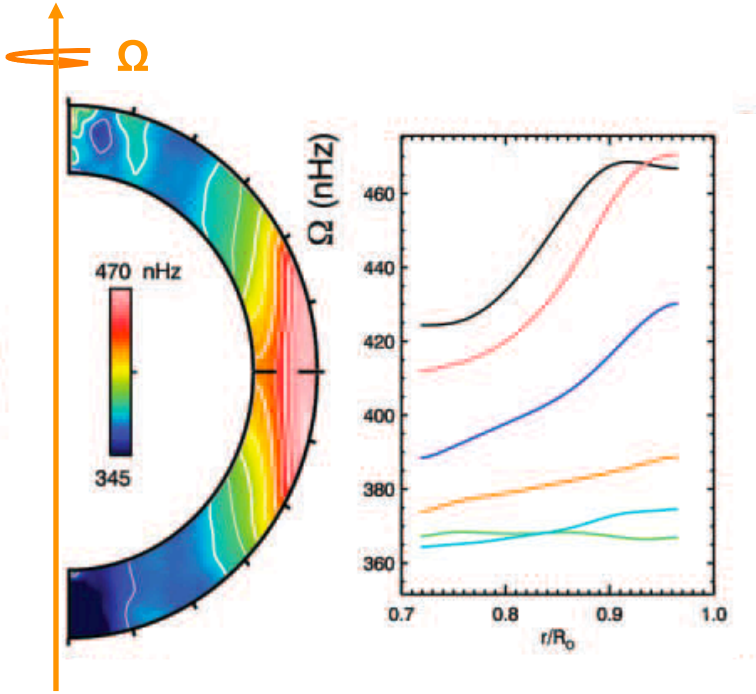
Cool stars:
C.E.: Dynamo field
R.C.: Fossil field

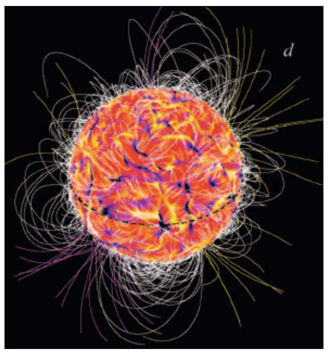
Hot stars:
C.C.: Dynamo field
R.E.: Fossil field

Turbulent transport in convective envelopes

Turbulence
- → Angular Momentum transport
  → differential rotation
- → Dynamo action
- → Convective penetration (overshoot); waves (& modes) excitation





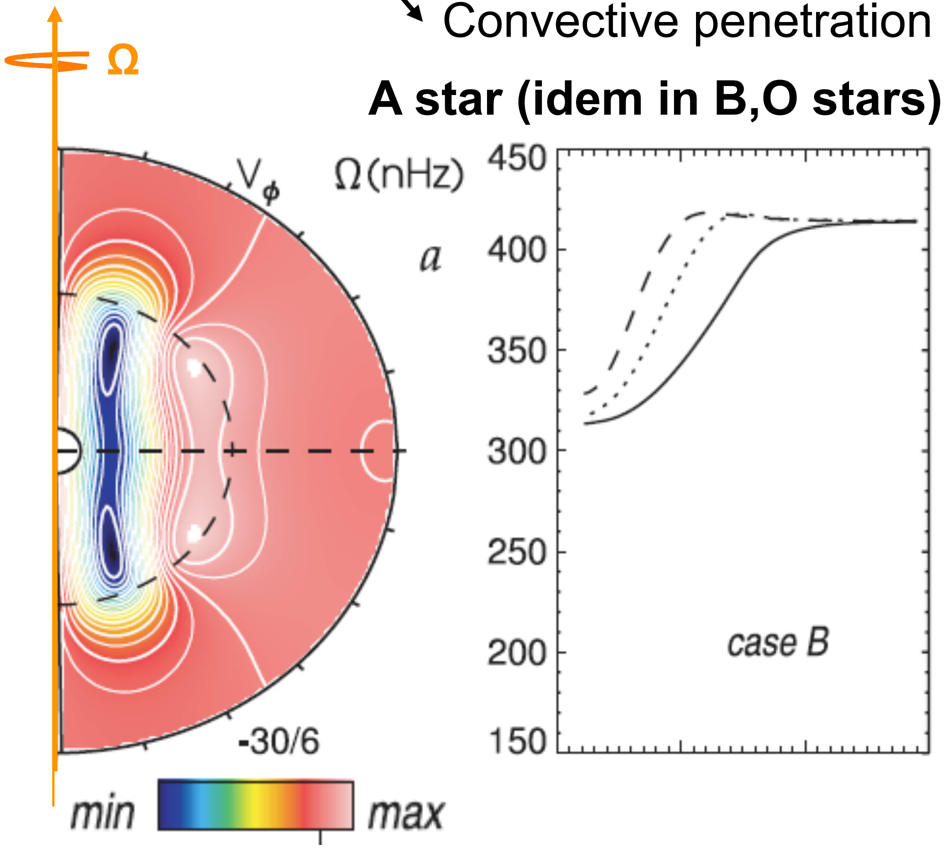


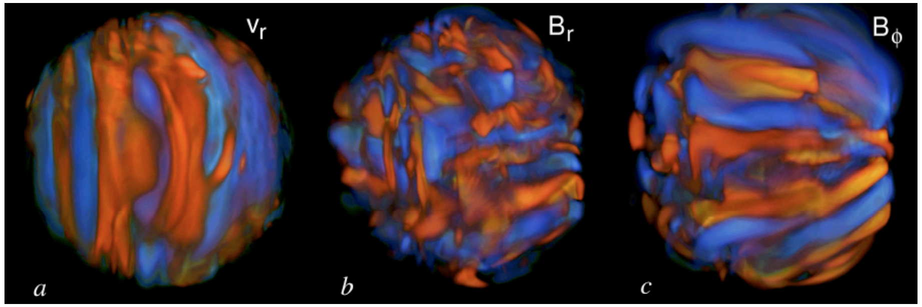
Brun & Toomre 2002; Brown et al. 2008; Brun, Miesch & Toomre 2011
Brun 2004; Brun, Miesch, Toomre 2004; Browning, Miesch, Brun, Toomre 2006

Transport in convective cores

Turbulence
- → Angular Momentum transport
  → differential rotation
- → Dynamo action (modified by the presence of a fossil field)
- → Convective penetration (overshooting); wave & (modes) excitation

A star (idem in B,O stars)

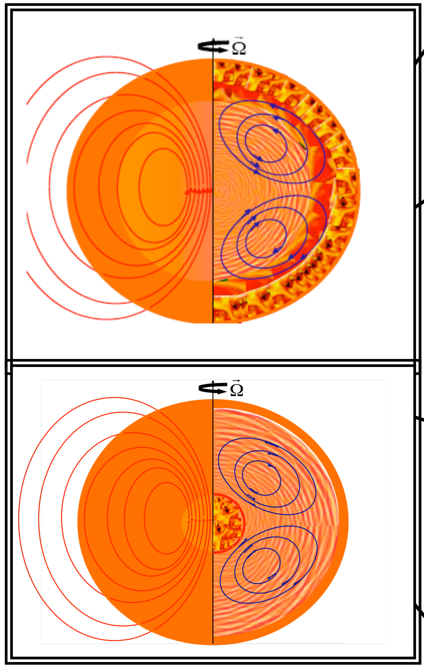




Browning, Brun & Toomre 2004

Brun, Browning & Toomre 2005
Featherstone et al. 2009

Secular transport processes in radiation zones



Meridional circulation (internal stresses and applied torques)

Busse 1981; Zahn 1992; Maeder & Zahn 1998; Garaud 2002; Mathis & Zahn 2004; Rieutord 2006; Espinosa Lara & Rieutord 2007; Decressin et al. 2009

Turbulence (shear of the diff. rot.)

Talon & Zahn 1997; Lignères et al. 1999; Garaud 2001; Maeder 2003; Mathis et al. 2004; Prat et al. 2013

Magnetic field

Secular torque

Charbonneau & Mac Gregor 1993; Rüdiger & Kitchatinov 1997; Gough & McIntyre 1998; Garaud 2002; Mathis & Zahn 2005; Brun & Zahn 2006; Garaud & Garaud 2008; Strugarek, Brun & Zahn 2011-12

Instabilities (and dynamo)

Tayler; Spruit 1999-2002; Braithwaite 2006; Arlt et al. 2007; Kitchatinov & Rüdiger 2007; Zahn, Brun & Mathis 2007

Internal waves

Press 1981;
Garcia-Lopez & Spruit 1991;
Schatzman 1993;
Montalban & S. 1994-96-2000;
Talon & Charbonnel 2005;
Rogers et al. 2005-2006-2008;
Mathis et al. 2008; Mathis 2009;
Mathis & de Brye 2011-12

excited at the borders with C. Z.

propagating inside R. Z.

A. M. settled where they are damped

or if corotation

Goldreich & Nicholson 1989

Transport equations in stellar radiation zones
(expanded on spherical harmonics)

- Dynamics equation (Navier-Stockes equation)

$$\rho\left[\partial_t V + \underline{(V\cdot\nabla)V}\right] = -\nabla P - \rho\nabla\phi + \underline{\nabla\cdot\|\tau\|} + \underline{\left|\frac{1}{\mu_0}(\nabla\wedge B)\right|\wedge B}$$

Advection

Turbulent stresses

Waves

Lorentz torque & Maxwell stresses

- Equation of continuity

$$\partial_t\rho + \nabla\cdot(\rho V) = 0$$

- Induction equation for magnetic field

$$\partial_t B - \nabla\wedge(V\wedge B) = -\nabla\wedge(\|\eta\|\otimes\nabla\wedge B)$$

- Equation for the transport of heat

$$\rho T\left[\partial_t S + V\cdot\nabla S\right] = \nabla\cdot(\chi\nabla T) + \rho\epsilon - \nabla\cdot F + \mathcal{J}(B)$$

Thermal diffusion perturbing force

Spherical thermal diffusion

Nuclear energy production and heatings due to gravitational adjustments, turbulence and ohmic effects

(+ Poisson equation and the transport equation for chemicals)

TI: Meridional circulation & shear-induced turbulence

Method

- Internal macroscopic velocity field:

$$\boldsymbol{V} = r\sin\theta\,\Omega(r,\theta)\,\widehat{\mathbf{e}}_\varphi + \dot{r}\,\widehat{\mathbf{e}}_r + \boldsymbol{\mathcal{U}}_M(r,\theta)$$

Differential rotation | **Contraction -dilatation** | **Meridional circulation**

where $\Omega(r,\theta) = \overline{\Omega}(r) + \widehat{\Omega}(r,\theta) = \overline{\Omega}(r) + \sum_{l>0} \Omega_l(r)\,Q_l(\theta)$

Average | **Fluctuation**

and $\boldsymbol{\mathcal{U}}_M = \sum_{l>0}\left[U_l(r)P_l(\cos\theta)\,\widehat{e}_r + V_l(r)\frac{\mathrm{d}P_l(\cos\theta)}{\mathrm{d}\theta}\widehat{e}_\theta\right]$ with $V_l(r) = \frac{1}{l(l+1)\rho r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\rho r^2 U_l\right)$

Anelastic approximation

- Temperature and mean molecular weight:

$$T(r,\theta) = \overline{T}(r) + \delta T(r,\theta) \quad \text{where} \quad \delta T(r,\theta) = \sum_{l\geq 2} \delta T_l(r) P_l(\cos\theta)$$

$$\mu(r,\theta) = \overline{\mu}(r) + \delta\mu(r,\theta) \quad \text{where} \quad \delta\mu(r,\theta) = \sum_{l\geq 2} \delta\mu_l(r) P_l(\cos\theta)$$

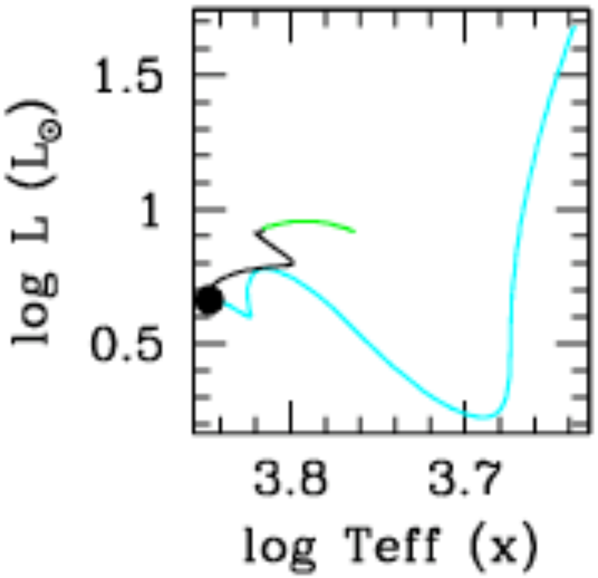
Numerical simulation of Type I Rotational Transport (I)

Hydrodynamical shellular case with $\Omega(r,\theta)=\Omega(r)$ (l=2)

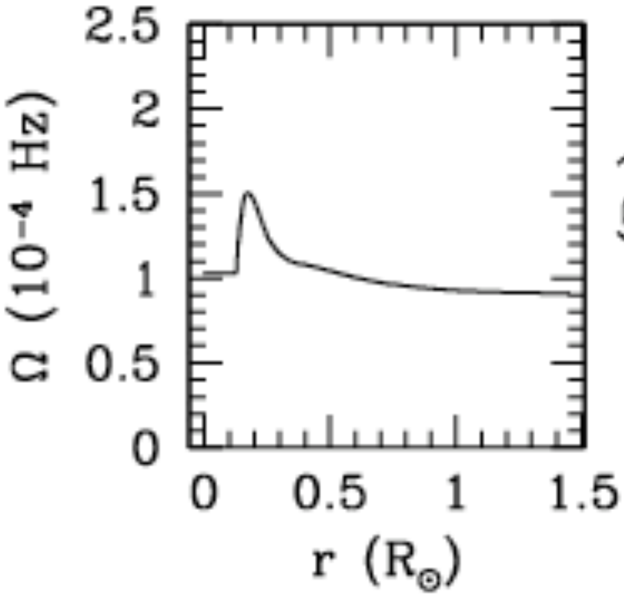
1.5 $M_\odot$
$Z=Z_\odot$
V_i=100 km.s^{-1}
Magnetic braking

Mathis & Zahn 2004
Decressin et al. 2009

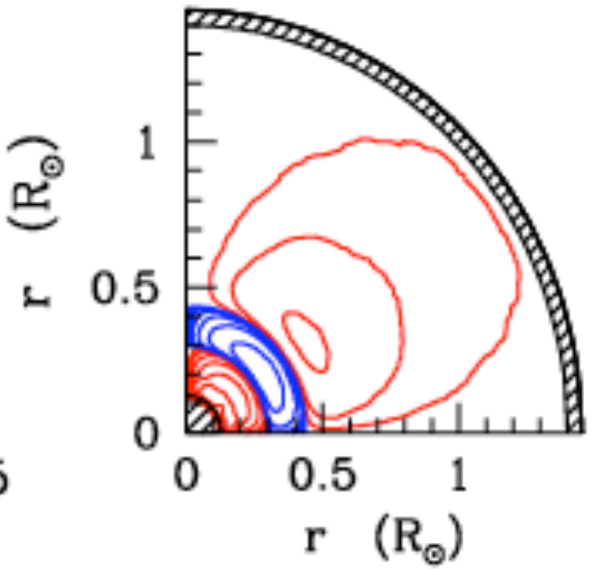
Hertzsprung-Russel diagram



Differential rotation



Meridional currents
driven by
viscous stresses
and **braking**

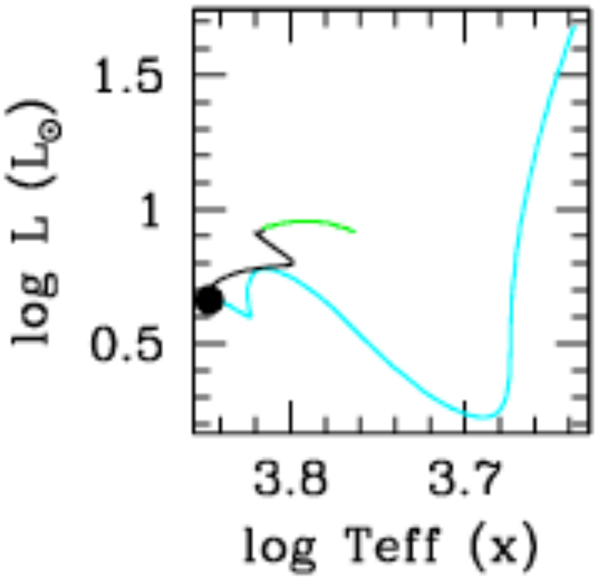


Numerical simulation of Type I Rotational Transport (II)

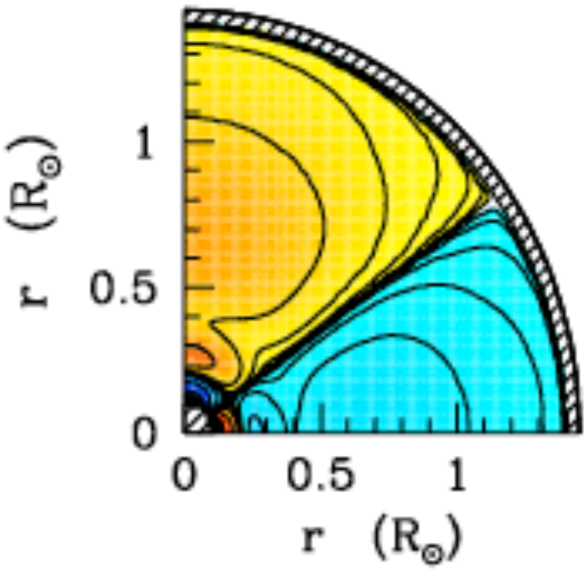
Hydrodynamical shellular case with $\Omega(r,\theta)=\Omega(r)$ (l=2)

Mathis & Zahn 2004
Decressin et al. 2009

Hertzsprung-Russel diagram

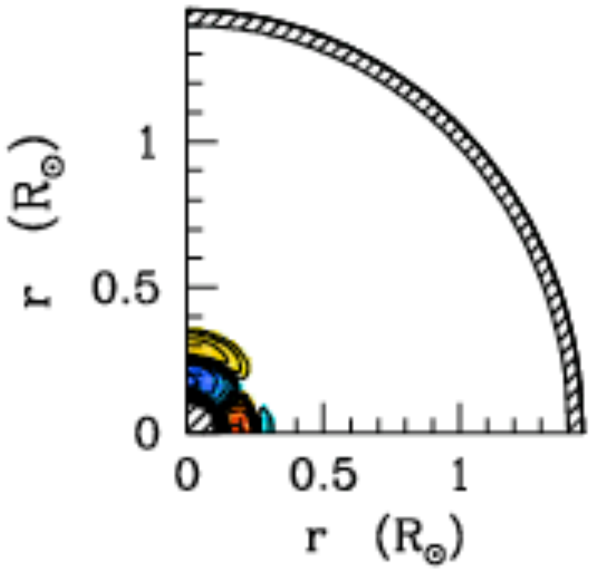


Temperature fluctuation
adjusts to entropy
advection by M. C.

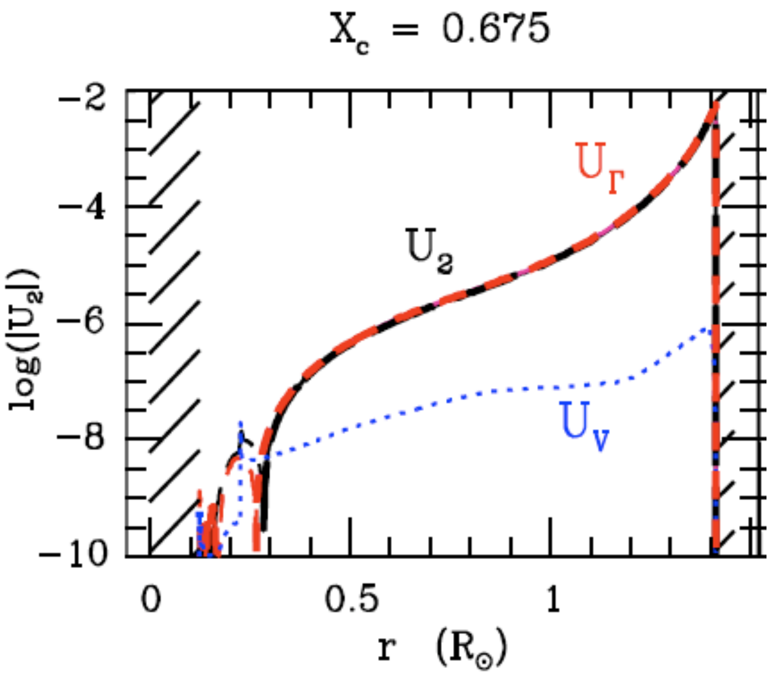


Latitudinal
temperature gradient
→ new **rotation** profile
because of **thermal wind**

Chemical fluctuation



Diagnosis and identification



Sustaining meridional circulation

$$U_2 = \frac{5}{\overline{\rho} r^4 \overline{\Omega}} \left[\Gamma(m) - \overline{\rho} \nu_v r^4 \partial_r \overline{\Omega} \right]$$

Extraction **Viscous**

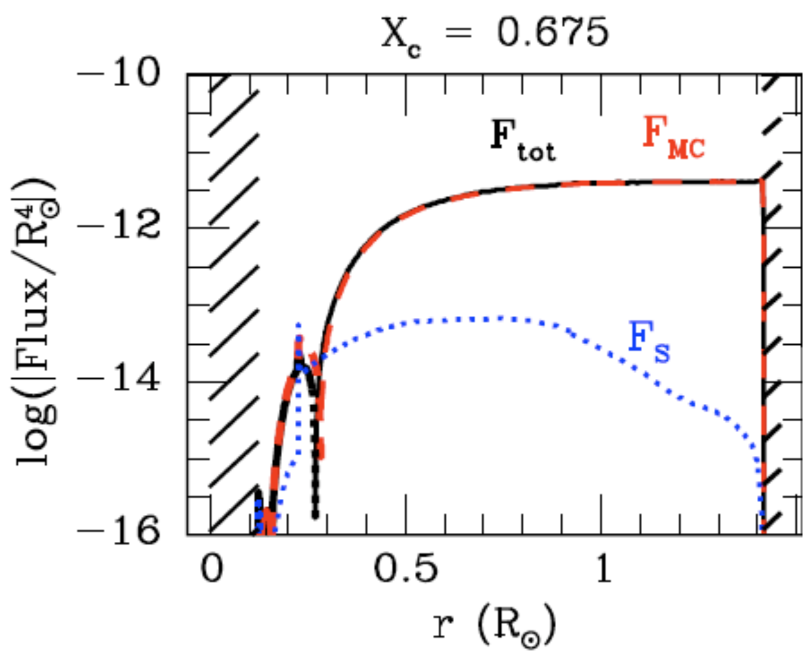
$$\Gamma(m) = \frac{1}{4\pi} \frac{\mathrm{d}}{\mathrm{d}t} \left[\int_0^{m(r)} r'^2 \overline{\Omega} \mathrm{d}m' \right]$$

→ **Wind-driven circulation** $t_{ES} = \frac{R}{U} \approx t_{KH} \left(\frac{1}{R\Omega^2} \frac{GM}{R^2} \right)$

Flux of Angular Momentum:

- **Meridional circulation** $$F_{MC}(r) = -\frac{1}{5} \overline{\rho} r^4 \overline{\Omega} U_2$$

- **Shear induced turbulence** $$F_S(r) = -\overline{\rho} r^4 \nu_V \partial_r \overline{\Omega}$$



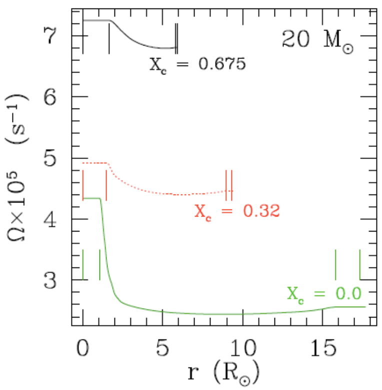
When the braking vanishes, a balance is established between the advective and the viscous transports of A. M.

Numerical simulation of Type I Rotational Transport (III)

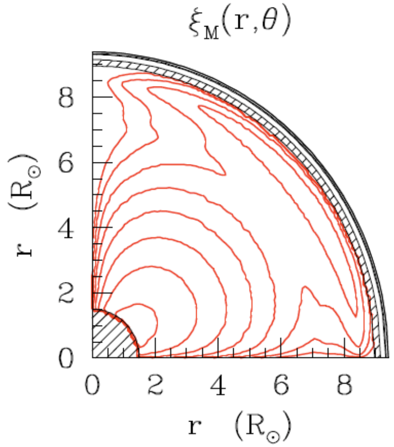
Shellular case with Ω(r,θ)=Ω(r) (l=2) – massive stars

Mathis & Zahn 2004; Decressin et al. 2009

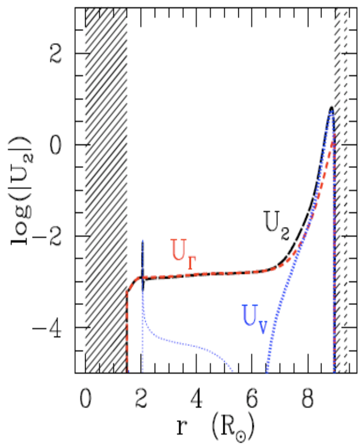
Differential rotation

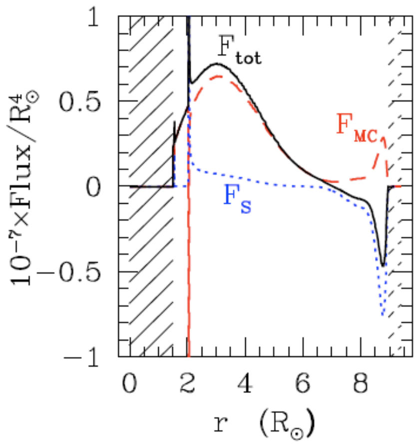


Stream lines M. C.

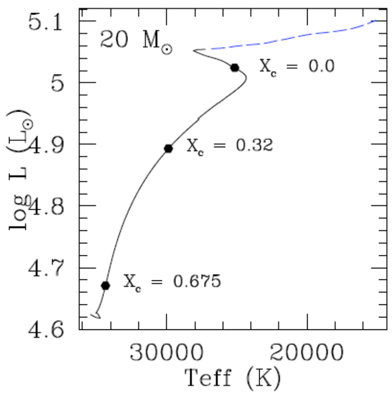


Value M. C.

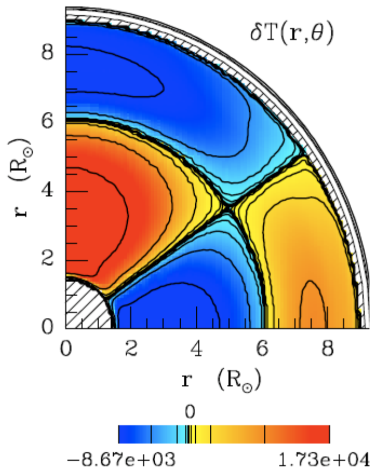




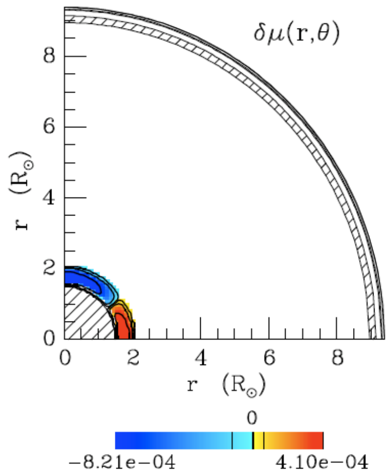
Hertzsprung-Russel diagram



Temperature fluctuation



Chemical fluctuation



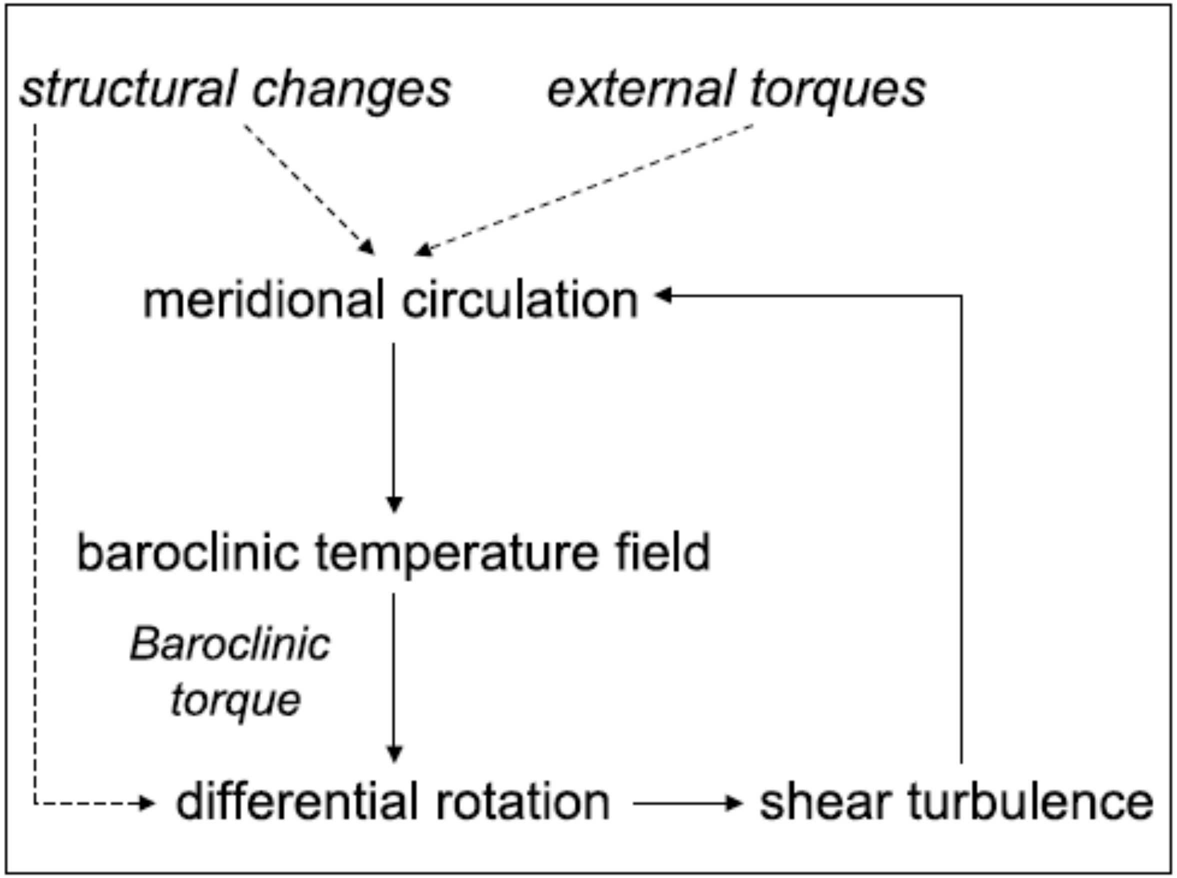
20 M⊙
Z=Z⊙
V_i=300 km.s^{-1}
X_c=0.32

The transport loop

$$U_2 = U_\Gamma + U_V = \frac{5}{\overline{\rho} r^4 \overline{\Omega}}\left[\Gamma(m) - \overline{\rho}\nu_v r^4 \partial_r \overline{\Omega}\right]$$

$$\underbrace{\overline{\rho}\,\overline{T} C_p \frac{d\Psi_2}{dt}}_{\overline{\rho}\,\overline{T}\partial_t \overline{S}} = \underbrace{-\overline{\rho}\,\overline{T} C_p \frac{N_T^2}{\overline{g}\delta} U_2}_{\overline{\rho}\,\overline{T} U_r \partial_r \overline{S}} + \underbrace{\overline{\rho}\frac{L}{M}\mathcal{T}_{2,\mathcal{B}} + \overline{\rho}\frac{L}{M}\mathcal{T}_{2,\mathrm{Th}}}_{\nabla\cdot(\chi\nabla T) - \nabla\cdot F_H} + \underbrace{\overline{\rho}\frac{L}{M}\mathcal{T}_{2,\mathrm{N-G}}}_{\rho\varepsilon}$$

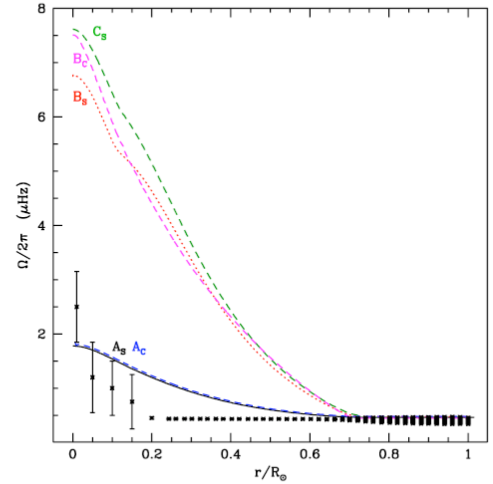
$$\frac{1}{3} r\,\partial_r \overline{\Omega}^2 = \frac{\overline{g}}{r}(\varphi\Lambda_2 - \delta\Psi_2)$$



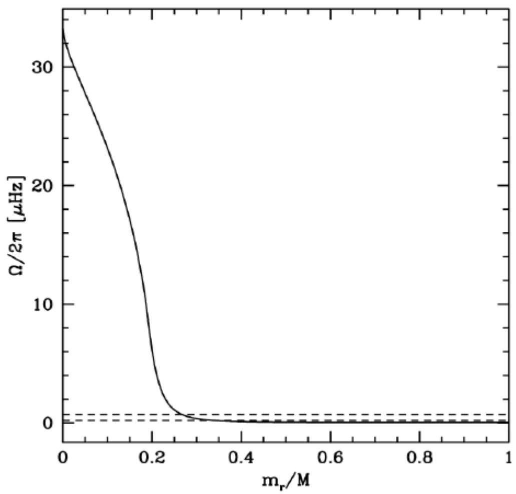
Mathis & Zahn 2004

Zahn 1992;
Rieutord 2006;
Decressin et al. 2009

Angular momentum transport in stars



Turck-Chièze et al. 2010



Cellier et al. 2012

→ **It doesn't reproduce the flat rotation profile of the solar radiative interior,**

(Pinsonneault et al. 1989; Talon & Zahn 1998; Talon & Charbonnel 2005; Turck-Chièze et al. 2010)

the rotation of the core of subgiant and red giant stars,

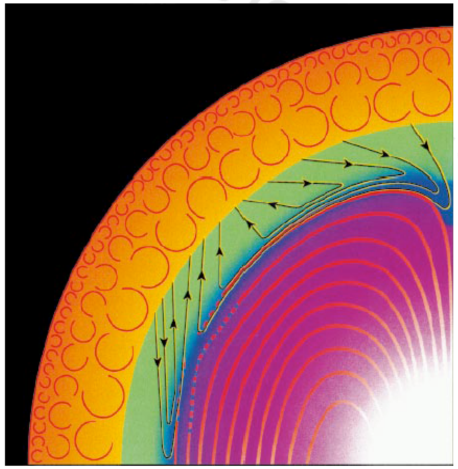
(Palacios et al. 2006; Eggenberger et al. 2012; Ceillier et al. 2012; Marques et al. 2013)

and **matter ejections in active massive stars**

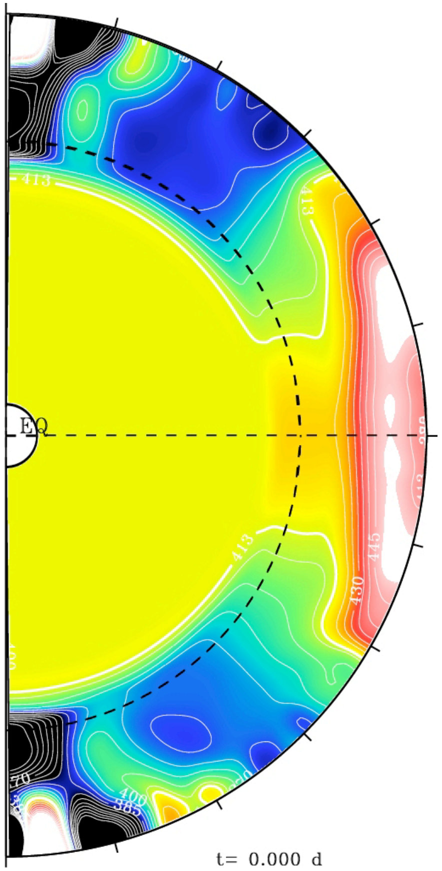
→ **Another process is responsible for the transport of angular momentum (TII)**

TII: Magnetic field

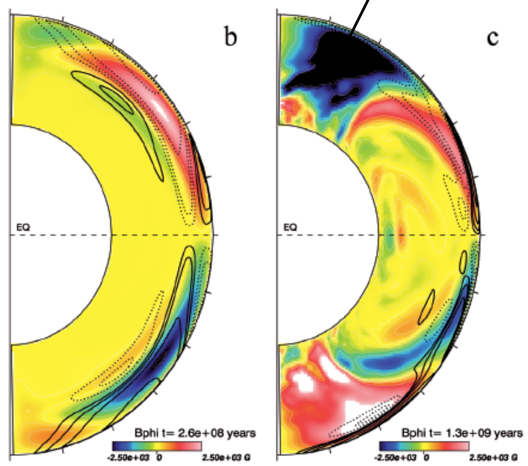
Transport of angular momentum in the axisymmetric case
3D-MHD models of the solar interior



Fossil poloidal magnetic field
Gough & McIntyre 1998



Ω-effect
+instabilities



$B\varphi$

Interaction between a poloidal fossil field and the inward propagation of a latitudinal shear

→ **Ferraro's law at solar age + N-A instabilities; other geometry?**

Brun & Zahn 2006;
Zahn, Brun & Mathis 2007;
Strugarek, Brun & Zahn 2011-2012

Garaud & Garaud 2008; Rogers 2011;
Wood et al. 2011; N. Brummell's talk

TII: Internal Waves

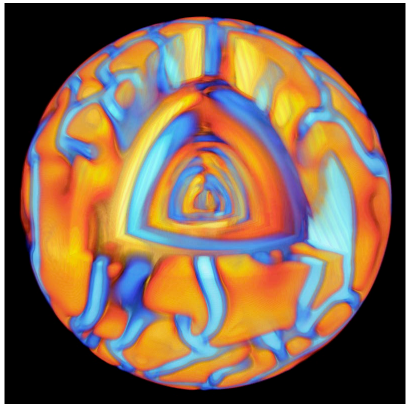
Internal waves in stellar interiors (I)

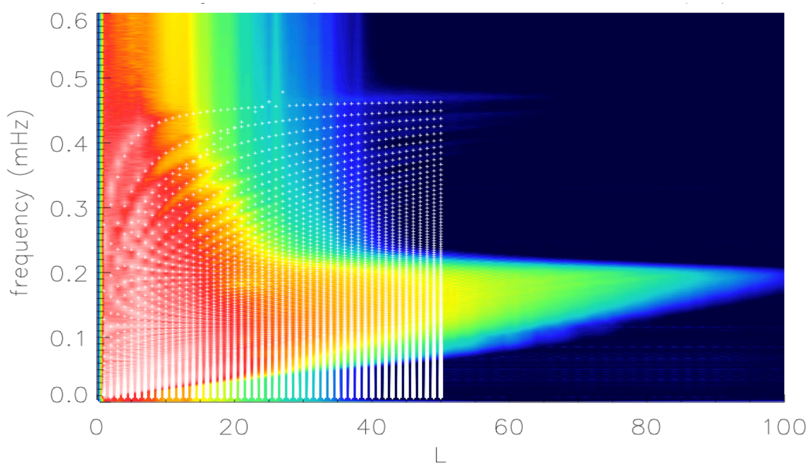
Convective excitation (PMS, MS & Advanced phases)

Goldreich, Murray & Kumar 1994;
Belkacem et al. 2009; Samadi et al. 2010; Lecoanet & Quataert 2012

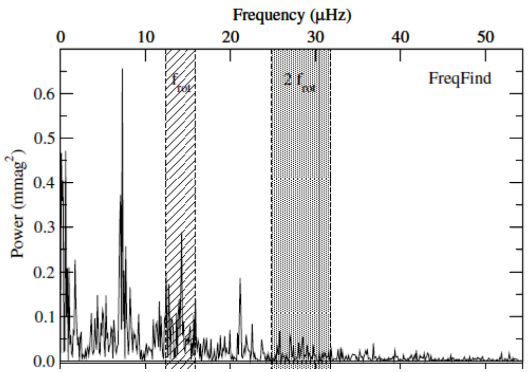
Kiraga et al. 2003-2005; Browning et al. 2004; Dintrans et al. 2005;
Rogers et al. 2005-2006-2008-2010-2012; Meakin et al. 2007;
Brun, Miesh, Toomre 2011;

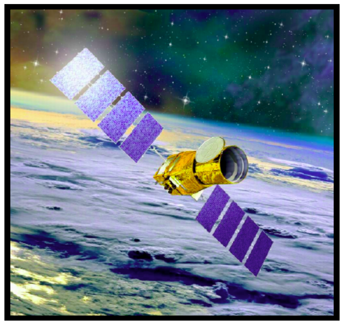
Brun, Miesch & Toomre 2011;
Alvan et al., in prep. (& poster)

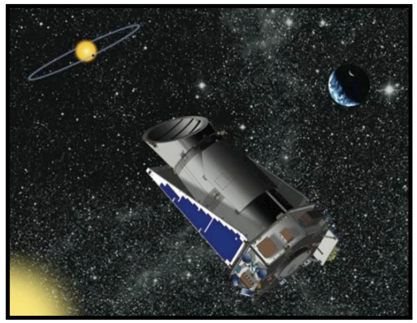




e.g. Garcia et al. 2007;
Beck et al. 2012;
Neiner et al. 2012



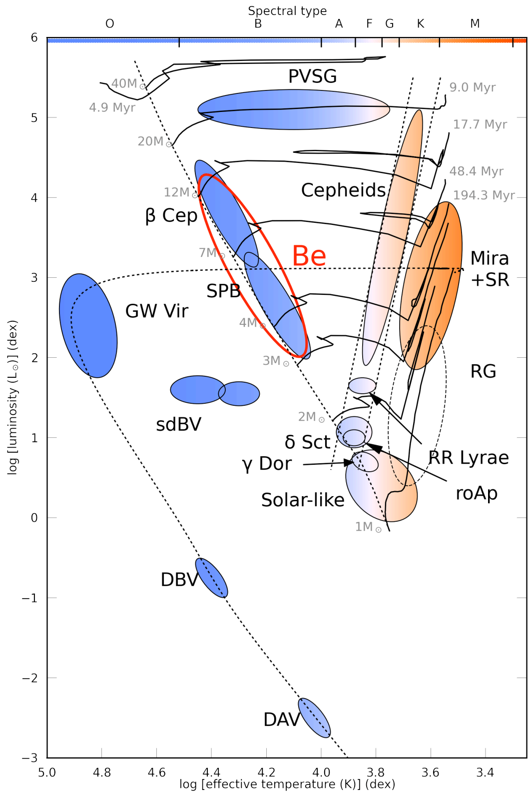




Internal waves in stellar interiors (II)

κ – mechanism

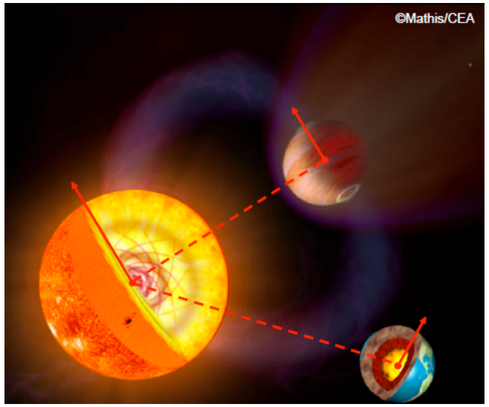
Lee & Saio 1993; Lee 2006;

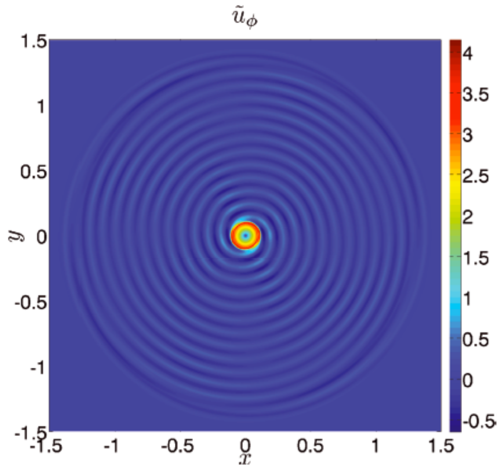


Courtesy P. Degroote

Tidal excitation

Zahn 1975; Witte & Savonjie 1999-2001-2002; Barker & Ogilvie 2010; Barker 2011





Barker & Ogilvie 2010

→ Angular momentum transport? (Help to probe it?)

The propagation equation (Ω=0, B=0)

The linearized equations

$$\begin{cases} D_t \boldsymbol{u} = -\dfrac{\boldsymbol{\nabla} p'}{\bar{\rho}} + \dfrac{\rho'}{\bar{\rho}} \boldsymbol{g}, \\ D_t \rho' + \boldsymbol{\nabla}.(\bar{\rho} \boldsymbol{u}) = 0, \\ D_t \left(\dfrac{\rho'}{\bar{\rho}} - \dfrac{1}{\Gamma_1} \dfrac{p'}{\bar{p}} \right) - \dfrac{N^2}{\bar{g}} u_r = 0 \end{cases}$$

Velocity field expansion

$$u_r(r,\theta,\varphi,t) = \sum_{l,m} \hat{u}_{r;l,m}(r) Y_{l,m}(\theta,\varphi) e^{i\sigma_w t}$$

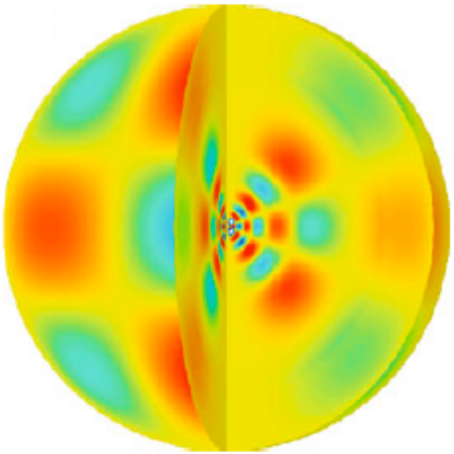
same form in the θ & φ directions

The propagation equation (Schrödinger like; e.g. B. Surtherland's talk)

$$\frac{\mathrm{d}^2 \Psi_{l,m}}{\mathrm{d}r^2} + k_V^2(r) \Psi_{l,m} = 0 \qquad k_V^2(r) = \left(\frac{N^2}{\sigma_w^2} - 1 \right) \underbrace{\frac{l(l+1)}{r^2}}_{k_H}$$

We have introduced $\Psi_{l,m}(r) = \bar{\rho}^{\frac{1}{2}} r^2 \hat{\xi}_{r;l,m}$ and $\boldsymbol{u} = D_t \boldsymbol{\xi}$

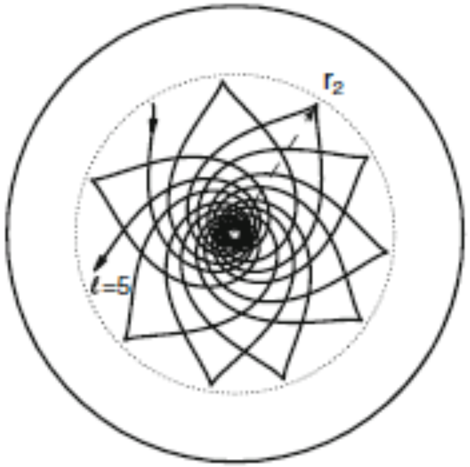
l=5, m=3, n=5



Low-frequency internal waves structure (Ω=0, B=0)

JWKB

$$\begin{cases} u_{r;l,m} = \mathcal{E}_{l,m}(r)\exp\left[i\int_r^{r_c} k_{V;l,m}(r')\,\mathrm{d}r'\right] Y_{l,m}(\theta,\varphi)\exp[i\sigma t] \\ k_{V;l,m}(r) = \left(\frac{N}{\sigma}\right)\frac{[l(l+1)]^{1/2}}{r} \end{cases}$$



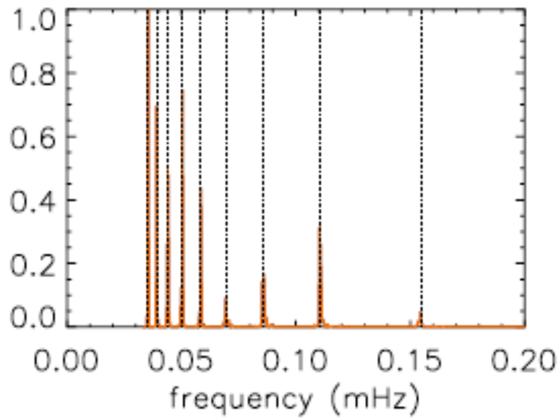
same form in the θ & φ directions

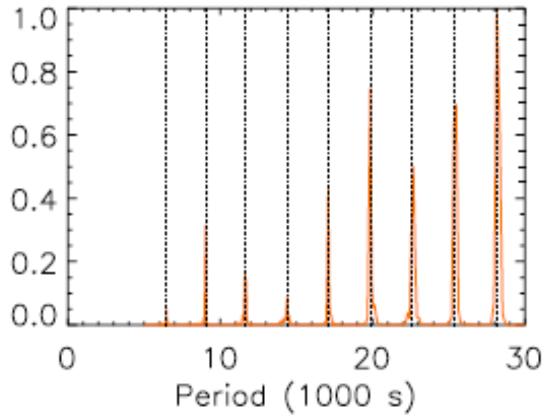
Normal modes: Bohr's quantization rule

$$\int_{r_1}^{r_2} k_{V;n,l,m}\mathrm{d}r = \pi(n+\alpha)$$

$$P_{n,l} = \frac{\pi(n+\alpha)}{[l(l+1)]^{1/2}\int_{r_1}^{r_2} N\frac{\mathrm{d}r}{r}}$$

l=1, m=0 (Alvan, Brun, Mathis 2012)





Transport: at the beginning,…

Astron. & Astrophys. 41, 329 – 344 (1975)

The Dynamical Tide in Close Binaries

J.-P. Zahn
Observatoire de Nice, and Joint Institute for Laboratory Astrophysics, University of Colorado, Boulder

Astron. Astrophys. 279, 431–446 (1993)

ASTRONOMY AND ASTROPHYSICS

Transport of angular momentum and diffusion by the action of internal waves

Evry Schatzman

Astron. Astrophys. 322, 320–328 (1997)

ASTRONOMY AND ASTROPHYSICS

Angular momentum transport by internal waves in the solar interior

Jean-Paul Zahn[1], Suzanne Talon[1], and José Matias[1,2]

THE ASTROPHYSICAL JOURNAL, 475:L143–L146, 1997 February 1
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ANGULAR MOMENTUM TRANSPORT BY GRAVITY WAVES AND ITS EFFECT ON THE ROTATION OF THE SOLAR INTERIOR

PAWAN KUMAR[1]
Institute for Advanced Study, Princeton, NJ 08540
AND
ELIOT J. QUATAERT
Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138
Received 1996 September 3; accepted 1996 November 18

Publ. Astron. Soc. Japan **35**, 343–353 (1983)

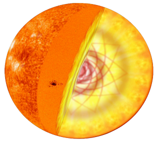
Wave–Rotation Interaction in Stars

Hiroyasu ANDO
Tokyo Astronomical Observatory, University of Tokyo, Mitaka, Tokyo 181

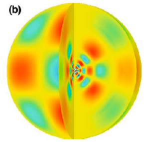
Mon. Not. R. Astron. Soc. **261**, 415–424 (1993)

Angular momentum transfer by non-radial oscillations in massive main-sequence stars

Umin Lee[1,2] and Hideyuki Saio[3]
[1] *Department of Astronomy, Faculty of Science, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan*
[2] *Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627-0011, USA*
[3] *Astronomical Institute, Faculty of Science, Tohoku University, Sendai, Miyagi 980, Japan*



Transport of Angular Momentum by internal waves



Equation for the transport of Angular Momentum:

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}(r^2\overline{\Omega}) = \underbrace{\frac{1}{5r^2}\partial_r\left(\rho r^4 \overline{\Omega} U_2\right)}_{\text{Advection}} + \underbrace{\frac{1}{r^2}\partial_r\left(\rho \nu_v r^4 \partial_r \overline{\Omega}\right)}_{\text{Viscous stresses}} - \underbrace{\frac{1}{r^2}\partial_r\left[r^2 \mathcal{F}_J(r)\right]}_{\text{Internal wave Reynolds stresses}}$$

Zahn et al. 1997

Advection | **Viscous stresses** | **Internal wave Reynolds stresses**

where:

$$4\pi r^2 \mathcal{F}_J(r) = -4\pi r_c^2 \sum \left\{ \frac{m}{\sigma(r_c)} \mathcal{F}_E(l, m, \sigma; r_c) \exp\left[-\tau_{l,m}\left(r, \overline{\Omega}(r)\right)\right] \right\}$$

spectrum excited by convection

Mean Energy flux at the base of the CZ

with the radiative damping:

$$\tau_{l,m}(r) = [l(l+1)]^{3/2} \int_r^{r_c} K \frac{N^3}{\sigma^4(r')} \frac{\mathrm{d}r'}{r'^3}$$

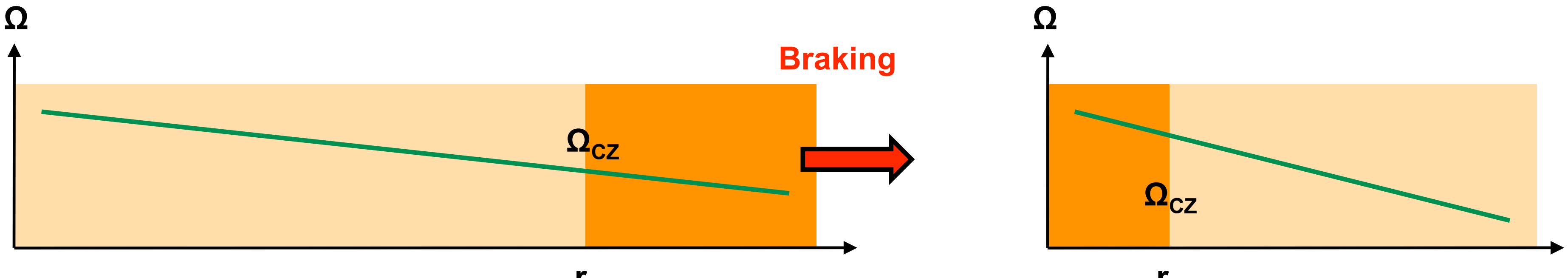
with $\sigma(r) = \sigma_{\rm exc} + m\Delta\overline{\Omega}(r)$

m>0 - retrograde (extraction)
m<0 - prograde (deposit)

If prograde and retrograde waves are equally excited:

- No differential rotation ⟶ no net deposition of A. M. $\quad \mathcal{F}_{\rm AM;m} + \mathcal{F}_{\rm AM;-m} = 0$
- Differential rotation ⟶ Doppler shift ⟶ net deposition of A. M.

Cases of spin-down/spin-up



The wave phase is expanded as $\exp\left[i\left(m\varphi + \sigma t\right)\right]$.

Then

$$\begin{cases} m > 0 \text{ corresponds to retrograde waves} \\ m < 0 \text{ corresponds to prograde waves.} \end{cases}$$

Moreover

$$\begin{aligned} \sigma &= \sigma_{\text{exc}} + m\Delta\overline{\Omega} \\ &= \sigma_{\text{exc}} + m\underbrace{\left(\overline{\Omega}(r) - \overline{\Omega}_{\text{CZ}}\right)}_{>0}. \end{aligned}$$

Thus

$$\sigma_{\text{p}} = \sigma_{\text{exc}} - |m|\Delta\overline{\Omega}$$
$$\sigma_{\text{r}} = \sigma_{\text{exc}} + |m|\Delta\overline{\Omega}$$

and

$$\sigma_{\text{p}} < \sigma_{\text{r}} \Rightarrow \sigma_{\text{p}}^{-4} > \sigma_{\text{r}}^{-4} \Rightarrow \tau_{\text{p}} > \tau_{\text{r}}.$$

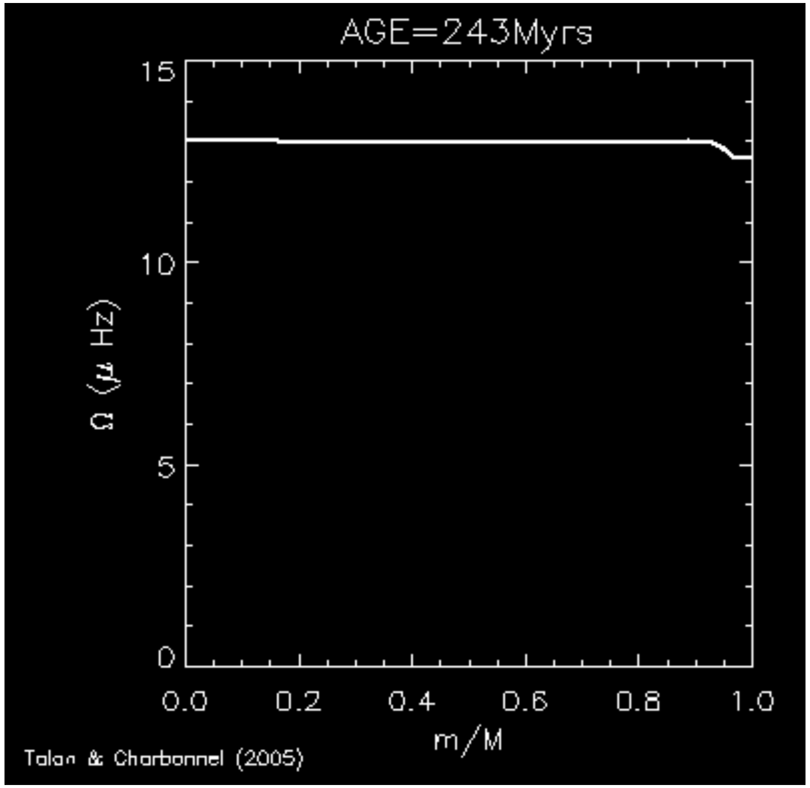
⇒ Prograde waves are thus damped closer to the excitation region than retrograde waves which propagate deeper in the core.

⇒ Net extraction of angular momentum in the core.

Here $\left(\overline{\Omega}(r) - \overline{\Omega}_{\text{CZ}}\right) < 0$.

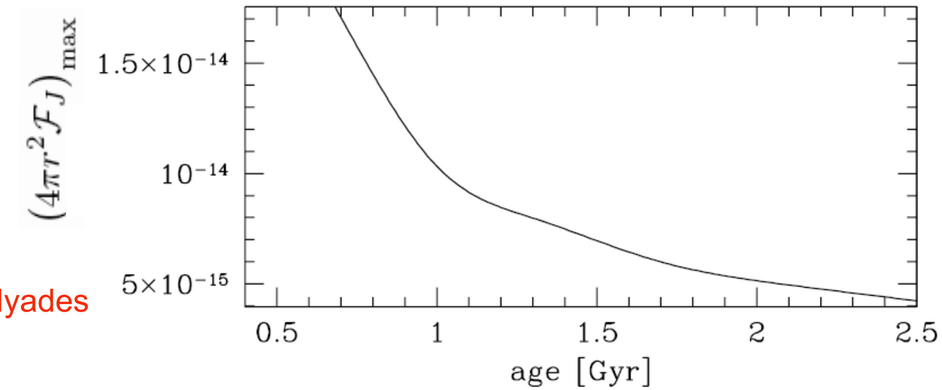
⇒ Net deposition of angular momentum in the envelope.

Transport by low degree, low frequency waves: the secular extraction

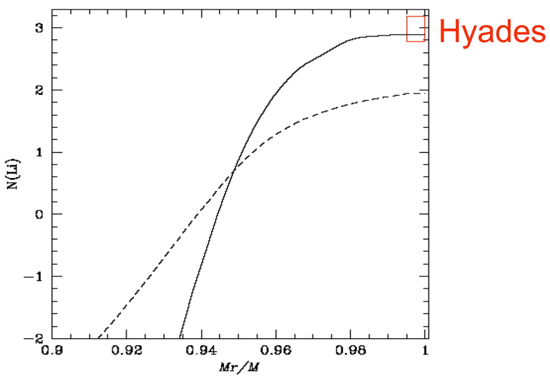


Dynamical vision of a $1M_{\odot}$ star with magnetic braking (V_i=50 $Km.s^{-1}$) and the advection

Low degree retrograde waves propagate in the interior
→ A. M. extraction driven by the wind



Li (0.7 Gyr)



Talon & Charbonnel 2005

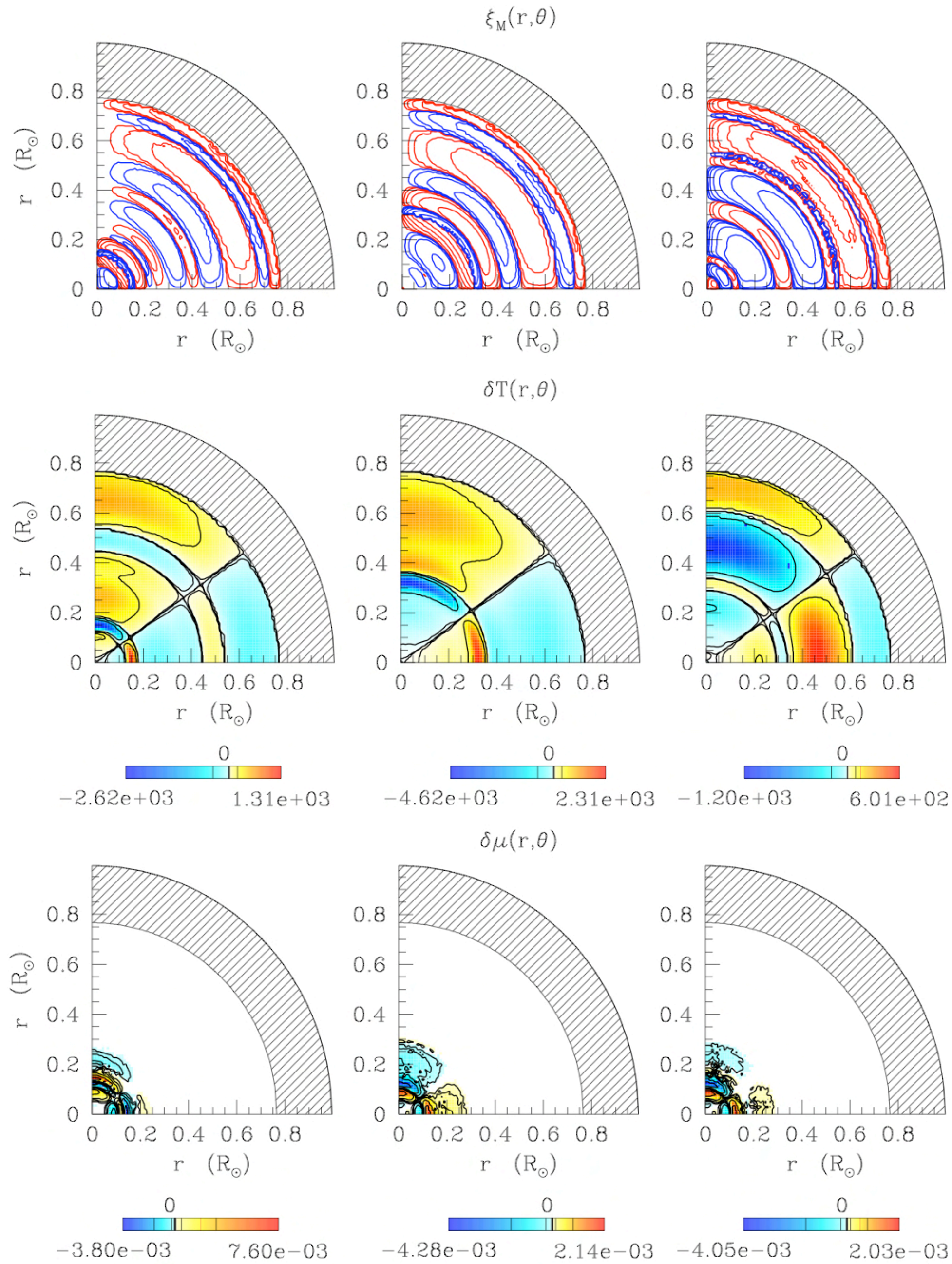
Able to lead to the rotation profile of the solar radiative interior

Diagnosis and identification

Stream lines M. C.

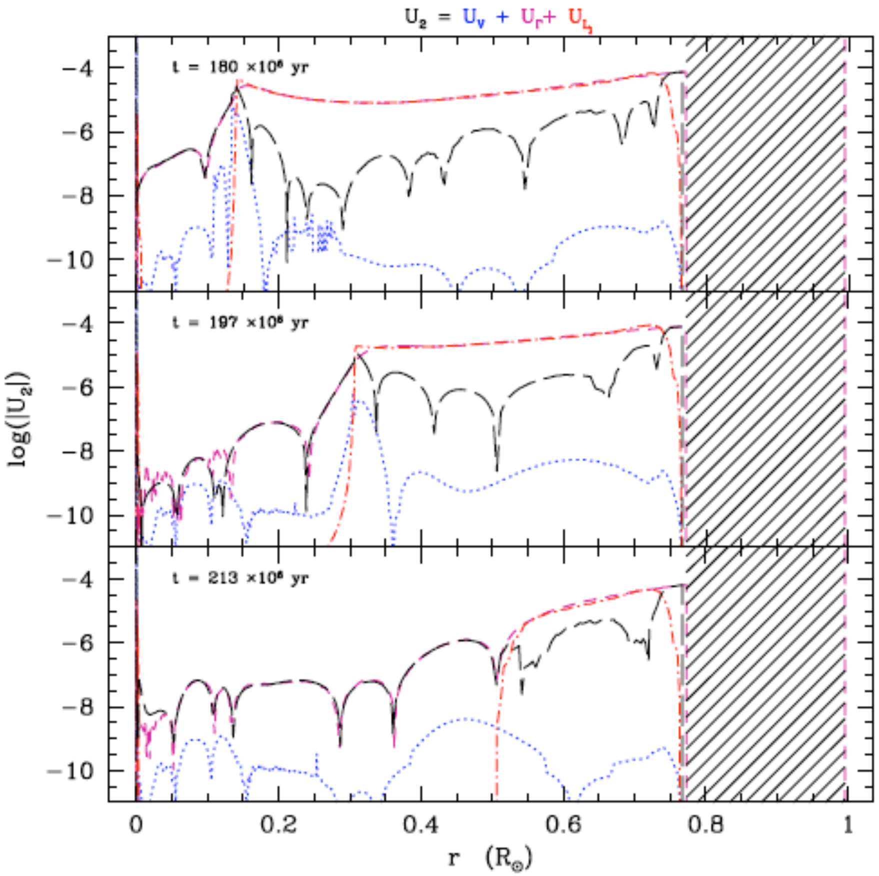
Temperature fluctuation

Chemical fluctuation



Secular extraction of angular momentum **driven by the braking** with a **highly multi-cellular** meridional circulation

Diagnosis and identification



Sustaining the meridional circulation

$$U_2 = \frac{5}{\overline{\rho} r^4 \overline{\Omega}} \left[\Gamma_M(r) - \overline{\rho} \nu_V r^4 \partial_r \overline{\Omega} + \frac{3}{8\pi} \mathcal{L}_J \right]$$

Extraction **Viscous** **IWs**

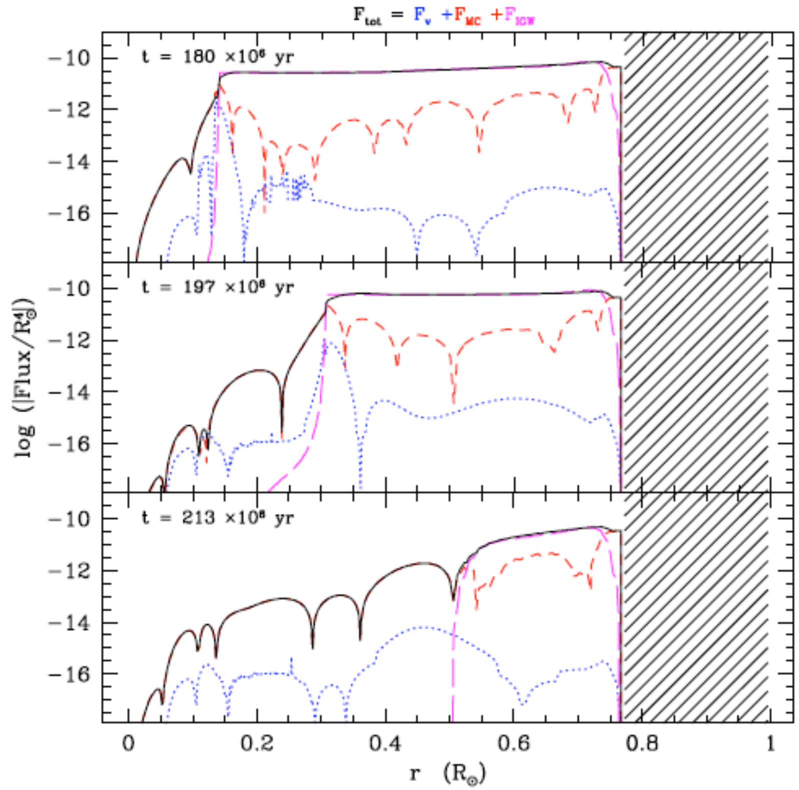
→ **Multi-cellular circulation driven by the wind and IWs Reynolds stresses (*C. Staquet's talk*)**

Fluxes of Angular Momentum:

- **Meridional circulation** $F_{MC}(r) = -\frac{1}{5} \overline{\rho} r^4 \overline{\Omega} U_2$

- **Shear induced turbulence** $F_S(r) = -\overline{\rho} r^4 \nu_V \partial_r \overline{\Omega}$

- **Internal waves** $F_J(r) = \frac{3}{8\pi} \mathcal{L}_J$



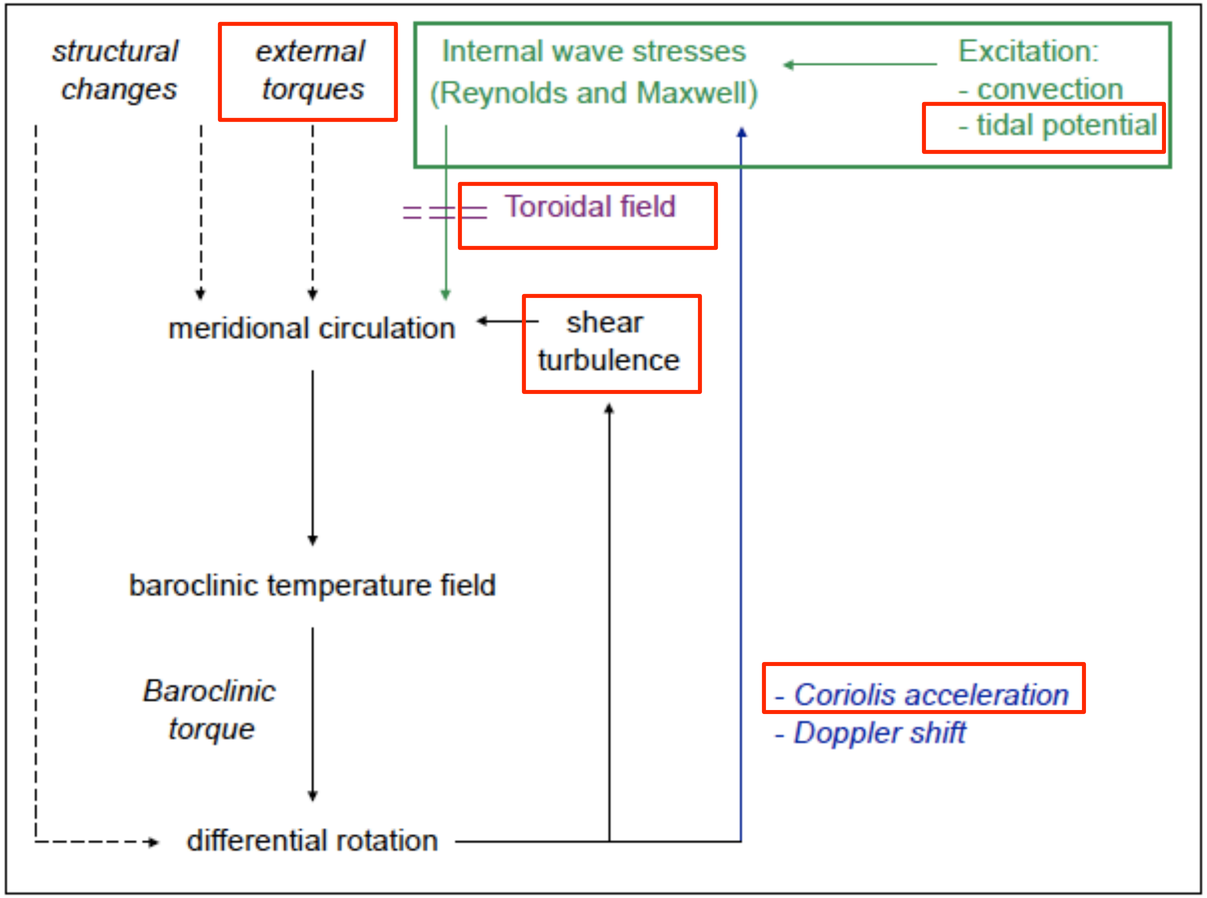
The transport loop with internal waves

Mathis & de Brye 2012

$$U_2 = \frac{5}{\overline{\rho} r^4 \overline{\Omega}}\left[\Gamma_M(r) - \overline{\rho}\nu_V r^4 \partial_r \overline{\Omega} + \frac{3}{8\pi}\mathcal{L}_J\right]$$

$$\underbrace{\overline{\rho}\,\overline{T}C_p \frac{d\Psi_2}{dt}}_{\overline{\rho}\,\overline{T}\partial_t \overline{S}} = \underbrace{-\overline{\rho}\,\overline{T}C_p \frac{N_T^2}{\overline{g}\delta}U_2}_{\overline{\rho}\,\overline{T}U_r\partial_r\overline{S}} \underbrace{+\overline{\rho}\frac{L}{M}\mathcal{T}_{2,\mathcal{B}} + \overline{\rho}\frac{L}{M}\mathcal{T}_{2,\mathrm{Th}}}_{\nabla\cdot(\chi\nabla T)-\nabla\cdot F_H} \underbrace{+\overline{\rho}\frac{L}{M}\mathcal{T}_{2,\mathrm{N-G}}}_{\rho\varepsilon}$$

$$\frac{1}{3}\, r\,\partial_r \overline{\Omega}^2 = \frac{\overline{g}}{r}\left(\varphi\Lambda_2 - \delta\Psi_2\right)$$



Zahn et al. 1997;
Talon & Charbonnel 2005;
Mathis et al. 2013

Internal waves interaction with shear induced turbulence: critical layers

Internal waves transport angular momentum if:
- dissipative processes
- corotation: critical layer

$$\sigma\left(r\right) = \sigma_{\rm exc} + m\Delta\overline{\Omega}\left(r\right) = 0$$

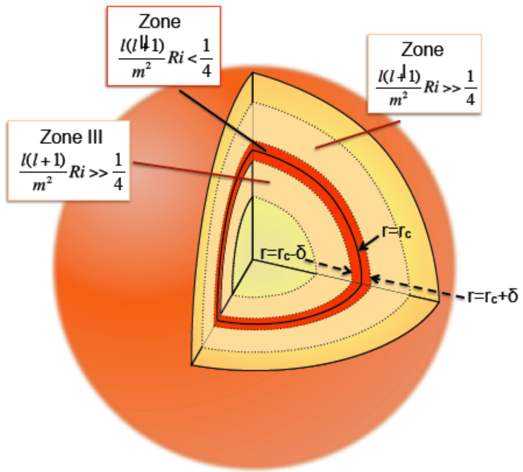
Case 1: non turbulent critical layer

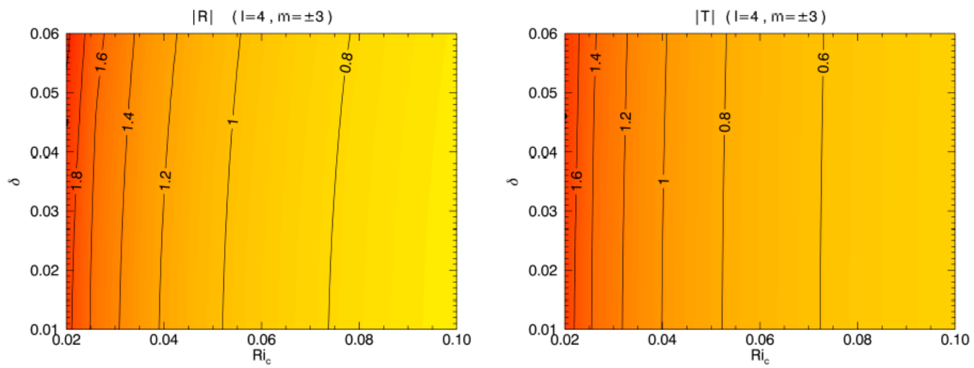
- a phase difference of $-\frac{\pi}{2}$,
- an attenuation by a factor $e^{-\pi\sqrt{\frac{l(l+1)}{m^2}Ri_c-\frac{1}{4}}}$

$$\mathrm{Ri}_c = \left(\frac{N^2}{\left(r\frac{\mathrm{d}\bar{\Omega}}{\mathrm{d}r}\right)^2}\right)_{r=r_c} \quad \alpha \quad \frac{\text{Stratification}}{\text{Shear}}$$

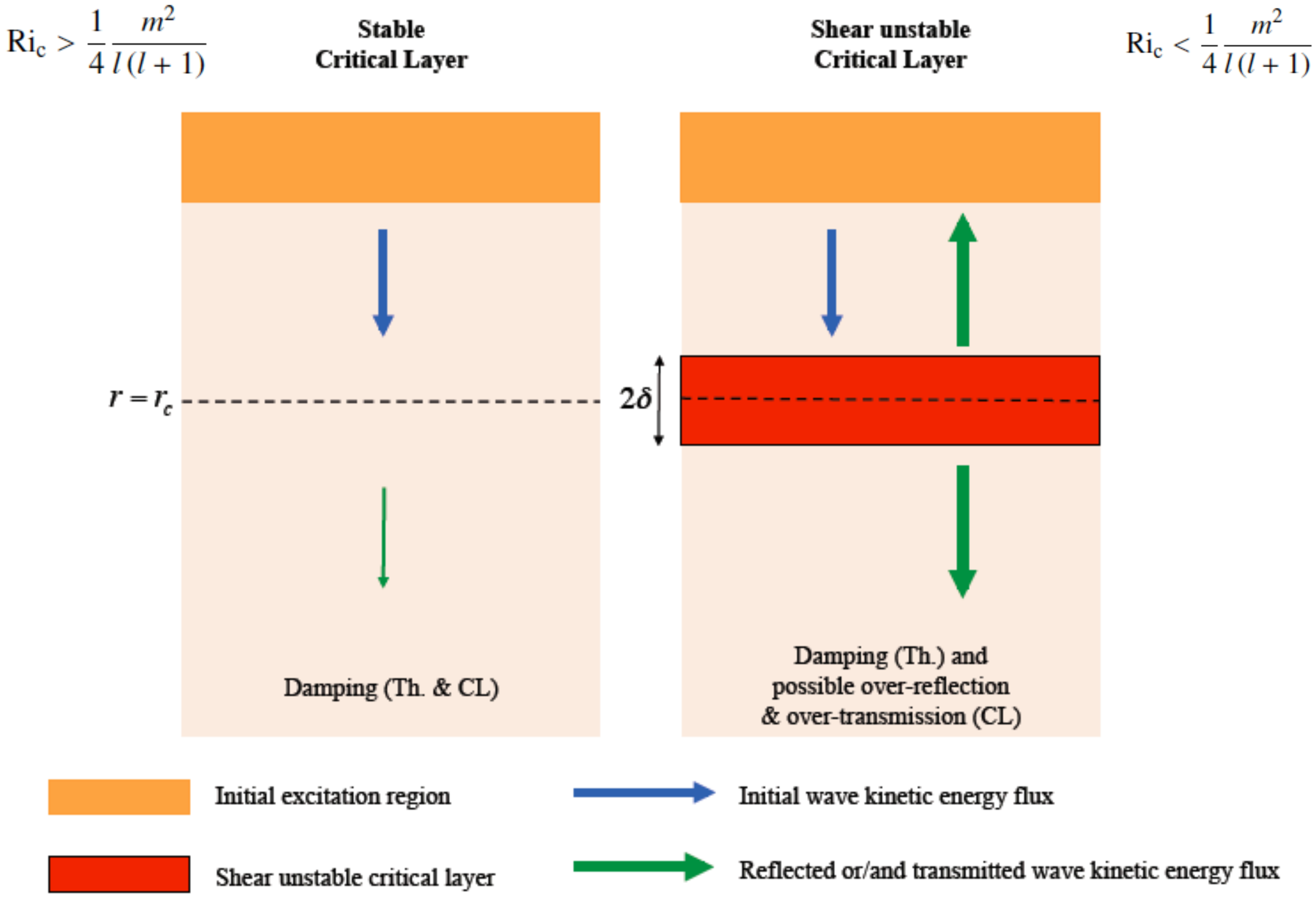
$$\mathrm{Ri}_c > \frac{1}{4}\frac{m^2}{l\left(l+1\right)}$$

Case 2: shear turbulent critical layer





Alvan, Mathis & Decressin 2013



Alvan, Mathis & Decressin 2013 (cf. Lindzen & Barker 1985; Nault & Sutherland 2007; D. Zimmerman's talk)

Internal waves region of excitation and propagation

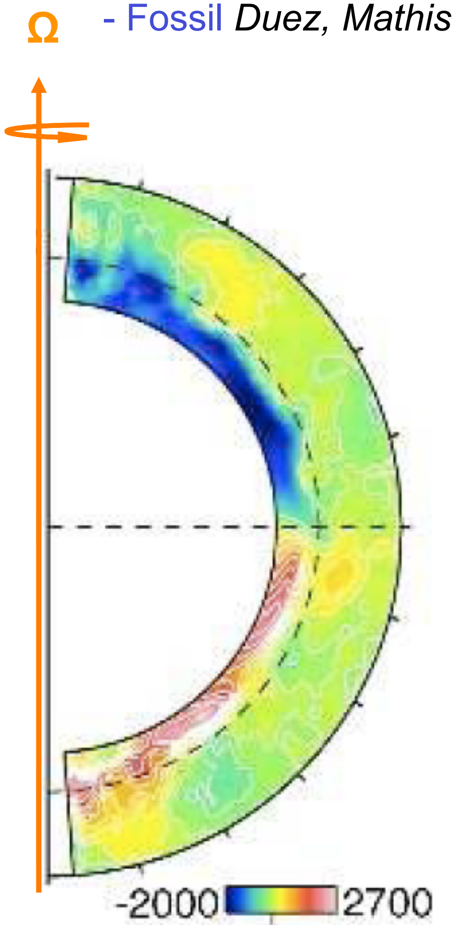
Complex magnetic fields

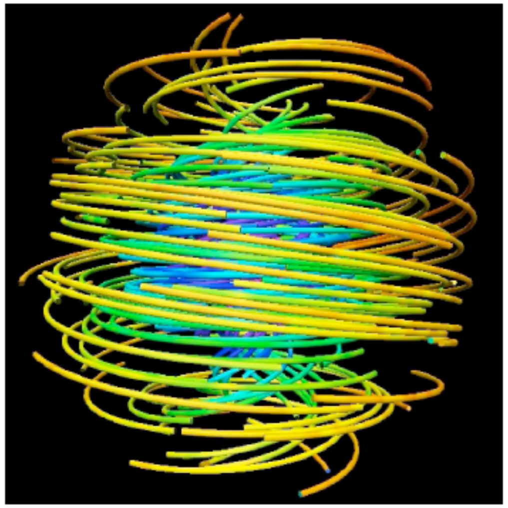
- Dynamo *Browning et al. 2006*

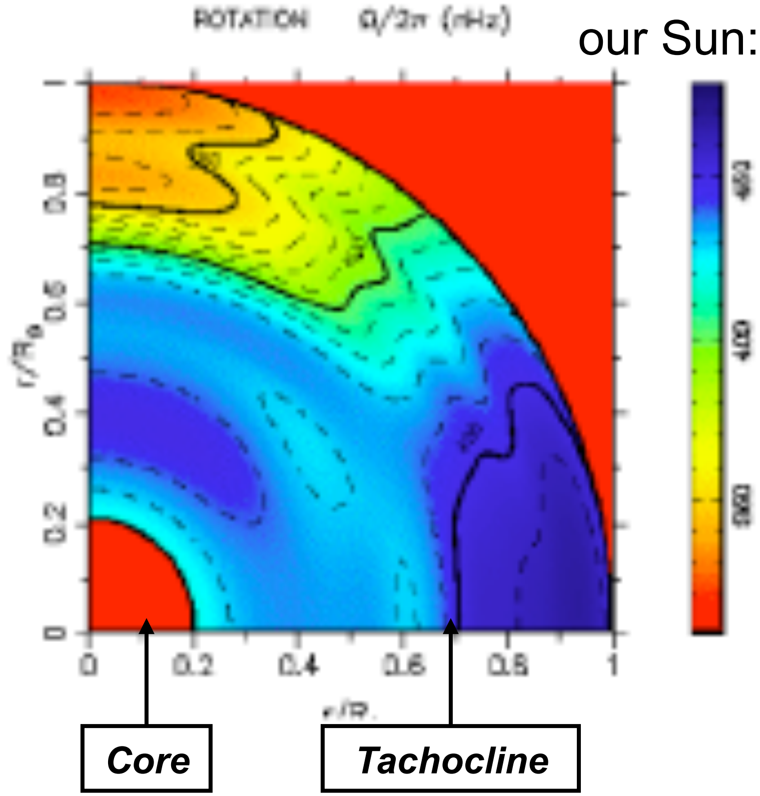
- Fossil *Duez, Mathis & Braithwaite 2010*

Differential rotation

Schou et al. 1998, Garcia et al. 2007, Eff-Darwich et al. 2008



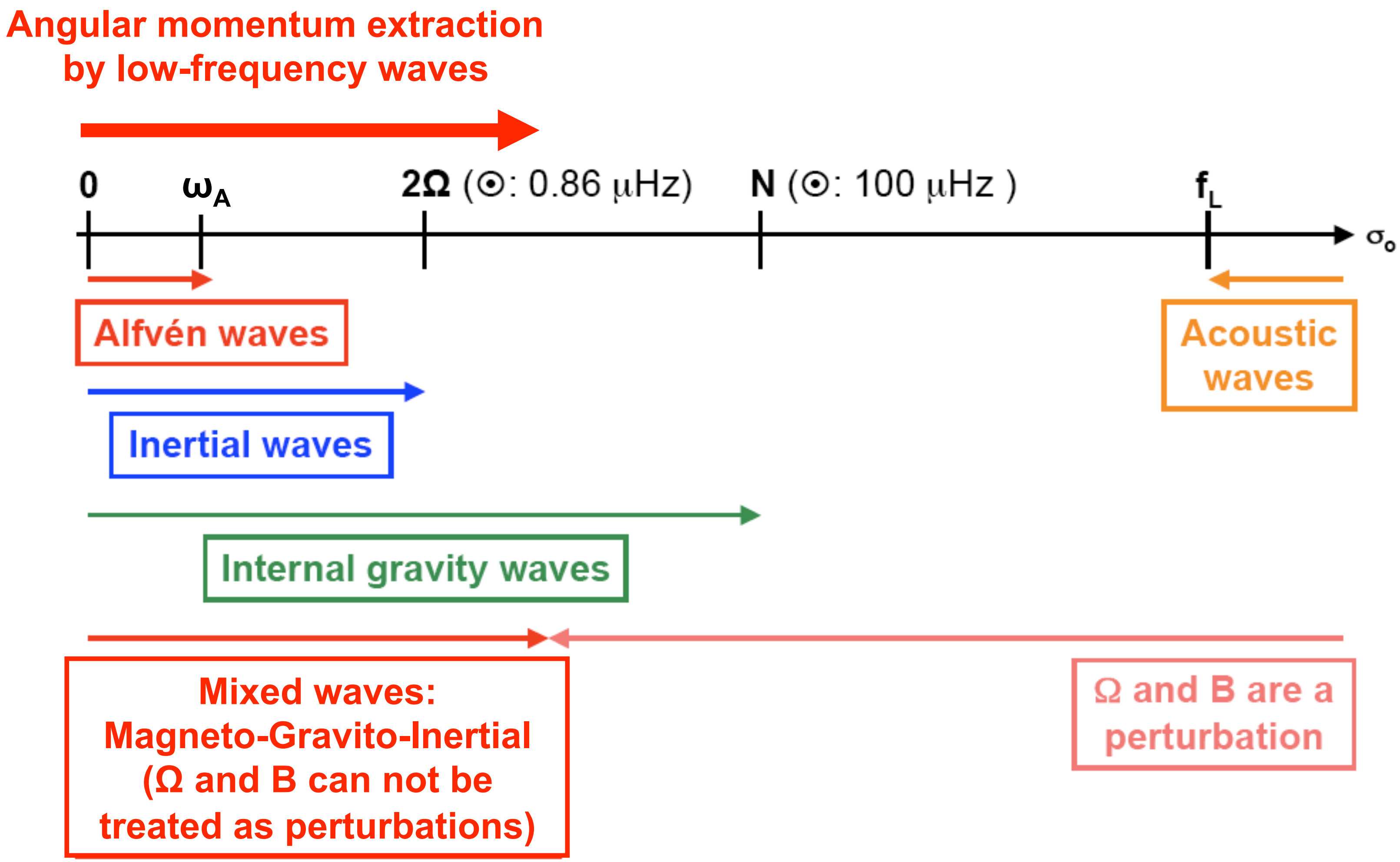




A coherent picture of internal wave machanisms

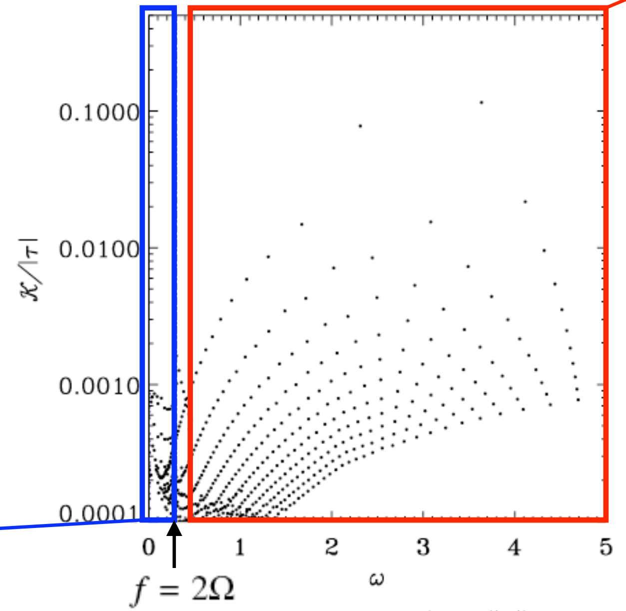
→ **needs to take into account the (differential) rotation and magnetic fields**

Ω & B effects in the treatment of internal waves: a necessity



Dintrans & Rieutord 2000; Mathis et al. 2008; Mathis 2009; Ballot et al. 2010, 11; Mathis & de Brye 2011, 12

The G.-I. waves: different families

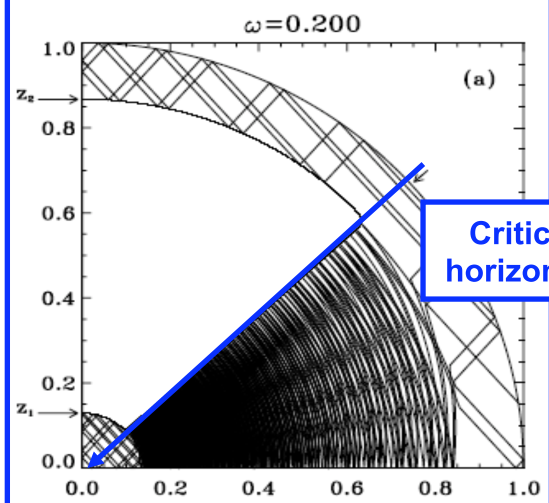


Hyperbolic waves

Elliptic waves

Fig. 9. Distribution in the complex plane $(\omega, \mathcal{K}/|\tau|)$ of the eigenvalues with $f = 0.3$, $m = 0^+$ and $\mathcal{K} = 5 \times 10^{-5}$. The resolution is $L = 22$ and $Nr = 100$.

$\widetilde{H}_2 : 0 < \omega < \min(2\Omega, N)$



Critical latitude: horizontal trapping

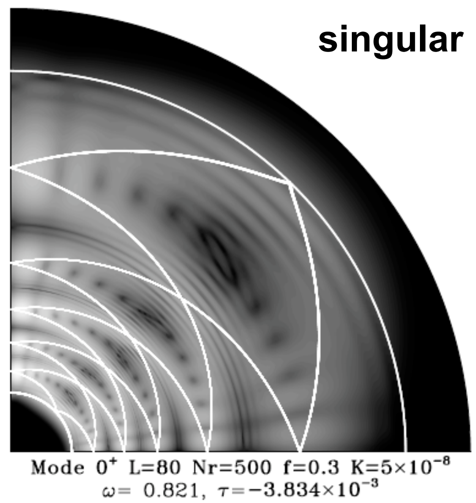
Dintrans & Rieutord 2000
(1.5 $M_\odot$; solid-body rotation)

$$\Lambda = \omega + i\tau$$

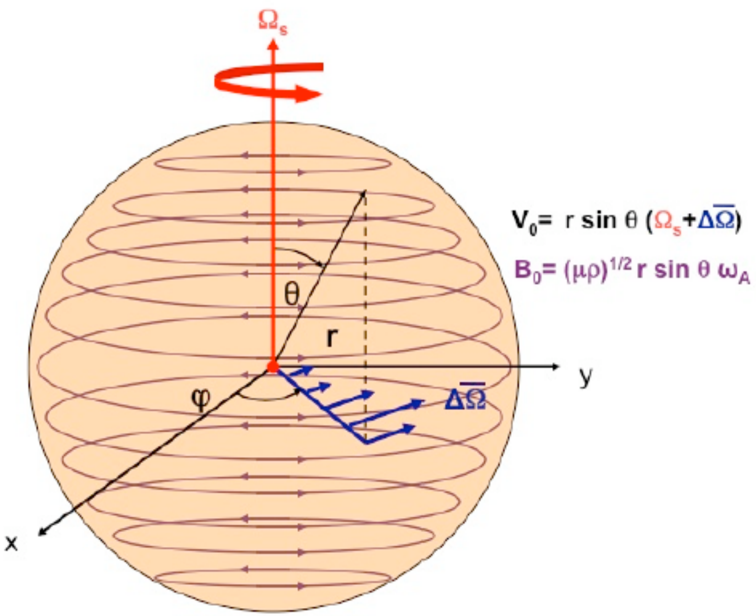
$\widetilde{E}_1 : \quad 2\Omega < \omega < N$

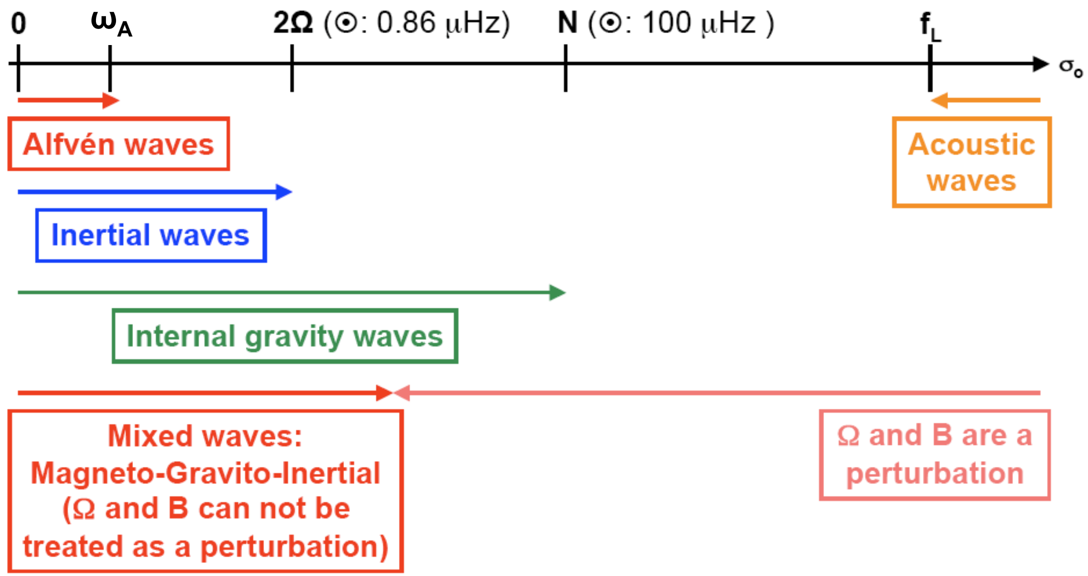
régular

singular



A first global Magneto-Gravito-Inertial waves set-up

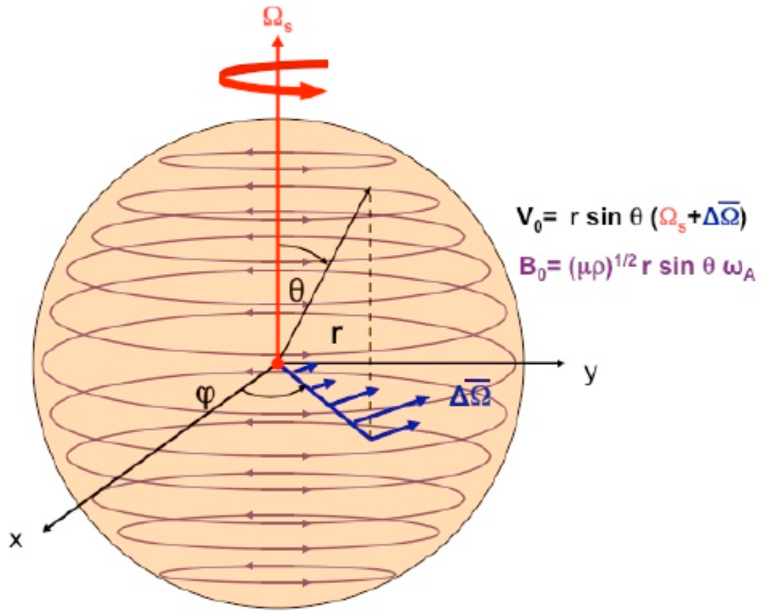




$$\sigma_s^2 \approx \left(\widehat{B}\cdot k\right)^2 V_A^2 + \left(N \times \widehat{k}\right)^2 + 4\left(\Omega\cdot\widehat{k}\right)^2$$

- Schatzman 1993; Kumar, Talon & Zahn 1999; Kim & McGregor 2003; Rogers & Mc Gregor 2010-2011
- Dintrans & Rieutord 2000; Mathis et al. 2008; Mathis 2009; Ballot et al. 2010, 11; Mathis & de Brye 2011, 12
- H.-C. Nataf's talk

A first global Magneto-Gravito-Inertial waves set-up



- Velocities:

$$V(r,t) = V_0(r,t) + \underline{u(r,t)} \text{ with } V_0 = r\sin\theta\,\Omega(r,\theta)\widehat{e_\varphi}$$

Wave's velocity field

$$\overline{\Omega}(r) = \underline{\Omega_s} + \underline{\Delta\overline{\Omega}(r)}, \text{ where } \Delta\overline{\Omega}(r) \ll \Omega_s$$

Uniform rotation: waves structure

Differential rotation: thermal diffusion

-Magnetic fields:

$$B(r,t) = B_0^T(r,t) + \underline{b(r,t)} \text{ with } B_0^T = \sqrt{\mu\rho}\,r\sin\theta\,\underline{\omega_A}\,\widehat{e_\varphi}$$

Wave's magnetic field

Uniform Alvén frequency

The Magneto-Gravito-Inertial waves dynamics - I

Friedlander 1987-1989;
Mathis & de Brye 2011

- Induction equation ($q = \eta/K \ll 1$)

$$b = \nabla \times \left(\xi \times B_0^{\mathrm{T}}\right) \longrightarrow b = \sqrt{\mu\overline{\rho}}\,\omega_{\mathrm{A}}\partial_\varphi \xi$$

- Momentum equation ($P_r = \nu/K \ll 1$)

$$\left(\partial_t + \Omega_s\,\partial_\varphi\right)\left[\left(\partial_t + \Omega_s\,\partial_\varphi\right)\xi + 2\,\Omega_s\,\widehat{e}_z \times \xi\right] =$$
$$-\frac{1}{\overline{\rho}}\nabla\,\Pi\,(r,t) - \nabla\,\widetilde{\Phi} + \frac{\widetilde{\rho}}{\overline{\rho}^2}\nabla\,\overline{P} + \frac{F_{\mathcal{L}}^{\mathrm{Te}}(\xi)}{\overline{\rho}}$$

Wave's total pressure

$$\Pi = \widetilde{P} + \frac{B_0^{\mathrm{T}} \cdot b}{\mu}$$

Wave's volumetric magnetic tension force

$$F_{\mathcal{L}}^{\mathrm{Te}}(\xi) = \frac{1}{\mu}\left[\left(B_0^{\mathrm{T}} \cdot \nabla\right) b + (b \cdot \nabla)\,B_0^{\mathrm{T}}\right]$$
$$= \overline{\rho}\omega_{\mathrm{A}}^2\left[\partial_{\varphi^2}\xi + 2\,\widehat{e}_z \times \partial_\varphi\,\xi\right]$$

- Continuity equation: *anelastic approximation*

- Energy equation: *regime dominated par thermal diffusion; i.e. P_r & $q \ll 1$*

- Poisson's equation: *the Cowling's approximation is assumed*

The Magneto-Gravito-Inertial waves dynamics - II

Using an expansion in Fourier's series $\exp(im\varphi)\exp(i\sigma t)$

$$u' = \mathrm{i}\,\sigma_{\mathrm{s}}\,\boldsymbol{\xi}'$$

$$b' = \mathrm{i}m\,\sqrt{\mu\overline{\rho}}\,\omega_{\mathrm{A}}\,\boldsymbol{\xi}'$$

$$-\mathcal{A}\boldsymbol{\xi}' + i\mathcal{B}\widehat{\boldsymbol{e}}_z \times \boldsymbol{\xi}' = -\boldsymbol{\nabla} W' + \frac{\rho'}{\overline{\rho}^2}\boldsymbol{\nabla}\overline{P}$$

Gravito-inertial waves like ⟶ Poincaré equation

0 < $\mathcal{A} = \sigma_{\mathrm{M}}^2 = \sigma_{\mathrm{s}}^2 - m^2\omega_{\mathrm{A}}^2$

Vertical trapping if *A<0*

$$\mathcal{B} = 2\left(\Omega_{\mathrm{s}}\sigma_{\mathrm{s}} - m\,\omega_{\mathrm{A}}^2\right)$$

Braginsky & Roberts 1975;
Friedlander 1987-1989; Mathis & de Brye 2011

The strong stratification case: the MHD Traditional Approximation

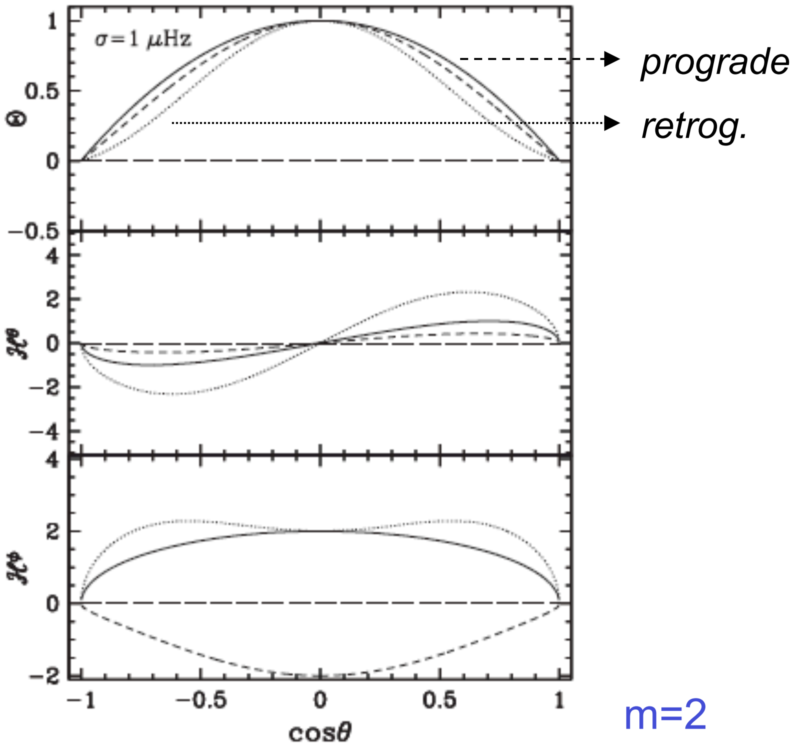
In stellar radiation zones $S_\Omega = \dfrac{N}{2\Omega_s}$ and $S_B = \dfrac{N}{\omega_{\mathrm{A}}}$ *<<1 → asymptotic expansion*

M.-G.-I. waves angular structure under MHD TA

M.-G.-I. waves horizontal eigenfunctions:
Hough functions (eigenfunctions of the Laplace Tidal Operator; *Laplace 1799, Hough 1898*)

$$\mathcal{L}_{\nu_{\mathrm{M};m}}\left[\Theta_{k,m}\left(x;\nu_{\mathrm{M};m}\right)\right] = -\Lambda_{k,m}\left(\nu_{\mathrm{M};m}\right)\Theta_{k,m}\left(x;\nu_{\mathrm{M};m}\right)$$

$$\left\{\begin{array}{l}
\mathcal{L}_{\nu_{\mathrm{M};m}} \equiv \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1-x^2}{1-\nu_{\mathrm{M};m}^2 x^2}\dfrac{\mathrm{d}}{\mathrm{d}x}\right) - \dfrac{1}{1-\nu_{\mathrm{M};m}^2 x^2}\left(\dfrac{m^2}{1-x^2} + m\nu_{\mathrm{M};m}\dfrac{1+\nu_{\mathrm{M};m}^2 x^2}{1-\nu_{\mathrm{M};m}^2 x^2}\right) \\[2ex]
\nu_{\mathrm{M};m} = R_{\mathrm{o}}^{-1}\dfrac{1-m\Lambda_{\mathrm{E}}}{1-\dfrac{m^2}{2}R_{\mathrm{o}}^{-1}\Lambda_{\mathrm{E}}} \qquad \left\{\begin{array}{l} R_{\mathrm{o}} = \dfrac{\sigma_s}{2\Omega_s} \\[2ex] \Lambda_{\mathrm{E}} = \dfrac{\omega_{\mathrm{A}}^2}{\Omega_s \sigma_s} \end{array}\right.
\end{array}\right.$$



m=2

Lee & Saio 1997; Townsend 2003;
Pantillon et al. 2007; Mathis et al. 2008;
Mathis & de Brye 2011

M.-G.-I. waves structure under MHD TA

Wave velocity and magnetic fields

$$\boldsymbol{u} = \sum_{j=\{r,\theta,\varphi\}} \left[\sum_{\sigma,m,k} u_{j;k,m}(\boldsymbol{r},t) \right] \widehat{\boldsymbol{e}}_j$$

$$u_{r;k,m} = -\mathcal{E}_{k,m}(r)\, \Theta_{k,m}(\cos\theta; \nu_{\mathrm{M};m}) \sin\left[\zeta_{k,m}(r,\varphi,t)\right] \times \exp\left[-\frac{\tau_{k,m}\left(r;\nu_{\mathrm{M};m},\Delta\overline{\Omega}\right)}{2} \right],$$

same form in the θ & φ directions

$$\boldsymbol{b} = \sum_{j=\{r,\theta,\varphi\}} \left[\sum_{\sigma,m,k} b_{j;k,m}(\boldsymbol{r},t) \right] \widehat{\boldsymbol{e}}_j$$

$$b_{j;k,m} = \sqrt{\mu\overline{\rho}}\, \omega_{\mathrm{A}} \frac{m}{\sigma_s} u_{j;k,m}.$$

- Wave propagation function

$$\zeta_{k,m}(r,\varphi,t) = \int_r^{r_c} k_{V;k,m}(r')\, \mathrm{d}r' + m\varphi + \sigma_s t \qquad k_{V;k,m} \equiv \left(\frac{N}{\sigma_{\mathrm{M}}}\right) \frac{\Lambda_{k,m}^{1/2}(\nu_{\mathrm{M};m})}{r}$$

$$\equiv F_r^{-1} \left(1 - \frac{m^2}{2} R_o^{-1} \Lambda_{\mathrm{E}}\right)^{-1/2} \frac{\Lambda_{k,m}^{1/2}(\nu_{\mathrm{M};m})}{r}$$

- Wave damping

$$\tau_{k,m}\left(r;\nu_{\mathrm{M};m},\Delta\overline{\Omega}\right) = \Lambda_{k,m}^{3/2}(\nu_{\mathrm{M};m}) \int_r^{r_c} K \frac{N_T^2 N}{\widetilde{\sigma}_m \widetilde{\sigma}_{M;m}^3} \frac{\mathrm{d}r'}{r'^3} \qquad \begin{cases} \widetilde{\sigma}_m(r) = \sigma_s + m\Delta\overline{\Omega}(r) \\ \widetilde{\sigma}_{M;m}(r) = \widetilde{\sigma}_m - m^2\omega_A^2 \end{cases}$$

M.-G.-I. waves propagation

Control parameters

- MHD local frequency: $\mathcal{A} = \sigma_{\rm M}^2 = \sigma_s^2 - m^2\omega_{\rm A}^2$

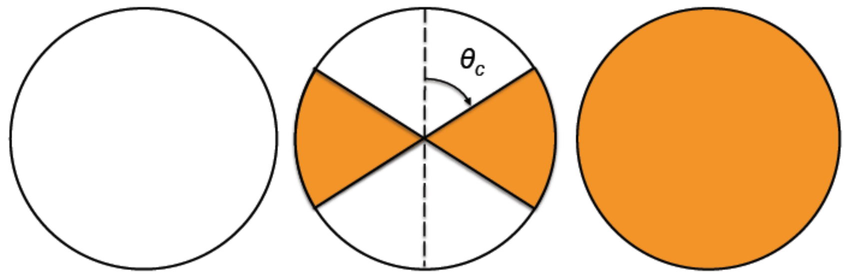
- Latitudinal trapping control parameter:

$$\nu_{{\rm M};m} = R_{\rm o}^{-1}\frac{1-m\Lambda_{\rm E}}{1-\frac{m^2}{2}R_{\rm o}^{-1}\Lambda_{\rm E}}$$

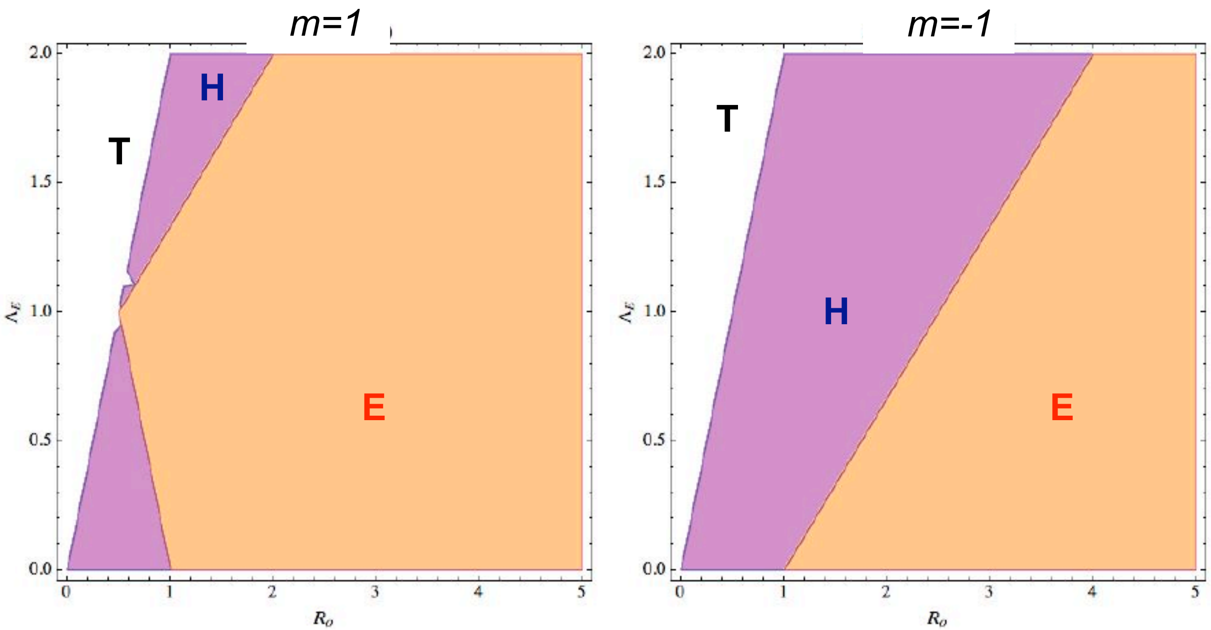
m>0 - retrograde
m<0 - prograde

$$R_{\rm o} = \frac{\sigma_s}{2\Omega_s} \qquad \Lambda_{\rm E} = \frac{\omega_{\rm A}^2}{\Omega_s\sigma_s}$$

Vert. trapping	Equatorial trapped waves	Regular waves & MHD T. A.
T($A\le0$)	H($A>0;\nu_M\ge1$)	E ($A>0;\nu_M<1$)



$$\theta_{c;m}(\nu_{{\rm M};m}) = \arccos\left(|\nu_{{\rm M};m}|^{-1}\right)$$



Mathis & de Brye 2011, 12

→ *If B≠0, net bias between pro & retrograde waves: θ_c(prograde)>θ_c(retrograde)*

Action of angular momentum

Definition:

$$\mathcal{L}_V^{\mathrm{AM}}(r,\theta) = \sum_{\sigma,k,m} \left\{ r^2 \mathcal{F}_{V;k,m}^{\mathrm{AM}} \right\} = \sum_{\sigma,k,m} \left\{ -\frac{m}{\sigma_s} \left(r^2 \mathcal{F}_{V;k,m}^{\mathrm{E}} \right) \right\}$$

$$= r^2 \sum_{\sigma,k,m} \left\{ \mathcal{F}_{V;k,m}^{\mathrm{Re}}(r,\theta) + \mathcal{F}_{V;k,m}^{\mathrm{Ma}}(r,\theta) \right\}$$

Energy flux at the borders with CZ

Lagrangian wave's Reynolds stresses

Lagrangian wave's Maxwell stresses

Grimshaw 1984
Mathis & de Brye 2012

$$\begin{cases} \mathcal{F}_{V;k,m}^{\mathrm{Re}} = \bar{\rho}\, r \sin\theta \left\langle u_{r;k,m} \left(u_{\varphi;k,m} + \sigma_s R_0^{-1} \cos\theta\, \xi_{\theta;k,m} \right) \right\rangle_\varphi \\ \mathcal{F}_{V;k,m}^{\mathrm{Ma}} = \\ \quad -\bar{\rho}\, r \sin\theta\, m R_0^{-1} \Lambda_{\mathrm{E}} \left\langle u_{r;k,m} \left(\frac{m}{2} u_{\varphi;k,m} + \sigma_s \cos\theta\, \xi_{\theta;k,m} \right) \right\rangle_\varphi \end{cases}$$

→ act against Reynolds stresses and scales as $(\omega_A/\sigma_s)^2$

The case of solar type stars: energy flux<0

- prograde waves (*m<0*) → angular momentum flux<0: deposit
- ondes rétrogrades (*m>0*) → angular momentum flux>0: extraction

Angular momentum transport

Angular momentum transport:

$$\overline{\rho}\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\overline{\Omega}\right)=-\frac{3}{2}\frac{1}{r^2}\partial_r\overline{\mathcal{L}_V^{\mathrm{AM}}}$$

$$\overline{\mathcal{L}_V^{\mathrm{AM}}}(r)=\left\langle\mathcal{L}_V^{\mathrm{AM}}\right\rangle_\theta$$
$$=\sum_{\sigma,k,m}\overline{\mathcal{L}_{V;k,m}^{\mathrm{AM}}}\left(r_c;\nu_{\mathrm{M};m}\right)\exp\left[-\tau_{k,m}\left(r;\nu_{\mathrm{M};m},\Delta\overline{\Omega}\right)\right]$$

Excited spectrum

A.-M. flux at the borders with CZ

Radiative damping

$$\tau_{k,m}\left(r;\nu_{\mathrm{M};m},\Delta\overline{\Omega}\right)=\Lambda_{k,m}^{3/2}\left(\nu_{\mathrm{M};m}\right)\int_r^{r_c}K\frac{N_T^2\,N}{\widetilde{\sigma}_m\,\widetilde{\sigma}_{M;m}^3}\frac{\mathrm{d}r'}{r'^3}$$

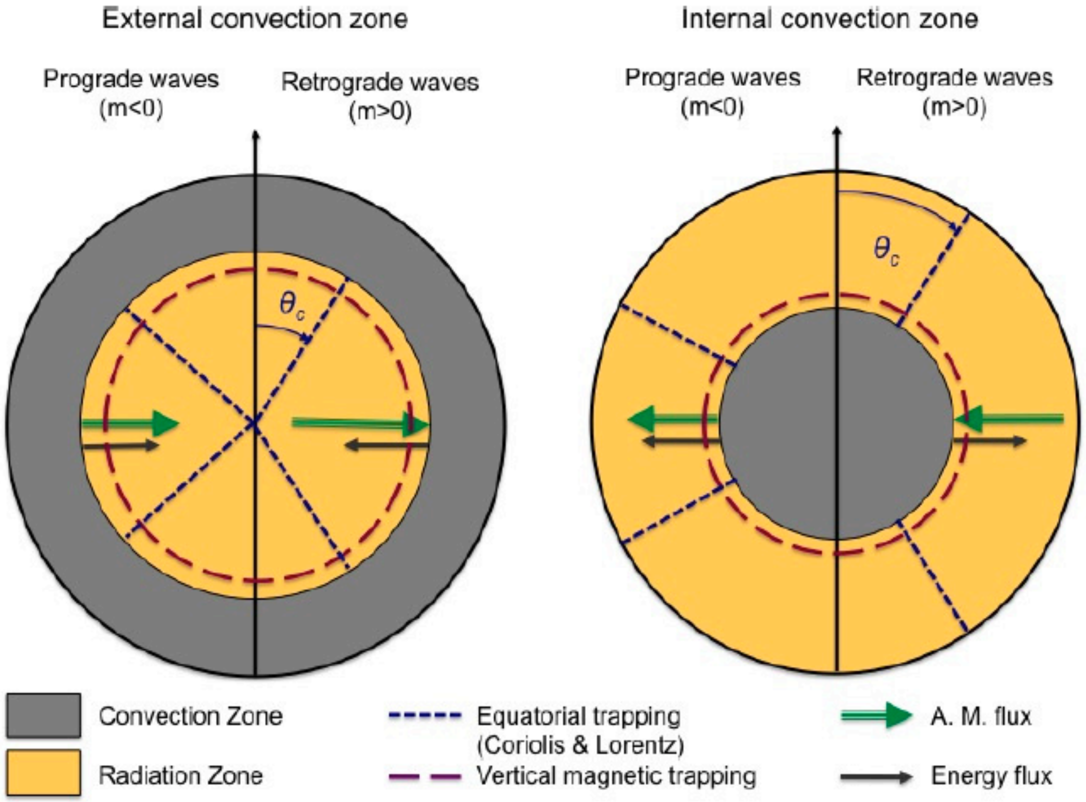
$$\begin{cases}\widetilde{\sigma}_m\left(r\right)=\sigma_s+m\Delta\overline{\Omega}\left(r\right)\\ \widetilde{\sigma}_{M;m}\left(r\right)=\widetilde{\sigma}_m-m^2\omega_A^2\end{cases}$$

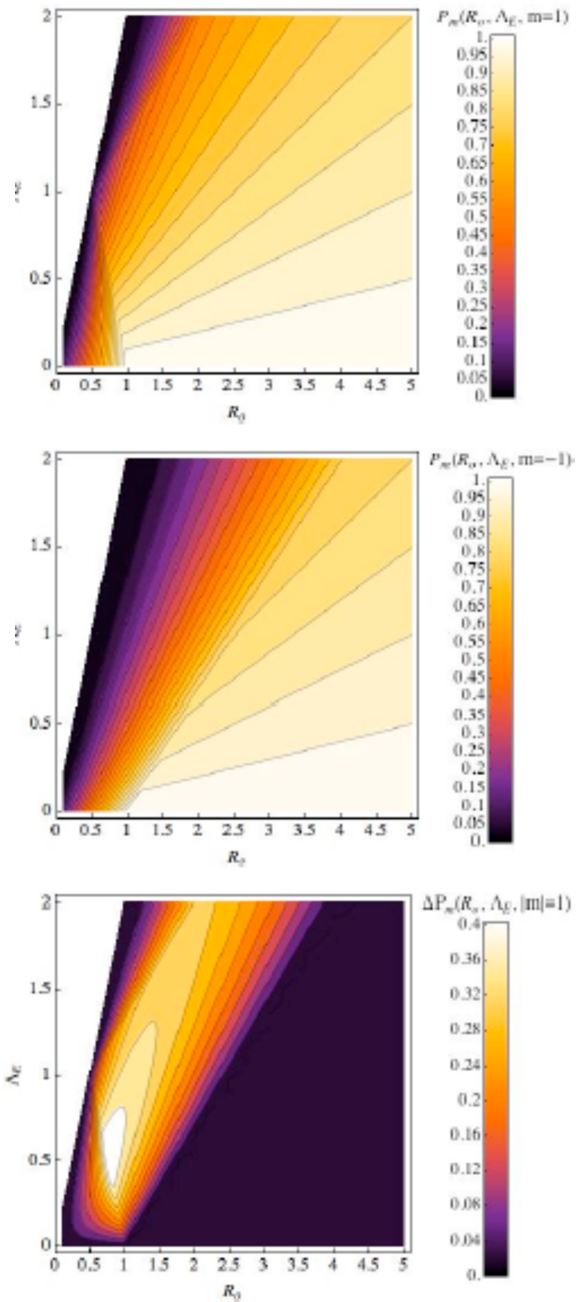
Radiative damping and Doppler effect:

- Doppler effect: σ_m(prograde)<σ_m(retrograde): prograde waves damped before retrograde waves

- $\Lambda_{k,m}$(prograde)<$\Lambda_{k,m}$(retrograde): reduces the bias between prograde and retrograde waves
- $\Lambda_{k,m}$>$\Lambda_{k,m}$(Ω & B_0=0): waves are damped closer to their excitation region

Excitation energy transmission





$$\mathcal{P}_m = \underbrace{\left(\frac{\sigma_{\rm M}}{\sigma_s}\right)^2}_{\text{vertical trapping}} \underbrace{\left[\frac{1}{2\pi}\int_{\theta_{c;m}}^{\pi/2}\sin\theta\,{\rm d}\theta\int_0^{2\pi}{\rm d}\varphi\right]}_{\text{equatorial trapping}}$$

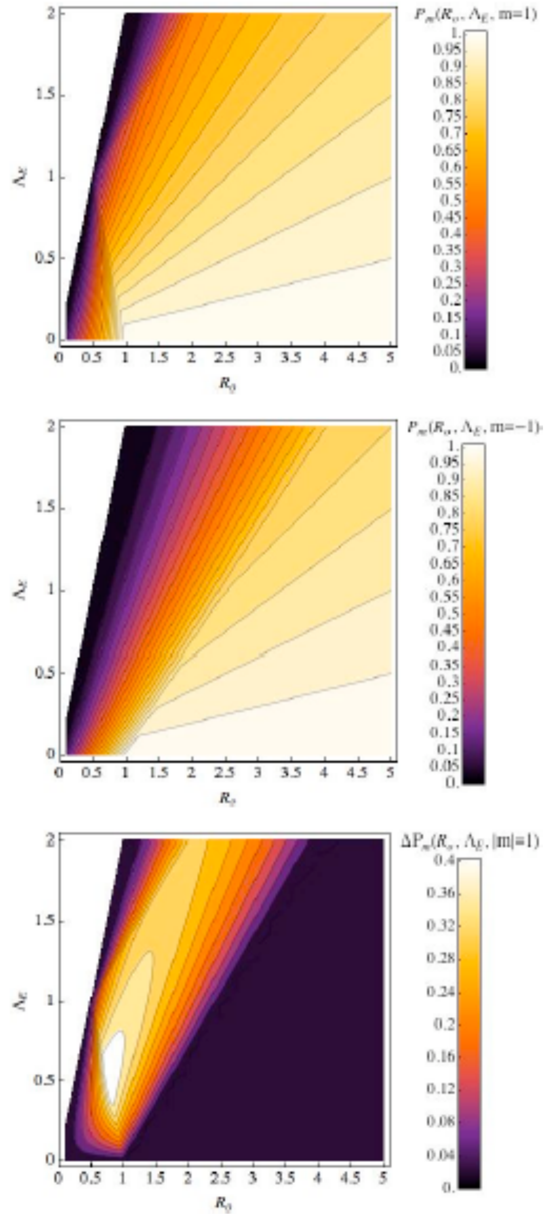
$$= \left(1-\frac{m^2}{2}R_{\rm o}^{-1}\Lambda_{\rm E}\right)\left[\cos\theta_{c;m}\,{\rm H_e}\left(|\nu_{{\rm M};m}|-1\right)+{\rm H_e}\left(1-|\nu_{{\rm M};m}|\right)\right]$$

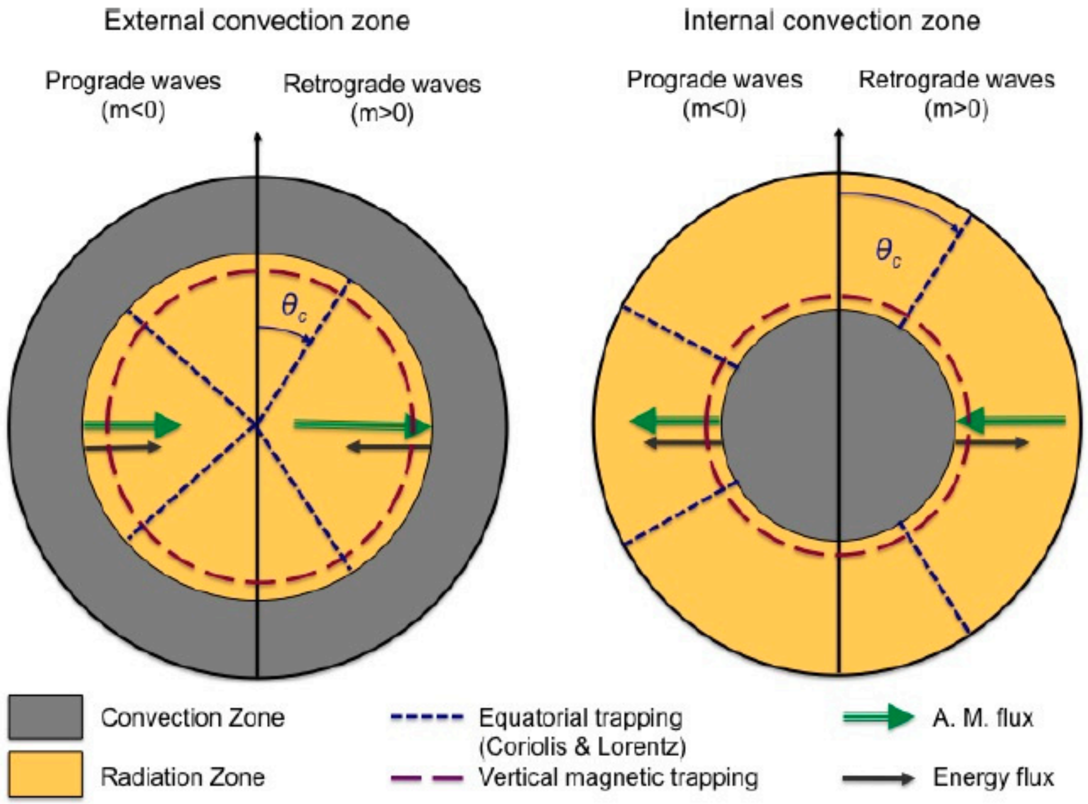
Transport of angular momentum

Mathis & de Brye 2012
(Mathis et al. 2008)

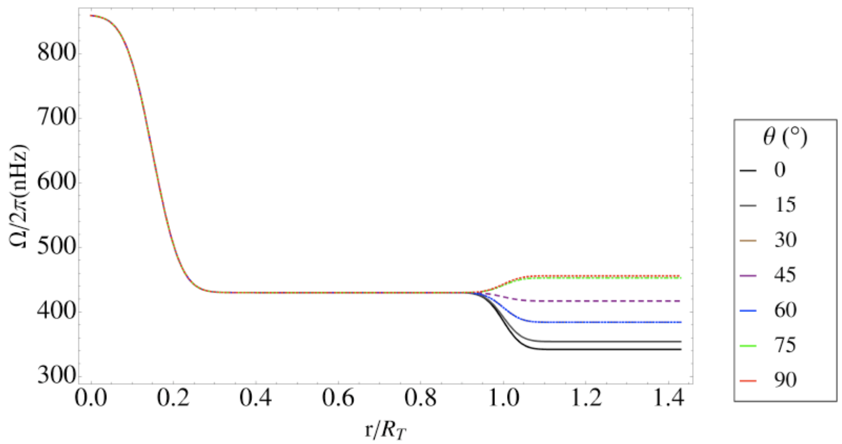
- For a given wave, the Radiative Damping is increased if Ω & $B_0 \neq 0$
- Modified horizontal structure → the bias between the RDs of retrograde and prograde waves decreases
- Magnetic field induces Maxwell stresses, acting against Reynolds ones, that scales as $(\omega_A/\sigma_s)^2$

-The difference in equatorial trappings → transmission of convective energy to retrograde waves is favoured

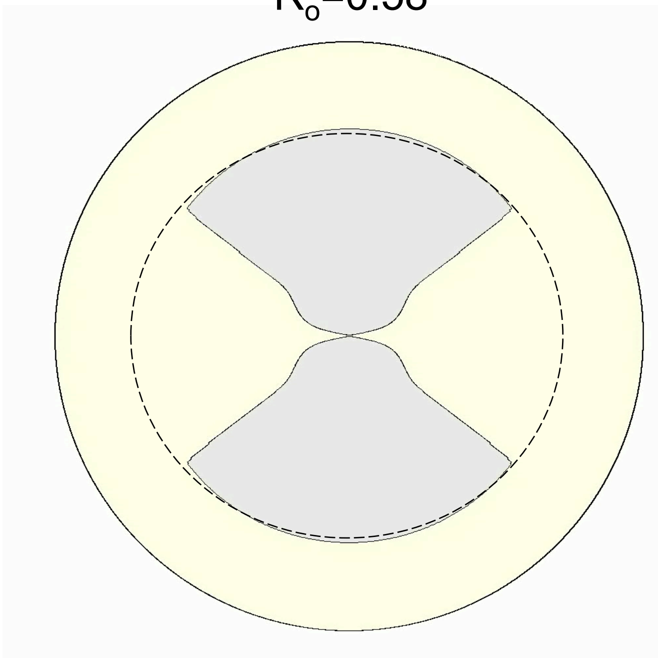




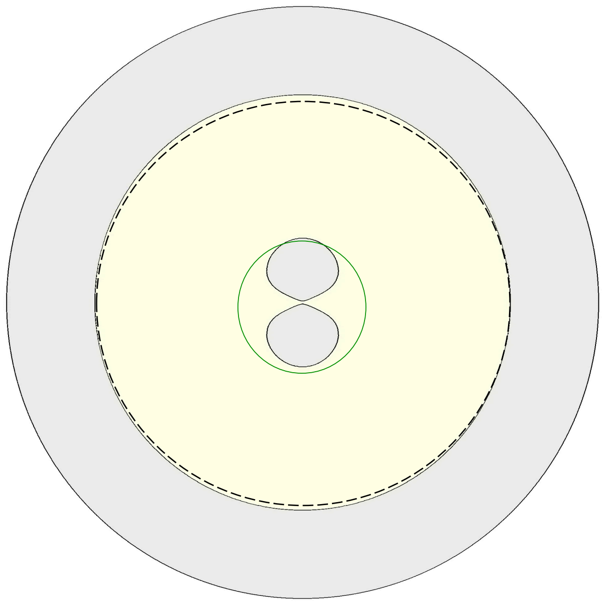
What's next: general differential rotation and B



R_o=0.58



R_o=1.16

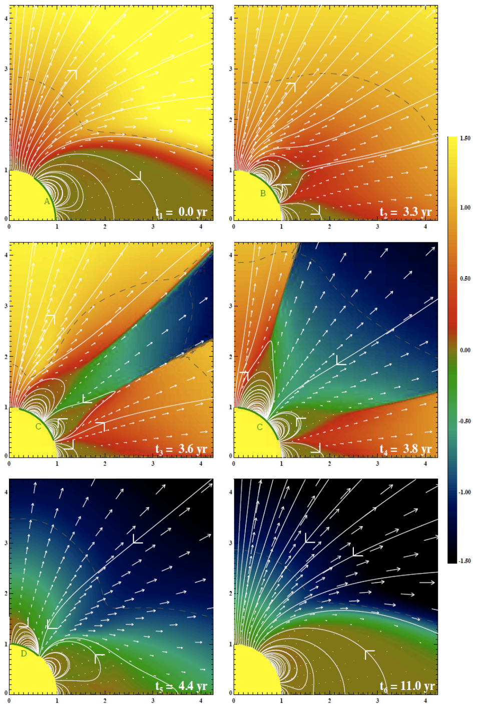


Mathis 2009
Mathis, Alvan & Brun 2013

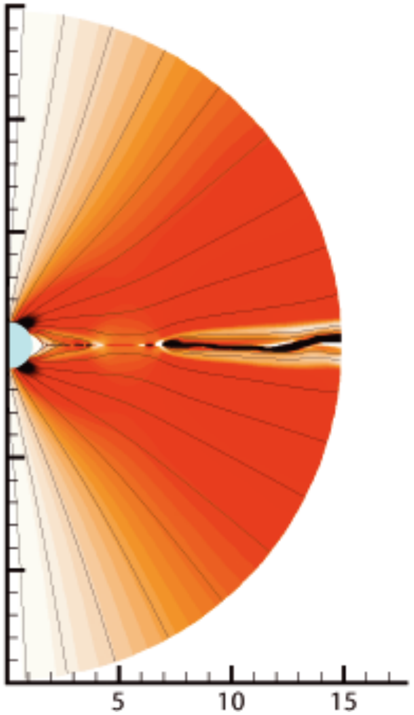
Interaction with stellar environment: winds, accretion disks

Winds: pressure-driven, line-driven

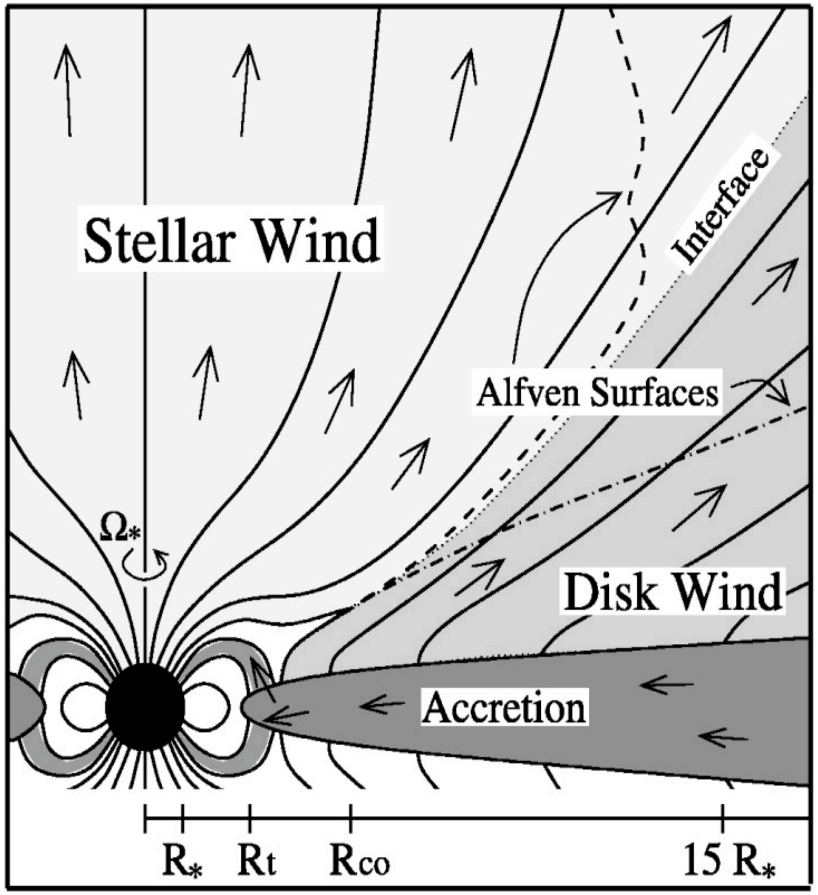
Pinto et al. 2011



Ud-Doula et al. 2008



Accretion disks

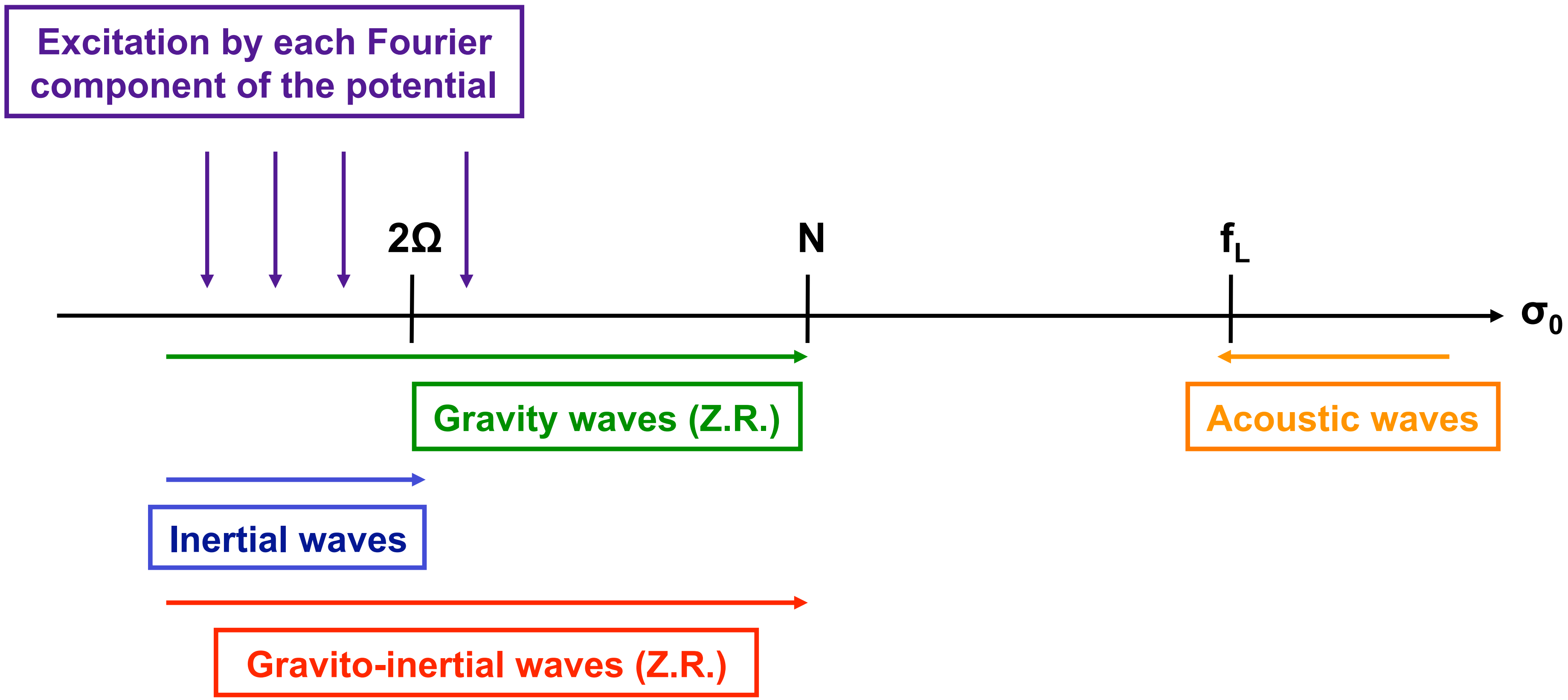


Matt & Pudritz 2005

Interaction with stellar environment: The tidal fluid velocity fields

Equilibrium tide: large-scale circulation induced by the hydrostatic adjustement to the tidal potential perturbation

Dynamical tide: waves excited by the tidal potential



The "engine" of the dynamical evolution of binary systems: energy dissipation

Dynamical evolution of a binary system:

Initial state:

- elliptic keplerian orbits of the two components
- non-synchronized rotations with the orbital movement
- non-aligned orbital and components' spins

Final state: **minimum energy state**

- circularized orbits

$$\frac{1}{t_{\rm circ}} = \frac{21}{2}\frac{k_2}{t_f} q\,(1+q)\left(\frac{R}{a}\right)^8 \qquad q = \frac{M_2}{M_1}$$

- components synchronized with the orbital movement

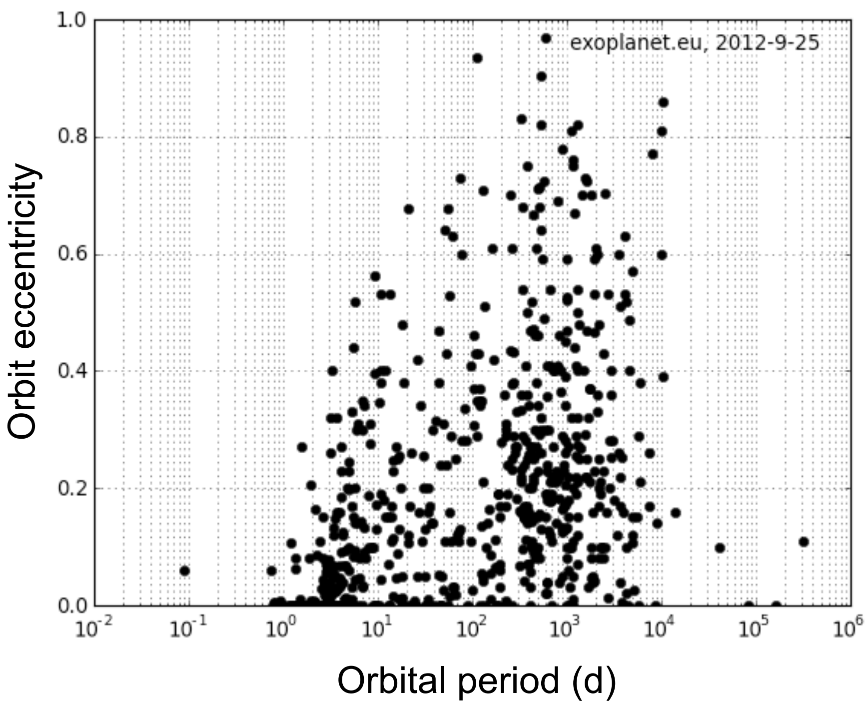
$$\frac{1}{t_{\rm sync}} = 6\frac{k_2}{t_f} q^2 \frac{M_1 R^2}{I}\left(\frac{R}{a}\right)^6$$

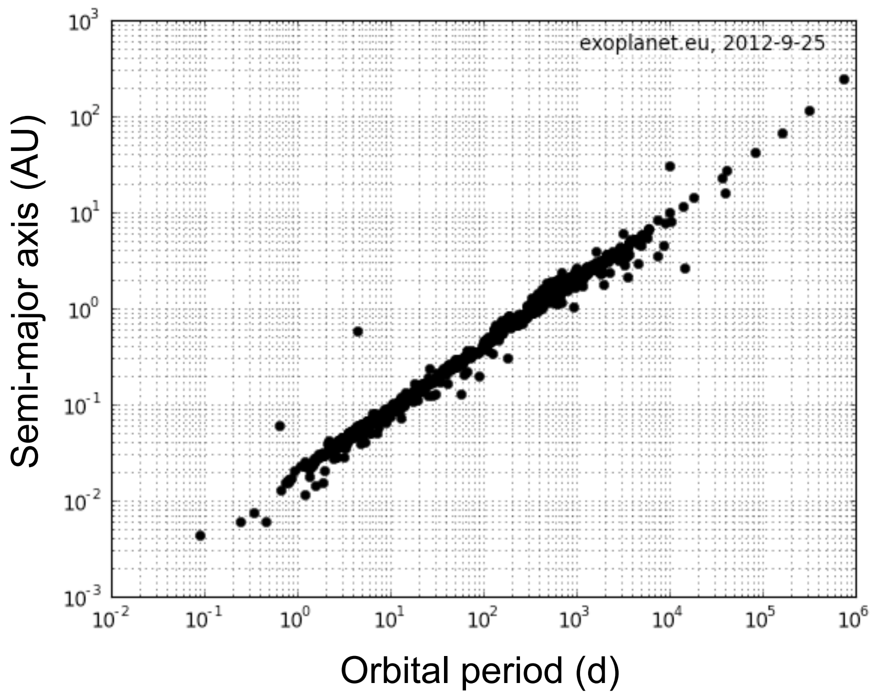
- aligned spins

or spiraling (*Hut 1980, 1981; Levrard et al. 2009*)

→ **Necessity to identify the dissipative processes that convert the kinetic energy into thermic one (→ time-scales for circularization, synchronization and alignment)**

A critical test of the theory for star-planet systems: the orbital state



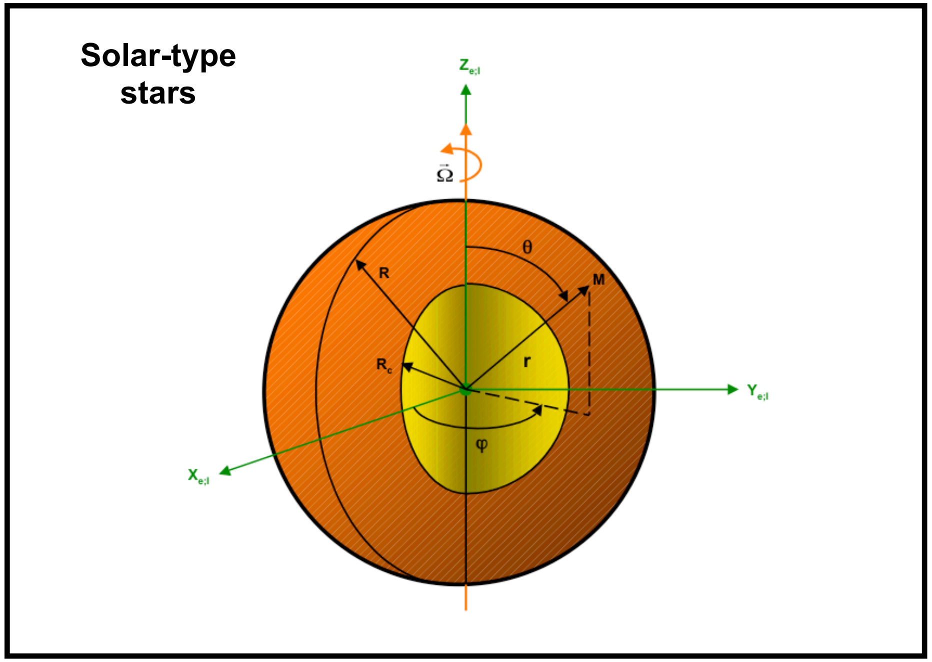


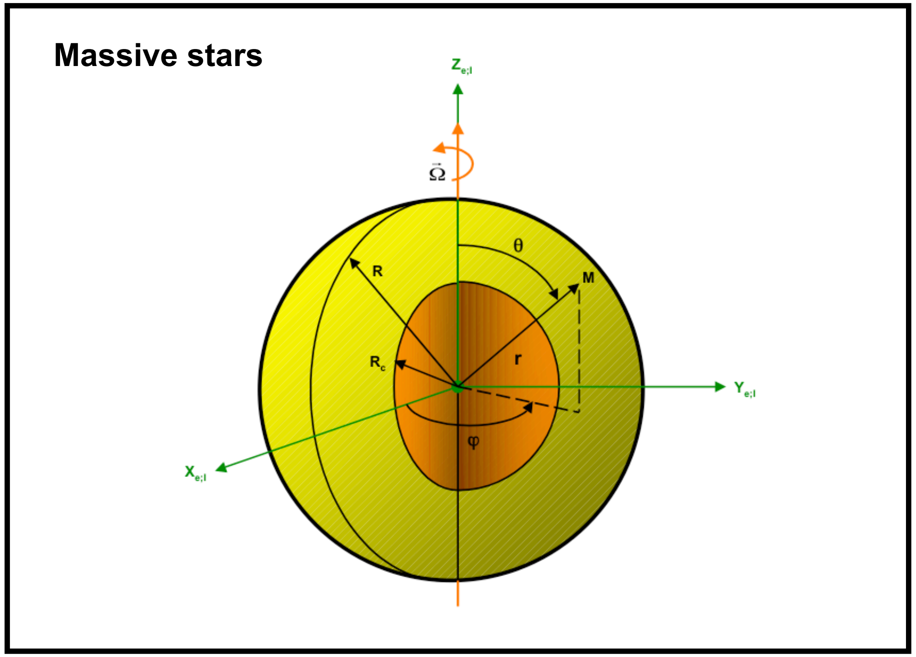
Schneider

Dissipative processes

C. E.: viscous friction
<u>equilibrium tide</u> and <u>dynamical tide:</u>
inertial waves

R. E.: radiative damping
<u>dynamical tide:</u>
gravity waves,
gravito-inertial waves





R. C.: radiative damping
<u>dynamical tide:</u>
gravity waves,
gravito-inertial waves

C. C.: viscous friction
<u>dynamical tide:</u>
inertial waves

Dynamical tide: the case of gravity (& inertial) waves

- Zahn 1975, Goldreich & Nicholson 1989: dynamical tide in stars with external radiative envelopes;

gravity waves
- → are excited at the border with the convective core by the tidal potential
- → propagate in the radiative envelope transporting angular momentum
- → the transported angular momentum settles where waves are damped

- Rocca 1987-1989, Savonije et al. 1995, Savonije & Papaloizou 1997, Papaloizou & Savonije 1997: generalization of previous studies taking into account the Coriolis acceleration in the case of a uniform rotation; supplementary resonances due to couplings introduced by the Coriolis acceleration → the applied torque is enhanced

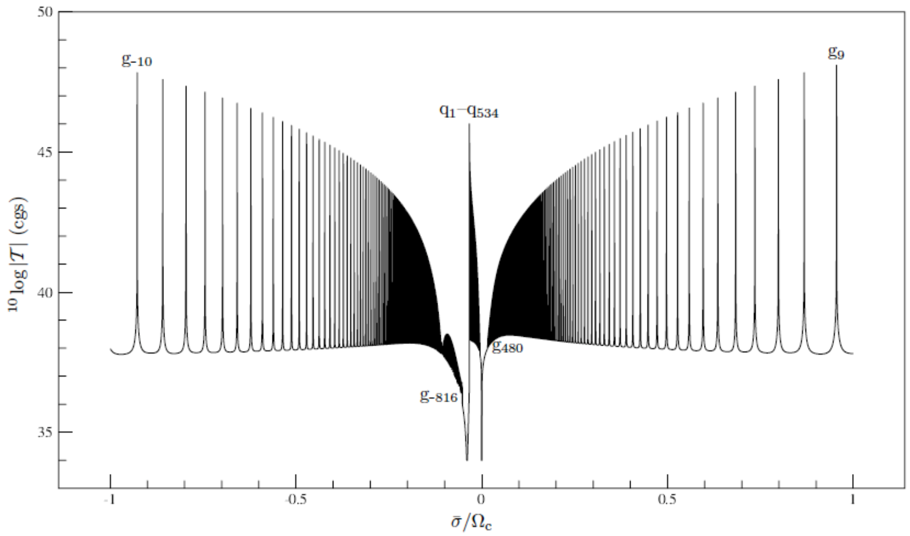
- Witte & Savonije 1999, 2001, 2002: take into account the Coriolis acceleration in the case of a uniform rotation; resonance locking: two modes, one which tends to accelerate the body, the other to decelerate it, lead by their combined action to lock the body in a resonant state → strong orbital evolution

- Barker & Ogilvie (2010-2011), Weinberg et al. (2012): breaking and parametric instabilities

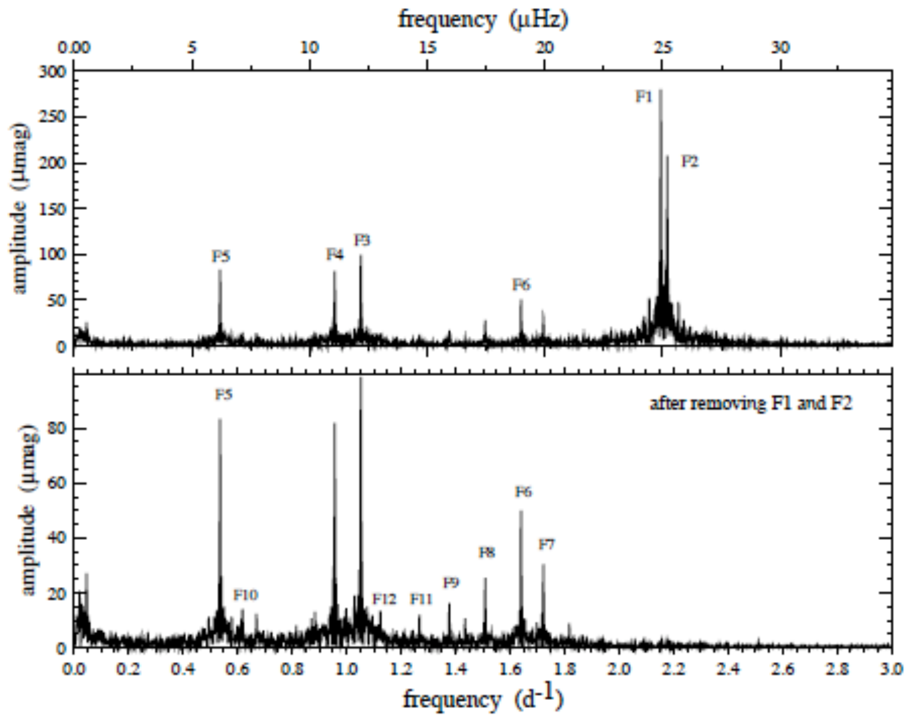
→ Mechanism which could explain the orbital state of close binaries of which components are main sequence solar-type or massive stars

Modification of global rotation

$1M_{\odot}$, $\Omega s=0.1\Omega_c$, $X_c=0.4$



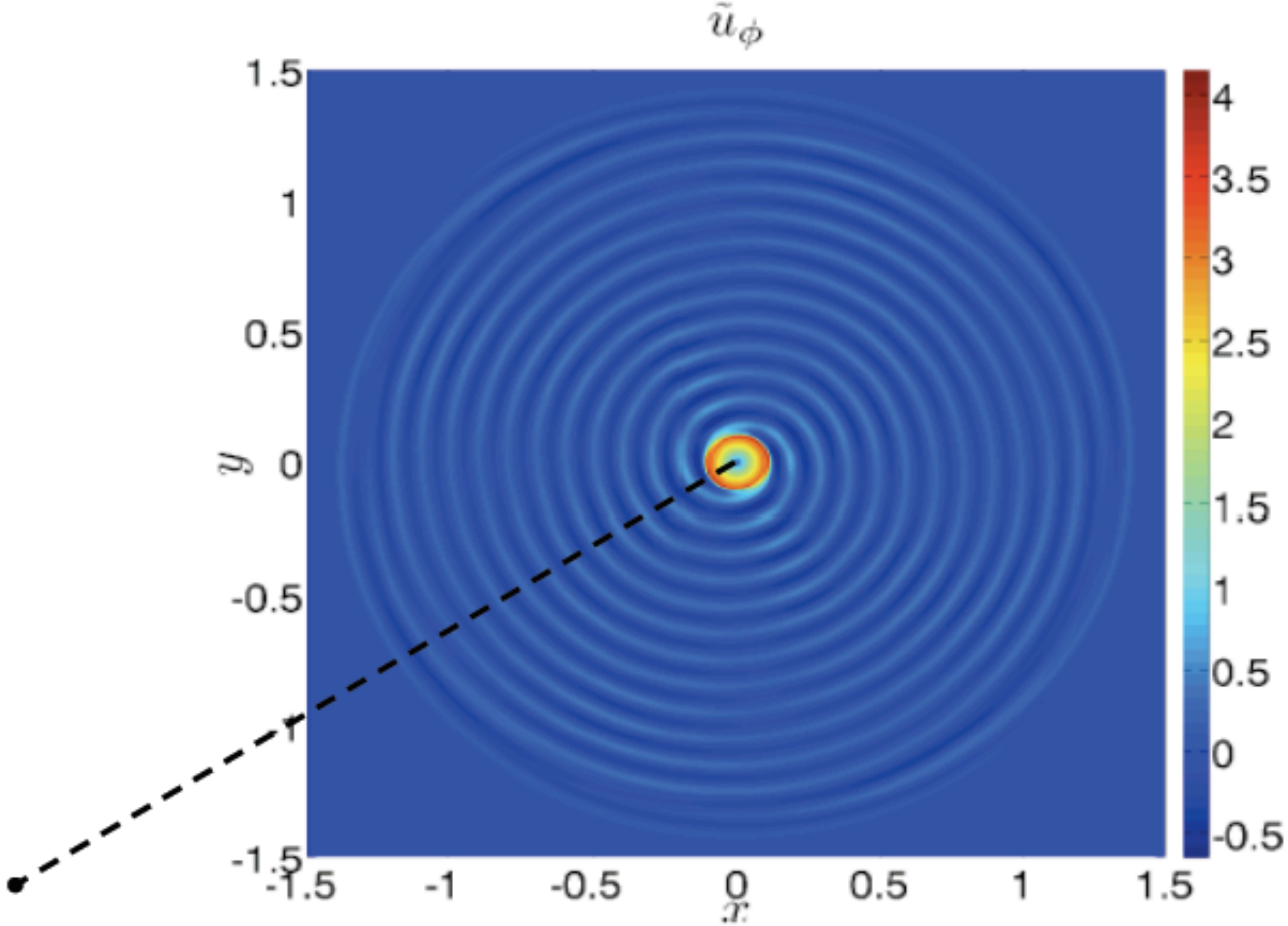
Witte & Savonije 2002;
see also Ogilvie & Lin 2007



KOI-54: Welsh et al. 2011

Modification of internal angular momentum

→ **tidal internal waves excited by a close planetary companion**



Barker & Ogilvie 2010

See also the elliptic instability for gravito-inertial waves, etc.

Internal waves are key players to understand angular momentum exchanges in stellar systems