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Waves and instabilities in a magnetized spherical Couette experiment

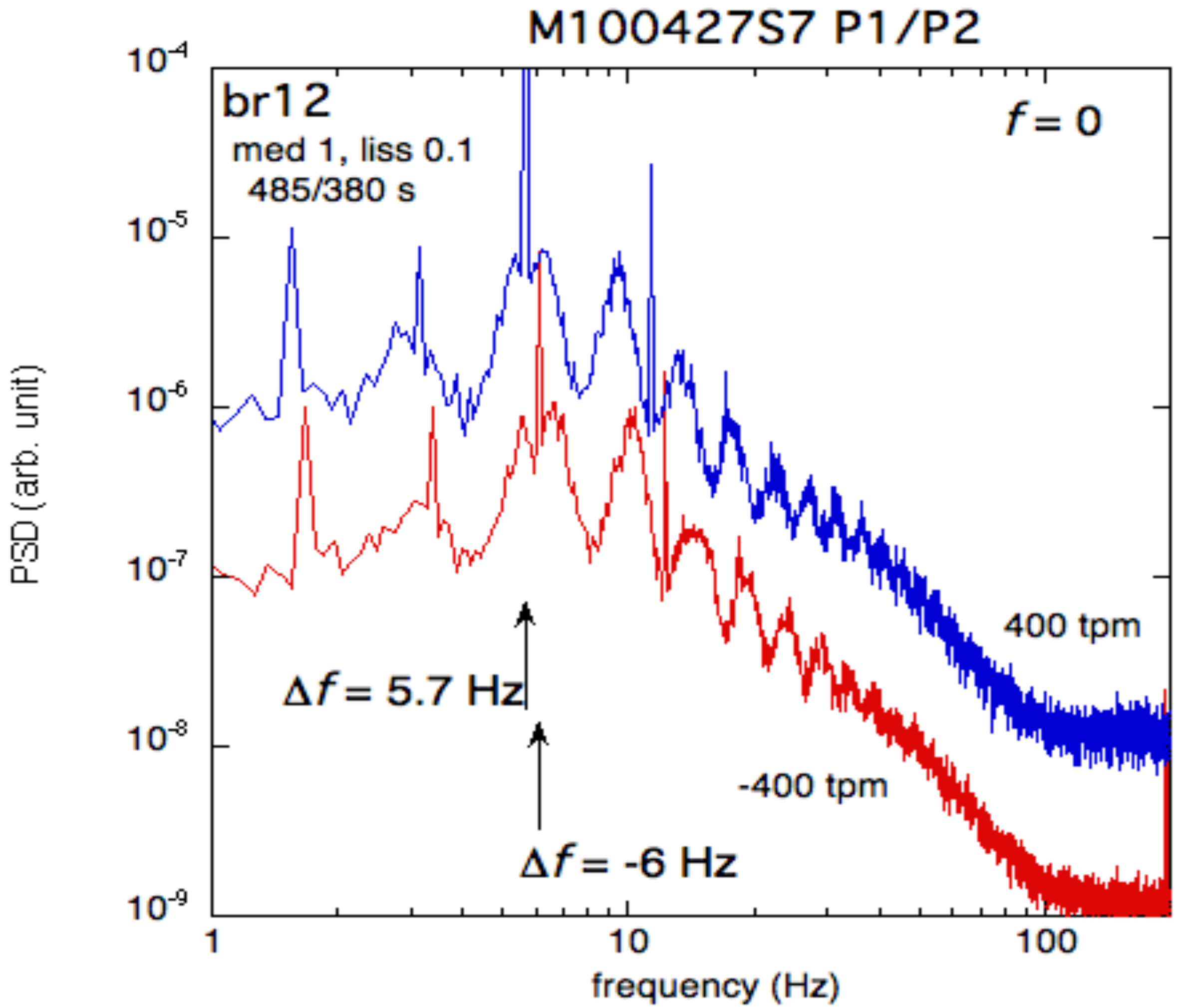
Henri-Claude NATAF
ISTerre, CNRS-Université de Grenoble

and the Geodynamo team of ISTerre…

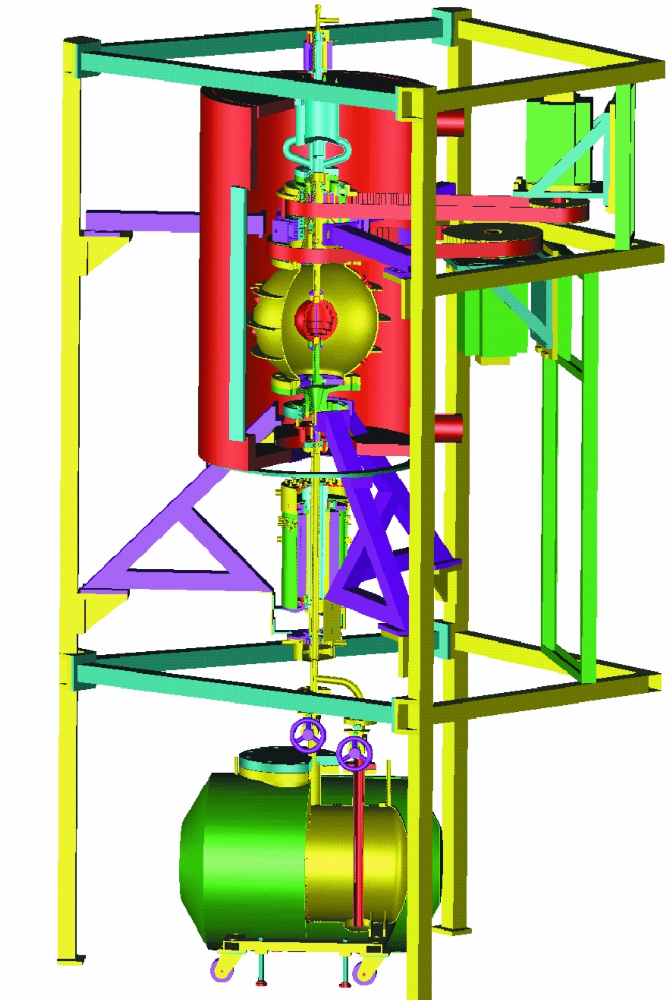
Take-away message

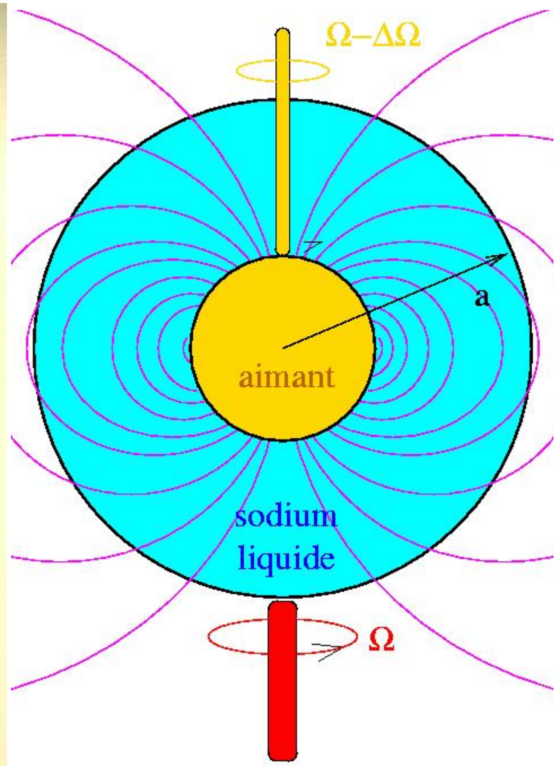
- Experimental magnetic field fluctuations display bumpy spectra.
- The bumps correspond to some sort of modes.
- Similar spectra are recovered in long enough numerical simulations.
- Fluctuations are linked to boundary layer instabilities.
- The imposed magnetic field severely hinders these instabilities.

Bumpy magnetic spectra: what do they mean?



The DTS experiment: spherical Couette flow in a dipolar magnetic field



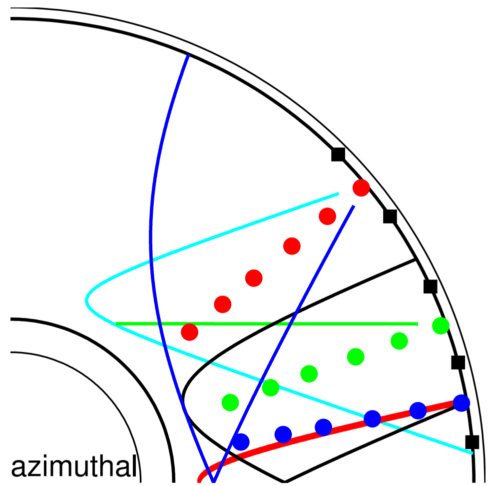


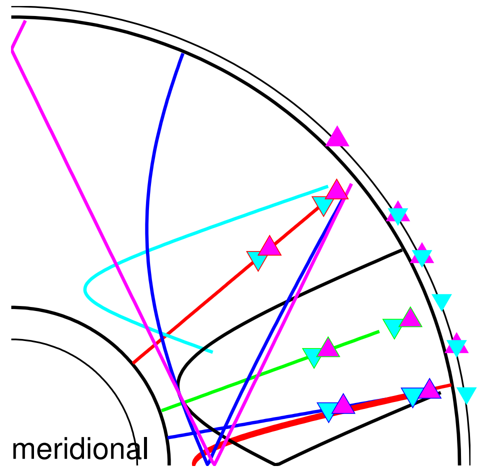
$$\Lambda = \frac{\sigma B^2}{\rho \Omega}$$

Cardin et al, 2002

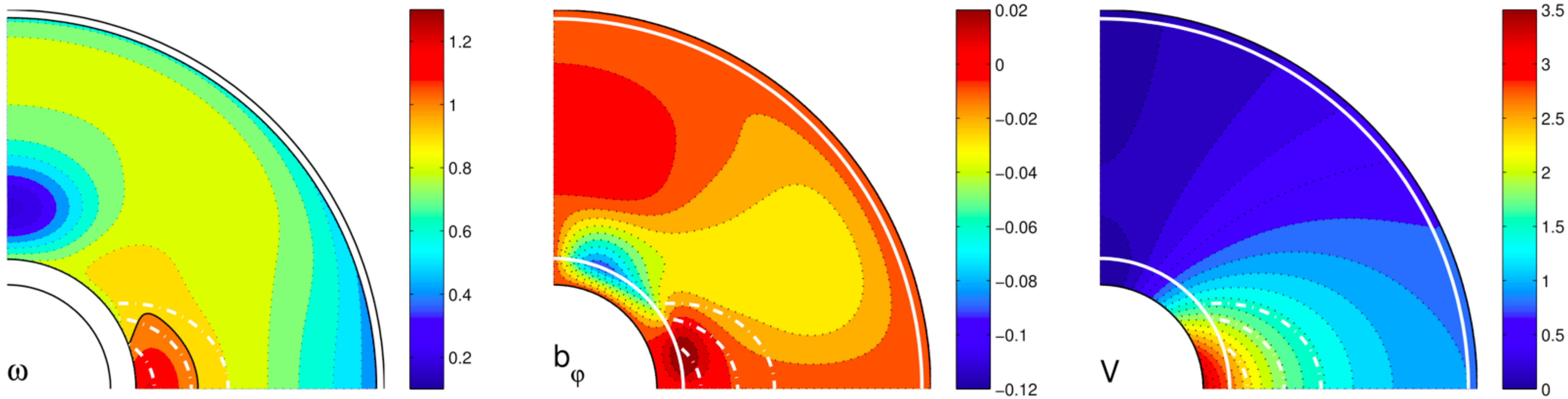
Inverting
for the mean axisymmetric state

- Velocity profiles (ultrasound Doppler)
- Induced magnetic field in a sleeve
- Torque
- Electric potentials at the surface
- Induced magnetic field at the surface

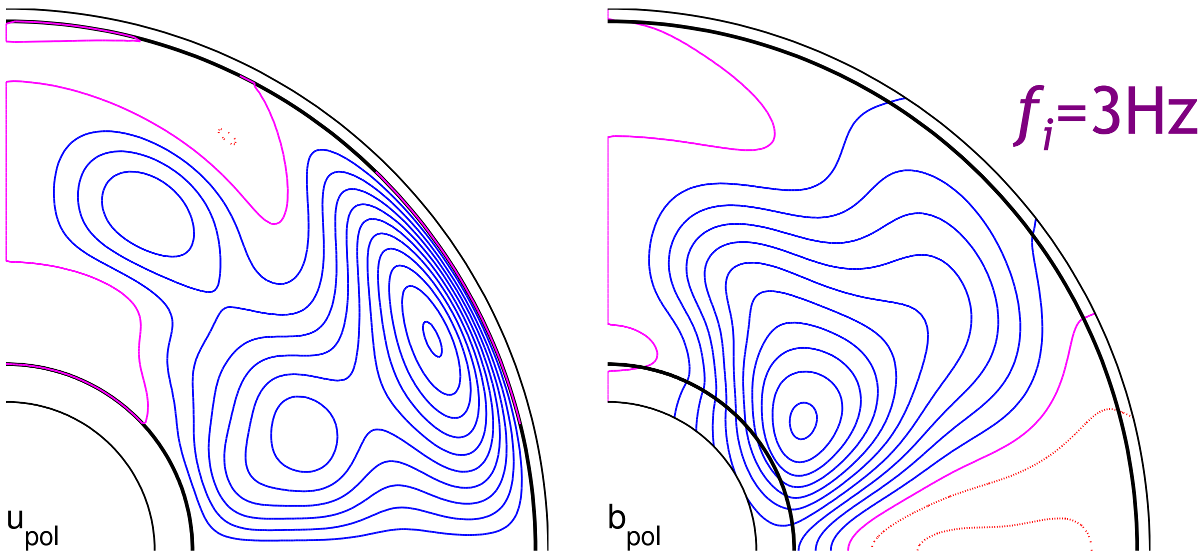




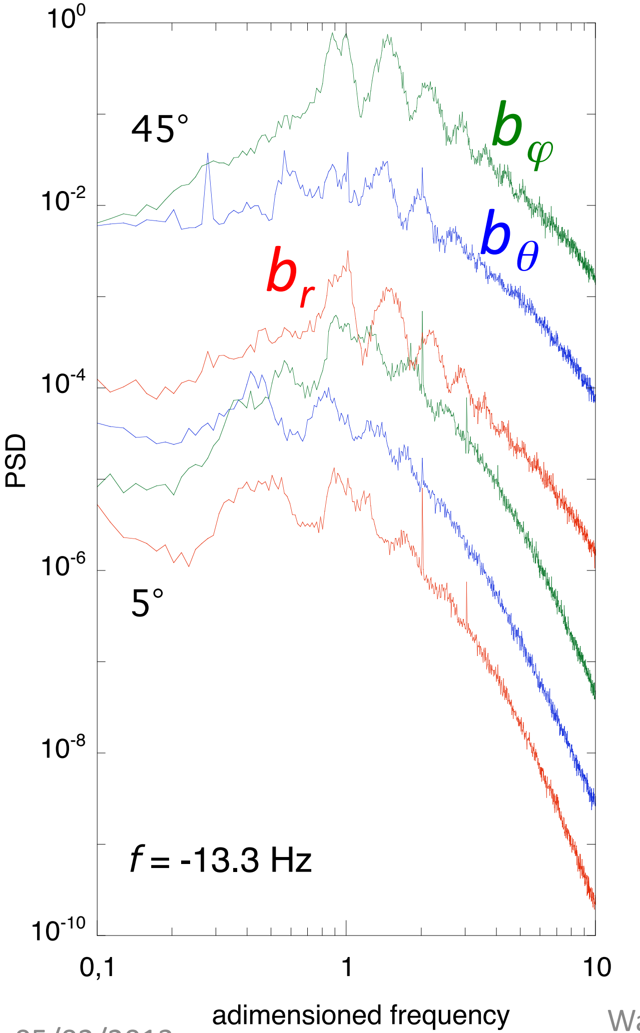
Nataf, 2013



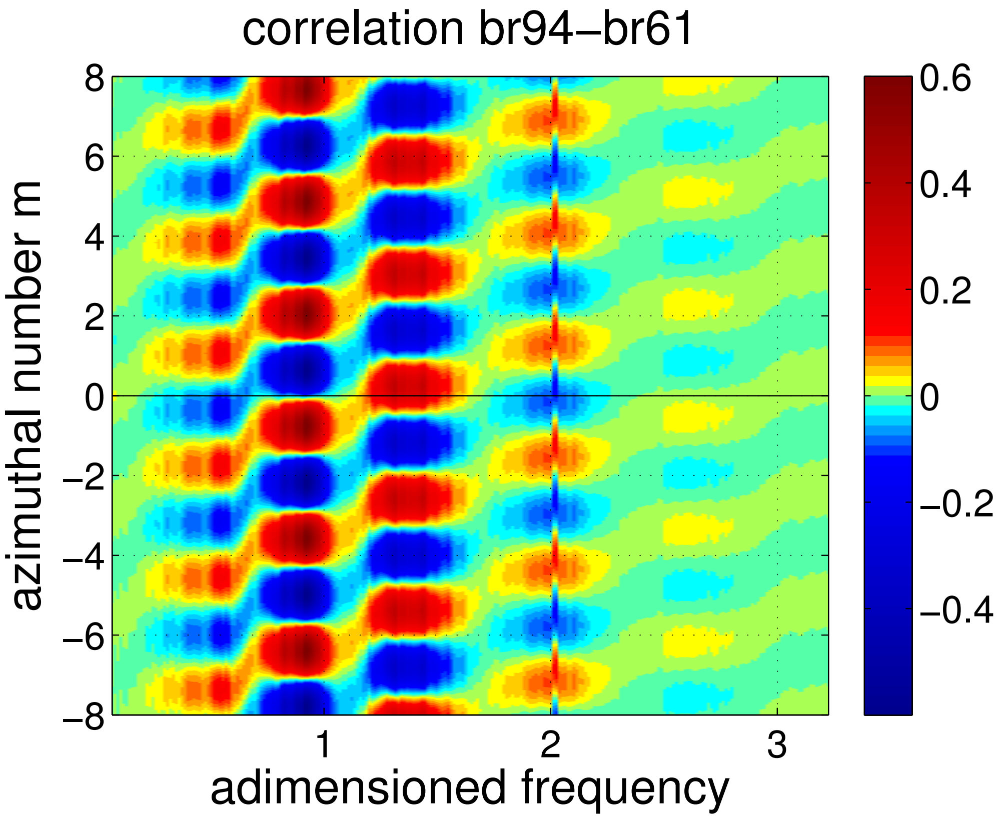
- Super-rotation
- Ferraro law region
  $\vec{B}_d \cdot \vec{\nabla}\omega = 0$
- Vortostrophic region
- Transition at $\Lambda_\ell=1$
- Non-Ferraro region
- Outer boundary layer not Hartmann



Magnetic field fluctuations

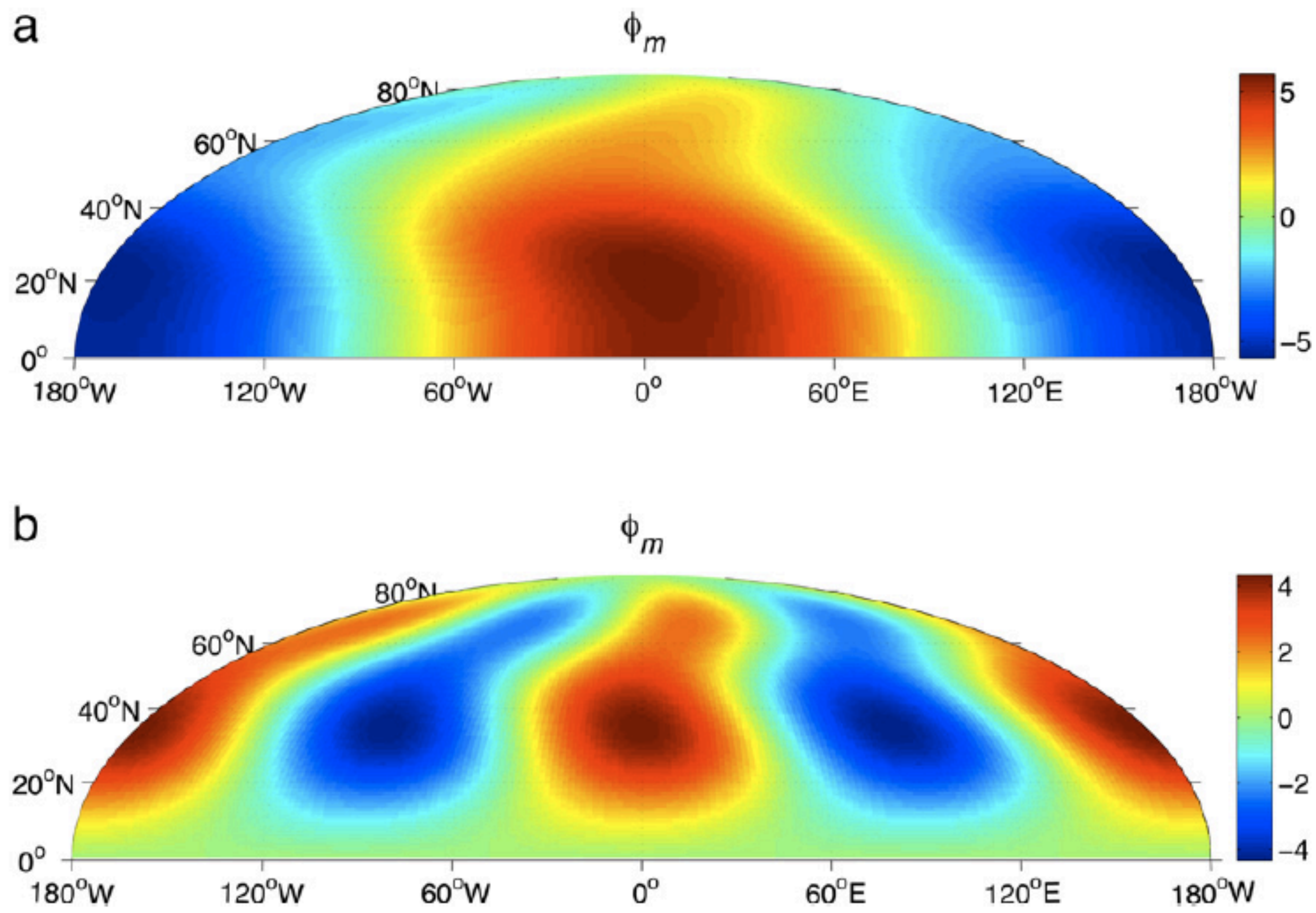


Each bump corresponds to a single azimuthal mode number *m*



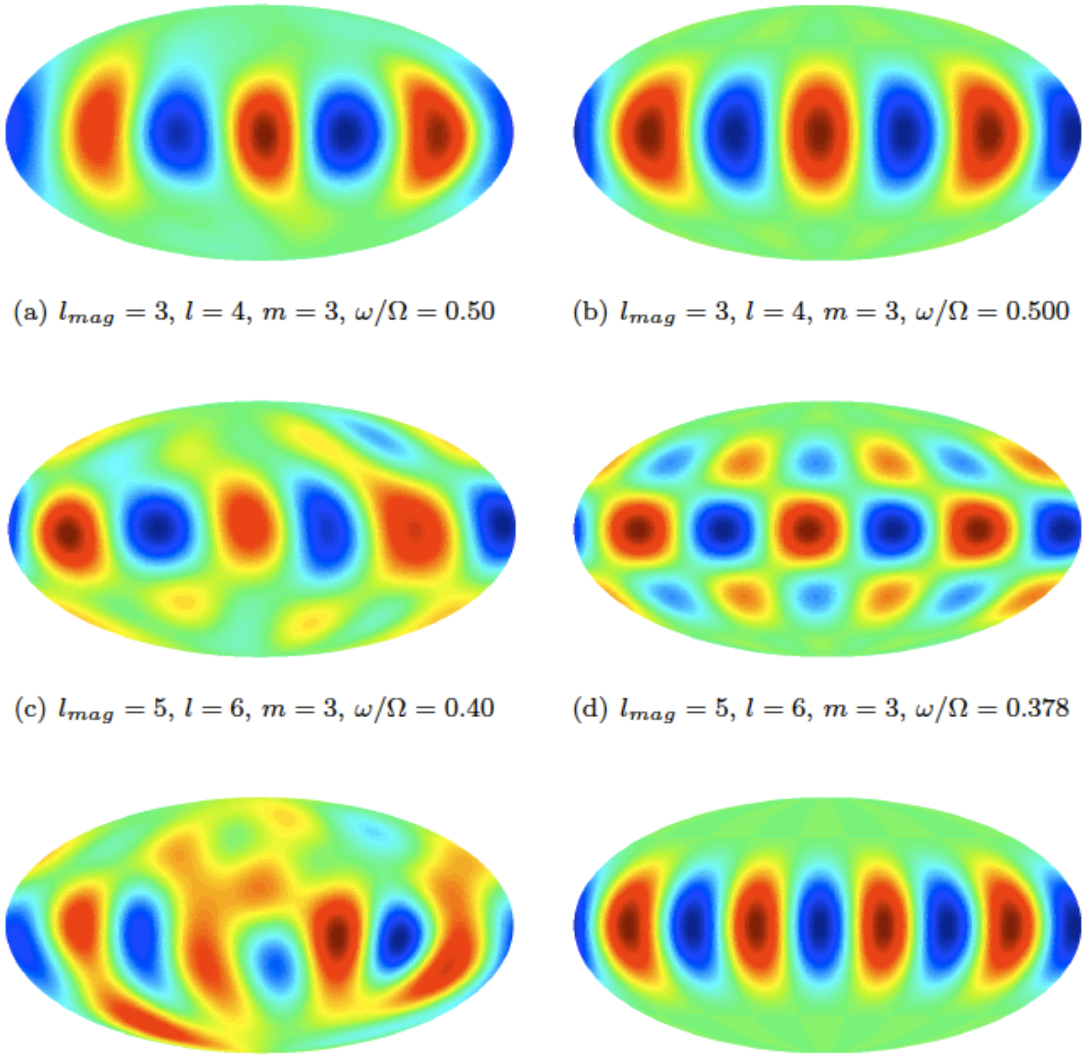
Figueroa et al, 2013

Mode structure at the surface



Schmitt et al, 2013

Are the modes we observe inertial modes as discovered by Dan Lathrop's group at the University of Maryland?



(a) $l_{mag} = 3, l = 4, m = 3, \omega/\Omega = 0.50$

(b) $l_{mag} = 3, l = 4, m = 3, \omega/\Omega = 0.500$

(c) $l_{mag} = 5, l = 6, m = 3, \omega/\Omega = 0.40$

(d) $l_{mag} = 5, l = 6, m = 3, \omega/\Omega = 0.378$

(e) $l_{mag} = 4, l = 5, m = 4, \omega/\Omega = 0.39$

(f) $l_{mag} = 4, l = 5, m = 4, \omega/\Omega = 0.400$

Observed and predicted magnetic signature of full sphere inertial modes

Kelley et al, 2006, 2007, 2010

Rieutord et al, 2012

→ *more in Dan Zimmerman's talk*

but in the DTS case:

- Strong imposed magnetic field
- Strong differential rotation

→ Are we seeing magneto-Coriolis modes?

$$\begin{cases} \dfrac{\partial u}{\partial t} + (U_0 \nabla) u + (u \nabla) U_0 + \nabla p = Le^2 \left[(\nabla \times B_0) \times b + (\nabla \times b) \times B_0 \right] + E\, \Delta u \\ \dfrac{\partial b}{\partial t} = \left[\nabla \times (U_0 \times b) + \nabla \times (u \times B_0) \right] + Em\, \Delta b \\ \nabla u = 0 \quad ; \quad \nabla b = 0 \end{cases}$$

$$Le = \frac{B_0^{ref}}{a \gamma \Delta\Omega \sqrt{\rho \mu}} = \frac{Ha}{\mathrm{Re} \sqrt{Pm}}$$

$$E = \frac{\nu}{a^2 \gamma \Delta\Omega}$$

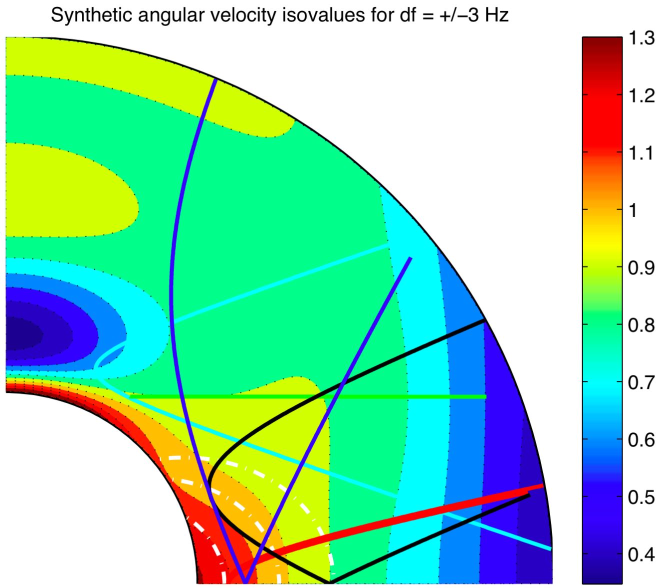
$$Em = \frac{\eta}{a^2 \gamma \Delta\Omega}$$

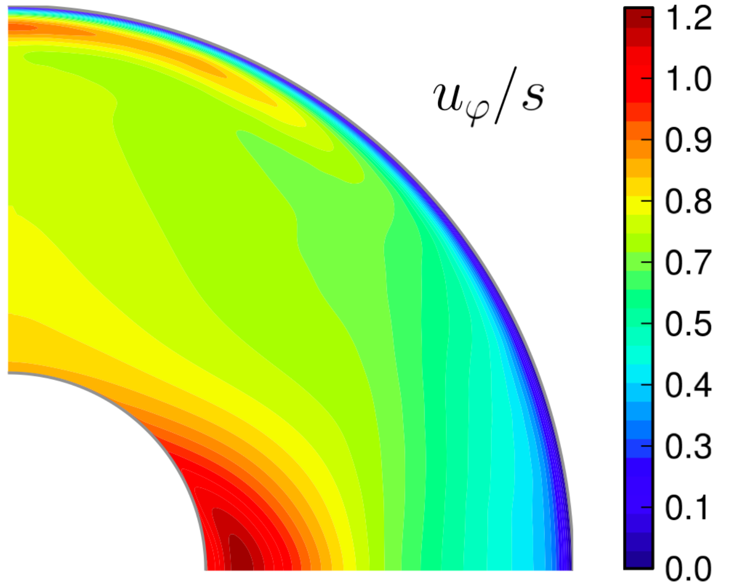
a

γ

Schmitt et al, 2013

Can we find these modes in full 3D numerical simulations?





DTS experiment:

Re = 450 000

Ha = 200

Λ = 0.03

Numerical simulation (XSHELLS):

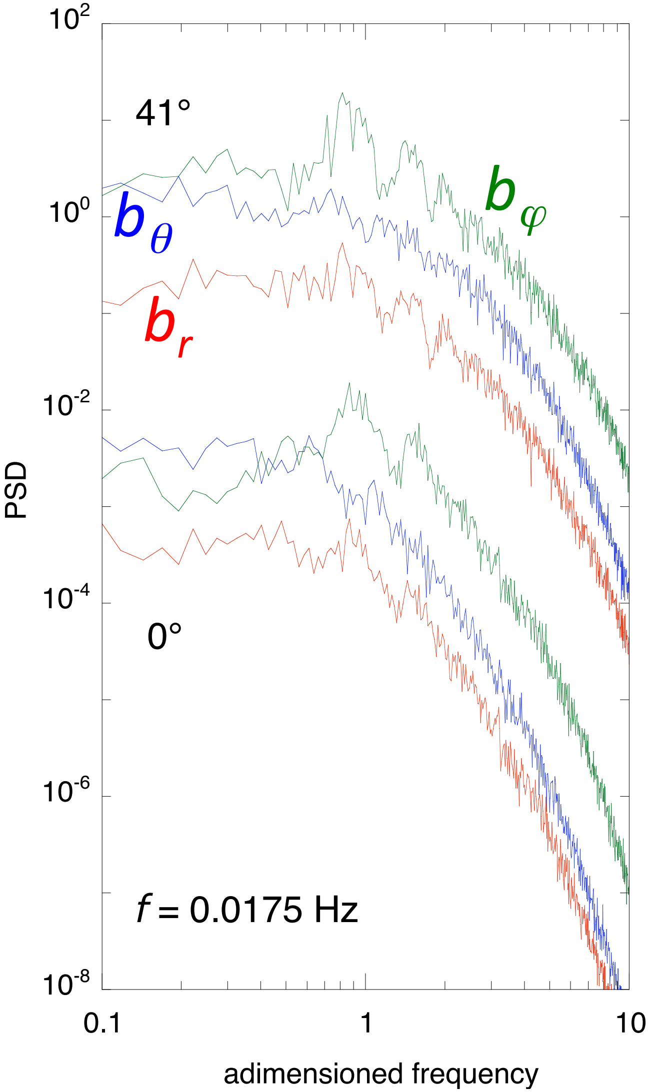
Re = 2 600

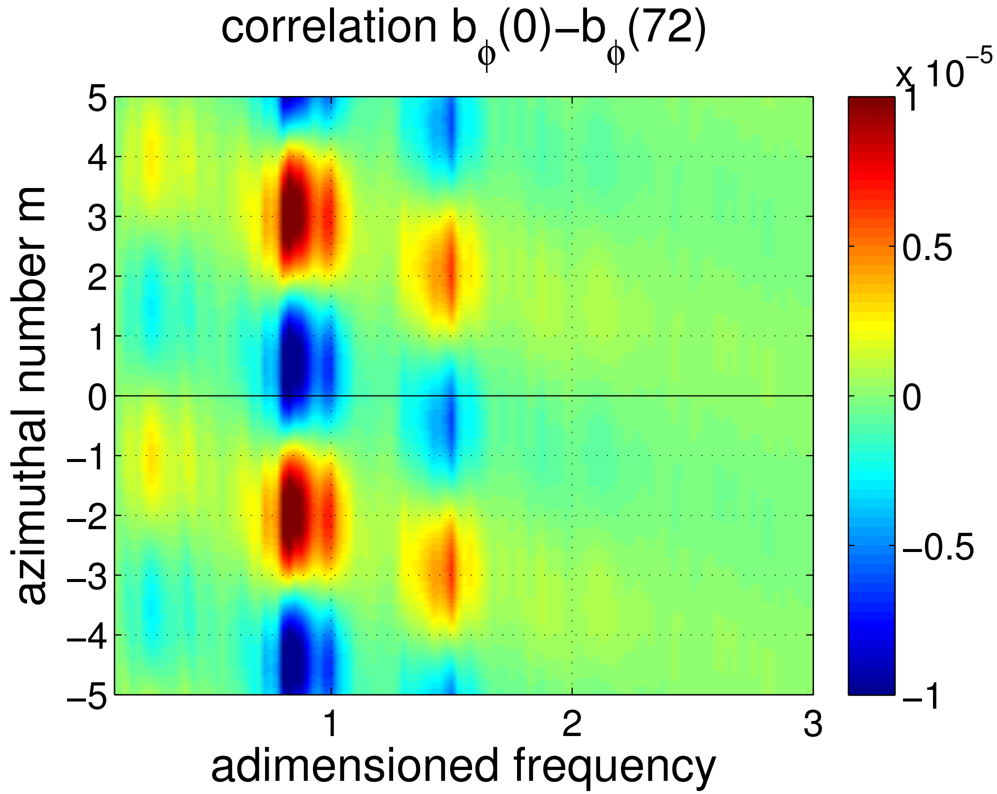
Ha = 16

Λ = 0.03

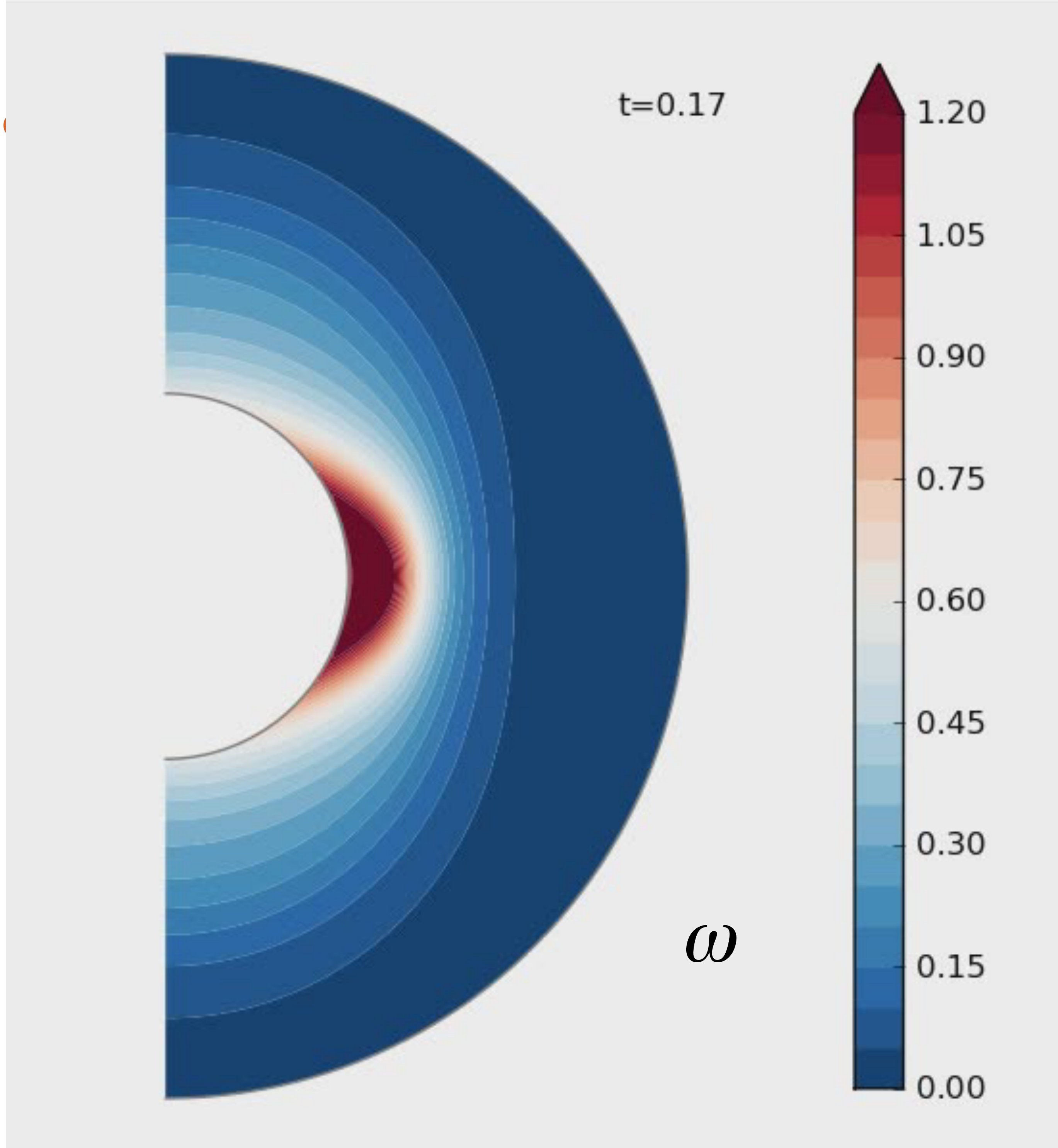
Figueroa et al, 2013

Yes, we can…





Figueroa et al, 2013



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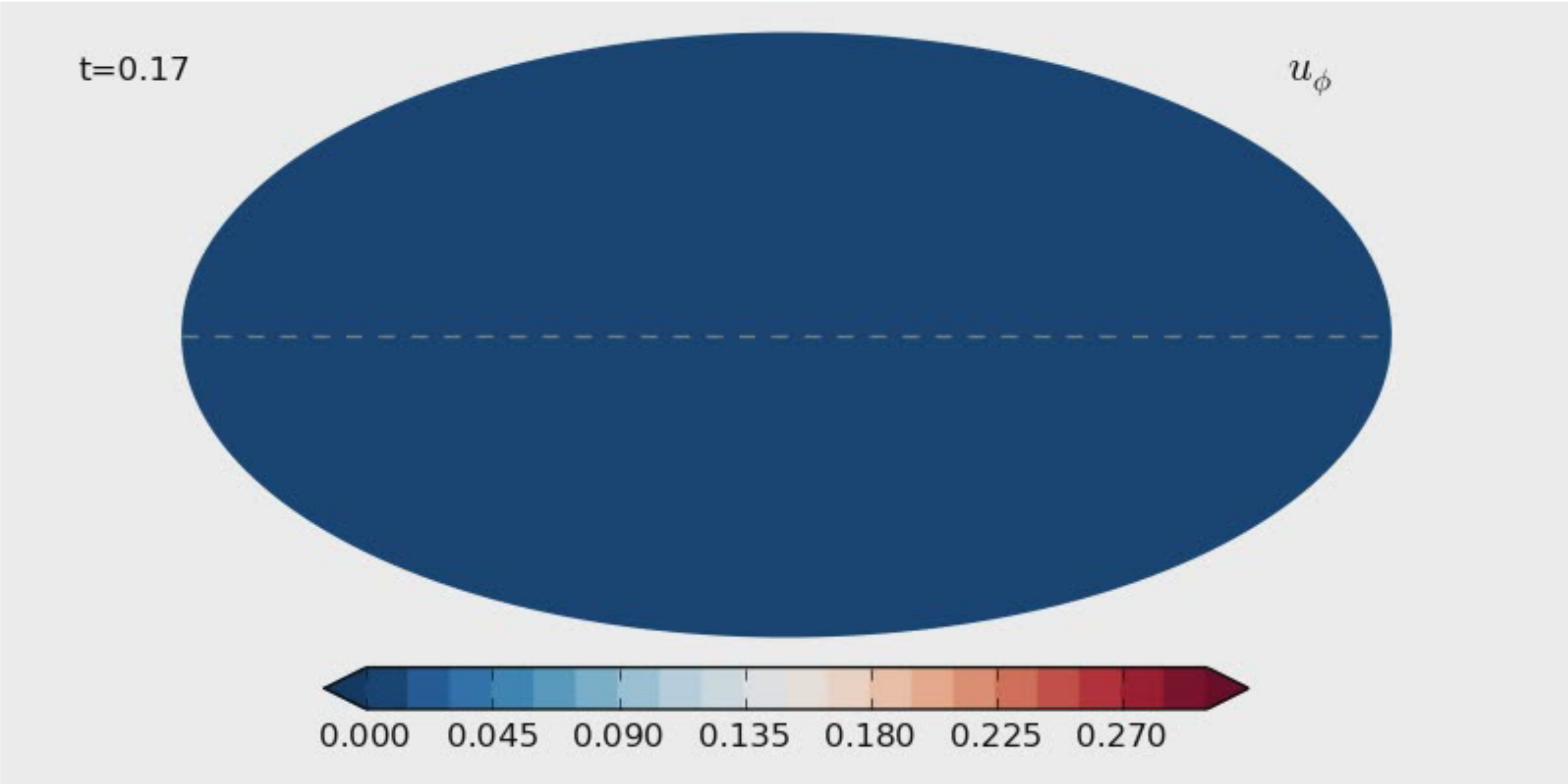
But what do they look like in the time-domain?

Figueroa et al, 2013

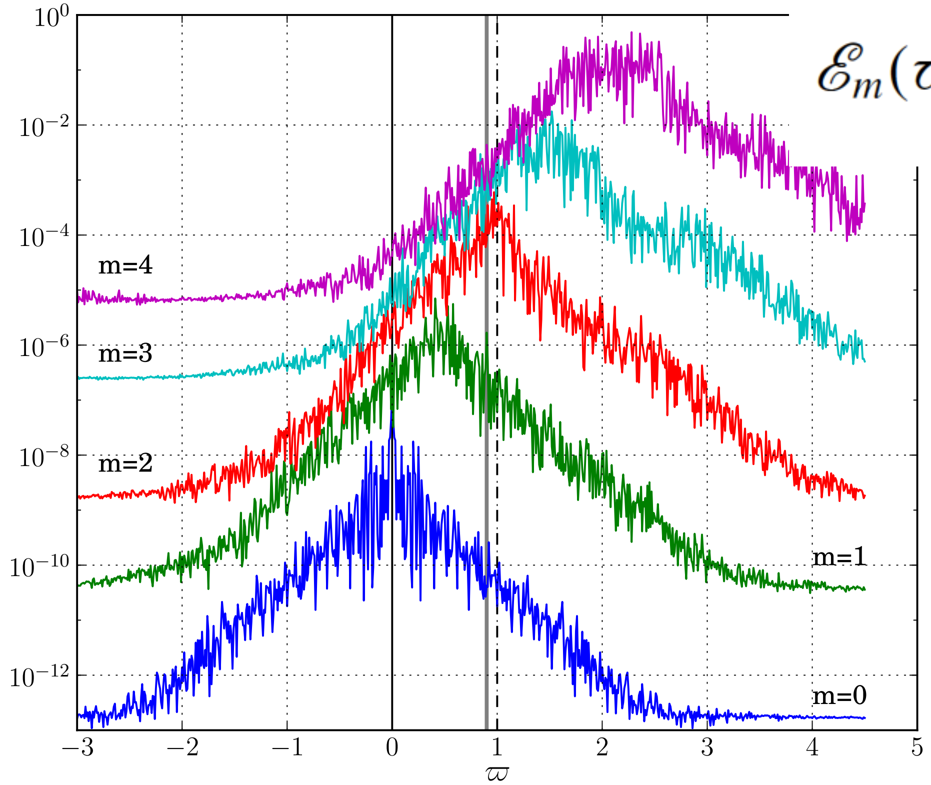


Figueroa et al, 2013

Full *fft* spectra for individual *m* (*r* < *0.55*)

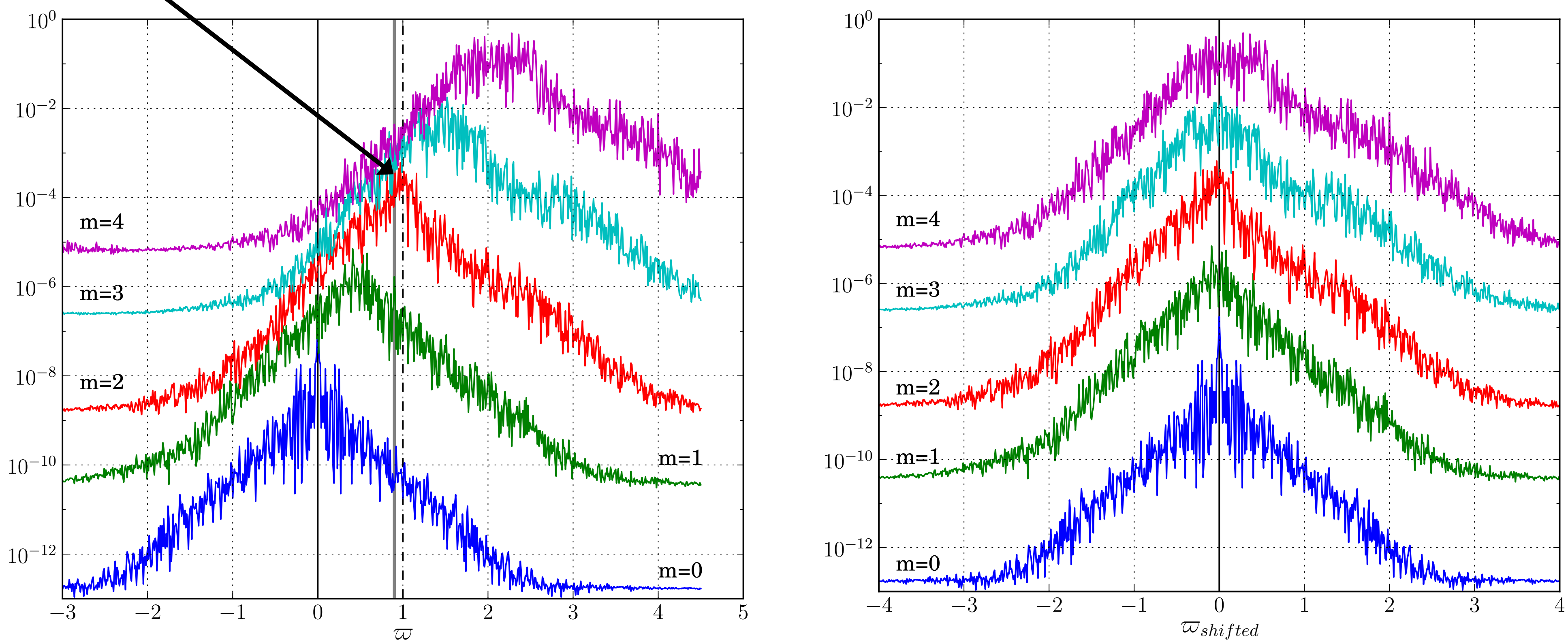
$$\boldsymbol{F}(r,\theta,\varphi,t)=\sum_m\sum_\varpi \boldsymbol{F}_m^\varpi(r,\theta)\mathrm{e}^{\mathrm{i}(m\varphi-\varpi t)}$$

$$\mathscr{E}_m(\varpi)=\int_{r_1}^{r_2}\int_{\theta_1}^{\theta_2}\|\boldsymbol{F}_m^\varpi(r,\theta)\|^2\, r\sin\theta\,\mathrm{d}\theta\,\mathrm{d}r$$



Full *fft* spectra for individual *m* (*r* < 0.55)

see mode structure

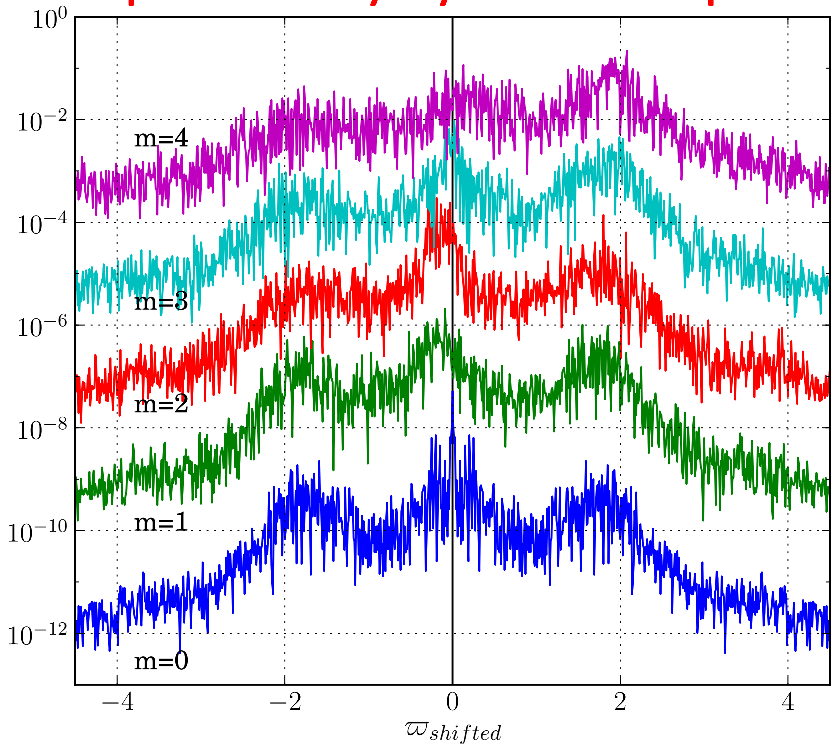


$$\varpi_{shifted} = \varpi - m f_{fluid}$$

Figueroa et al, 2013

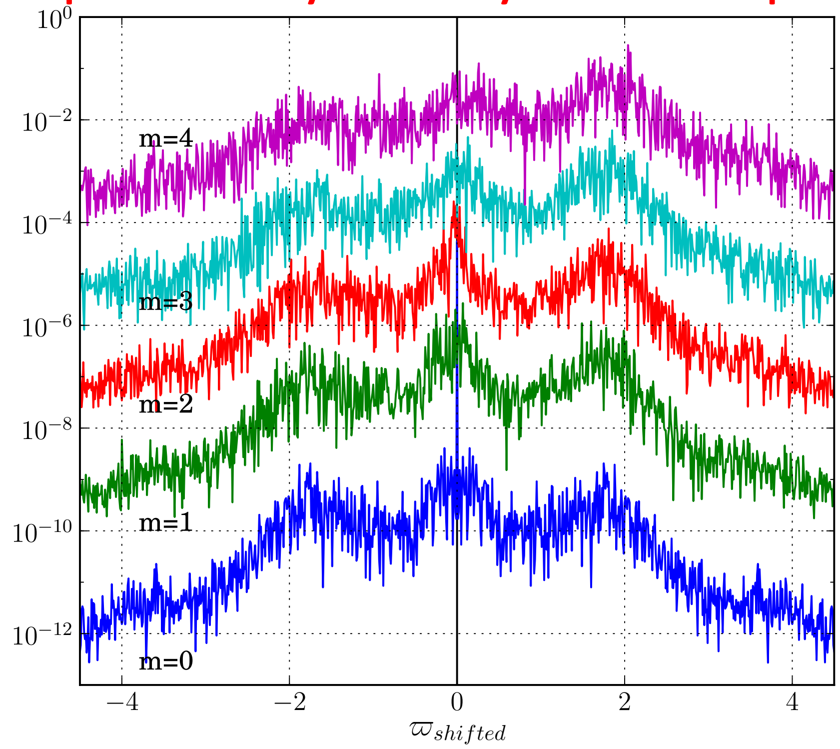
Full *fft* spectra for individual *m* (*surface - high latitude*)

Equatorially symmetric part



Shifted spectra

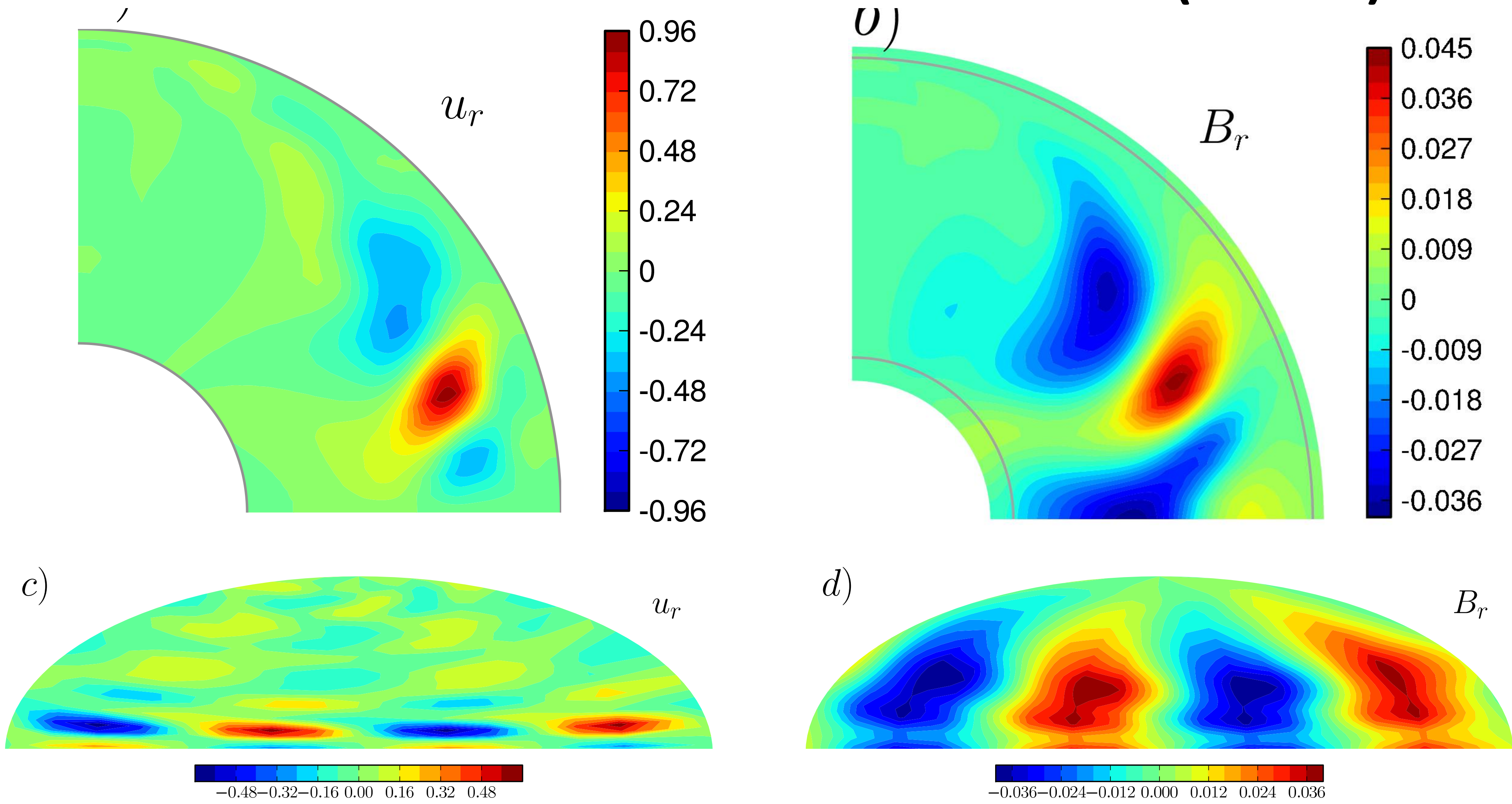
Equatorially anti-symmetric part



Shifted spectra

Figueroa et al, 2013

Non-linear mode structure (m=2)



Figueroa et al, 2013

Conclusions *1/2*

- Experimental magnetic field fluctuations display bumpy spectra.
- The bumps correspond to some sort of modes.
- Similar spectra are recovered in long enough numerical simulations.
- Fluctuations are linked to boundary layer instabilities.
- The imposed magnetic field severely hinders these instabilities.

Conclusions 2/2

- The Lorentz force controls both the axisymmetric mean state and the fluctuations, *BUT* the magnetic energy is much smaller than the kinetic energy, and the Lorentz force is *not* the restoring force.
- The critical layer approach of Rieutord *et al* could apply here as well.
- The full *fft* technique that we introduced can help identifying non-linear modes.

encore…

$$\left\{\begin{array}{ll} \Delta_1 b_\varphi & = -s\boldsymbol{B}_d \cdot \boldsymbol{\nabla}\omega + \mathrm{Rm}\left\{-s\boldsymbol{b}_p \cdot \boldsymbol{\nabla}\omega + s\boldsymbol{u}_p \cdot \boldsymbol{\nabla}\left(\frac{b_\varphi}{s}\right) - \left[\boldsymbol{\nabla}\times\langle\tilde{\boldsymbol{u}}\times\tilde{\boldsymbol{b}}\rangle\right]_\varphi\right\} \\ \Delta_1 a & = \frac{\boldsymbol{u}_p}{s}\cdot\boldsymbol{\nabla}(sA_d) + \mathrm{Rm}\left\{\frac{\boldsymbol{u}_p}{s}\cdot\boldsymbol{\nabla}(sa) - \langle\tilde{\boldsymbol{u}}\times\tilde{\boldsymbol{b}}\rangle_\varphi\right\}. \end{array}\right.$$