ÉCOLE DE PHYSIQUE des HOUCHES

Université Joseph Fourier GRENOBLE

Waves, turbulence, rotation and dissipation in the Earth's dynamo

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dédicace

I dedicate this lecture to my father, Roger Nataf. Exactly sixty years ago, in 1953, he was attending one of the very first Summer Schools in Les Houches, about Quantum Mechanics. At that time, there were only 20-30 students, and the School was 8 weeks long. My mother liked to recall these happy days, as she had been able to break the rule and come along with their two children: my elder brother and sister. I was not born at that time.

Take-away message

- Numerical simulations of convective dynamos have played a major role in demonstrating the crucial role of rotation in the *generation* of the Earth's magnetic field.
- Rotation could also play a major role in limiting the *dissipation* of the geodynamo.

Alfvén waves (1942)

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla p + \left(\frac{\mathbf{B}}{\mu} \cdot \nabla \right) \mathbf{b} + \rho \nu \nabla^2 \mathbf{u},$$

linearized Navier-Stokes eq^n

$$\frac{\partial \mathbf{b}}{\partial t} = (\mathbf{B} \cdot \nabla)\, \mathbf{u} + \frac{1}{\mu \sigma} \nabla^2 \mathbf{b}$$

linearized induction eq^n

Introducing Elsasser variables: $\mathbf{u}^{\pm} = \mathbf{u} \pm \mathbf{b}/\sqrt{\rho \mu}$ yields:

$$\frac{\partial \mathbf{u}^+}{\partial t} = -\nabla \frac{p}{\rho} + \left(\frac{\mathbf{B}}{\sqrt{\rho \mu}} \cdot \nabla \right) \mathbf{u}^+ + \nu \nabla^2 \mathbf{u} + \frac{1}{\mu \sigma} \nabla^2 \frac{\mathbf{b}}{\sqrt{\rho \mu}};$$

$$\frac{\partial \mathbf{u}^-}{\partial t} = -\nabla \frac{p}{\rho} - \left(\frac{\mathbf{B}}{\sqrt{\rho \mu}} \cdot \nabla \right) \mathbf{u}^- + \nu \nabla^2 \mathbf{u} - \frac{1}{\mu \sigma} \nabla^2 \frac{\mathbf{b}}{\sqrt{\rho \mu}}$$

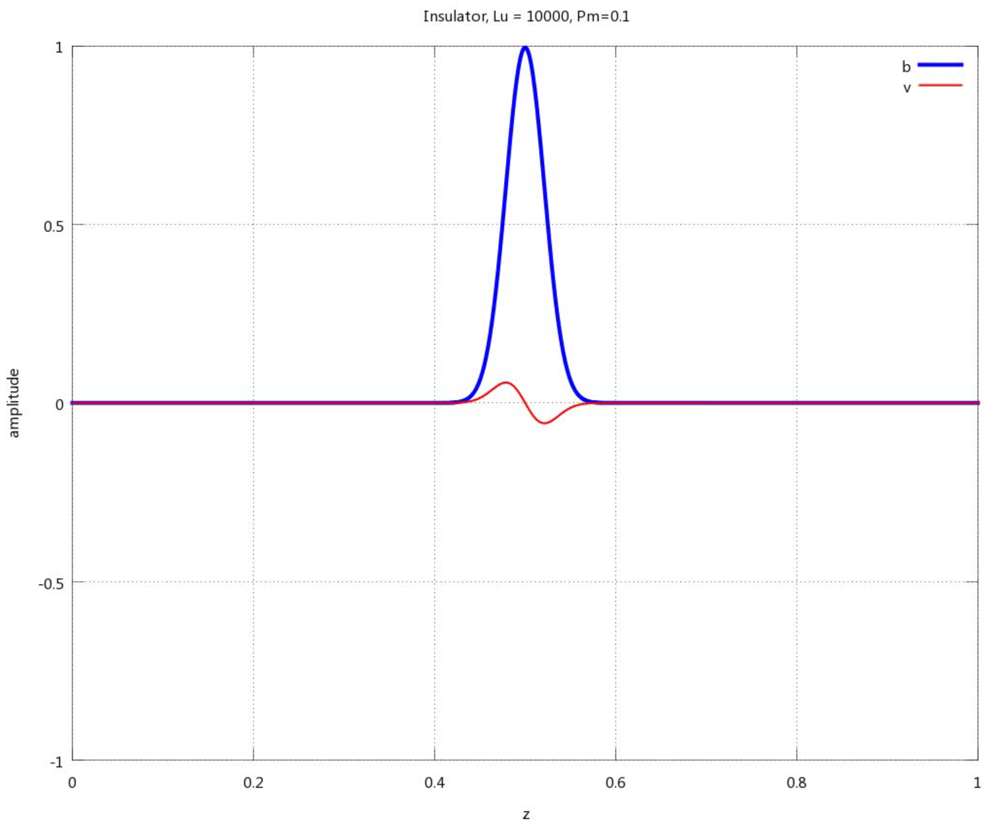
from Alboussière et al, 2011

Properties of ideal Alfvén waves

$$\frac{\partial \mathbf{u}^{\pm}}{\partial t} = \pm \left(\frac{\mathbf{B}}{\sqrt{\rho\mu}} \cdot \nabla \right) \mathbf{u}^{\pm}$$

- Transverse
- Non-dispersive
- Alfvén velocity
- Energy equipartition

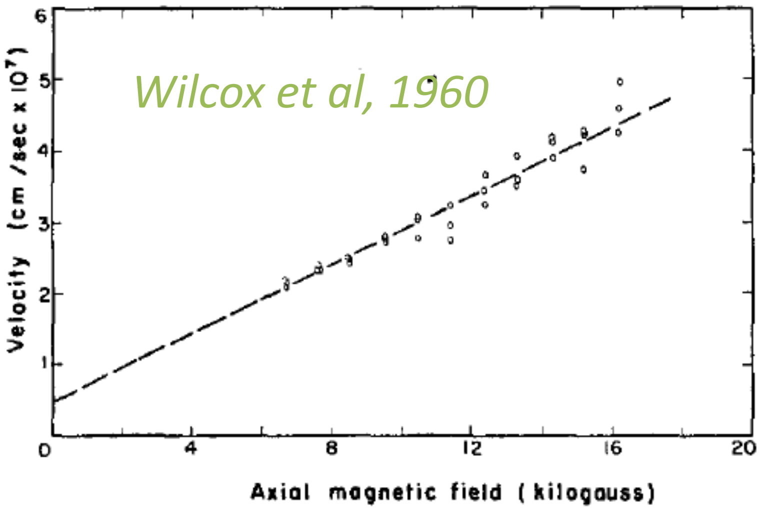
With vanishing diffusivities

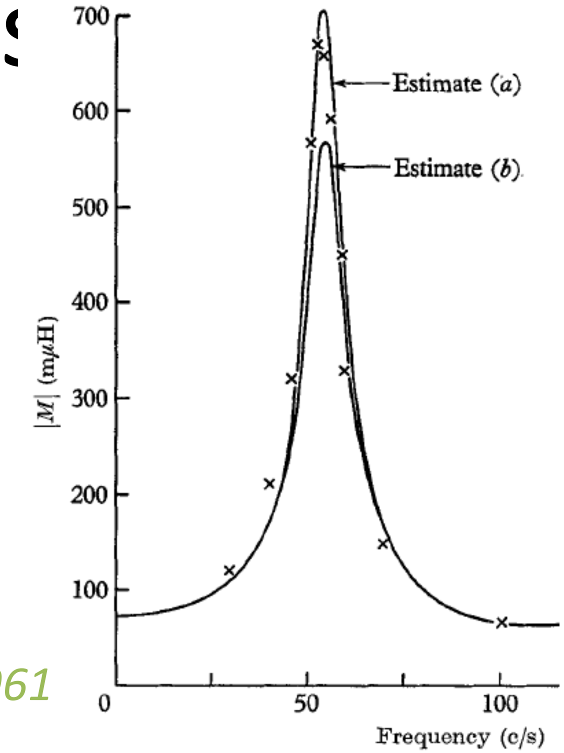


courtesy Ph. Cardin

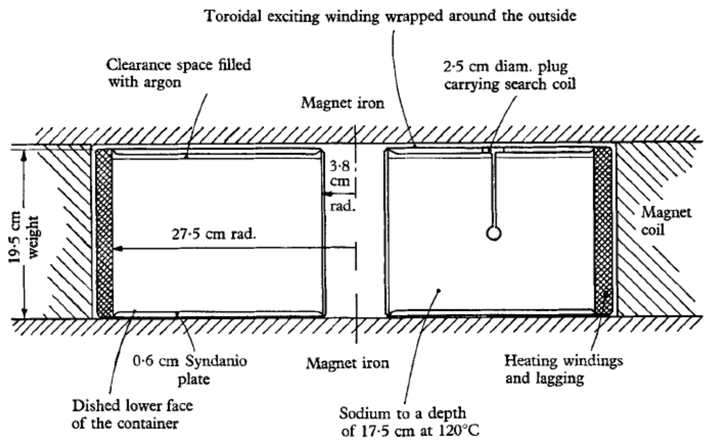
Early experimental demons

- Lundquist, 1949 (mercury)
- Lehnert, 1953 (sodium)
- Wilcox et al, 1960 (H plasma)
- Jameson, 1961 (sodium)

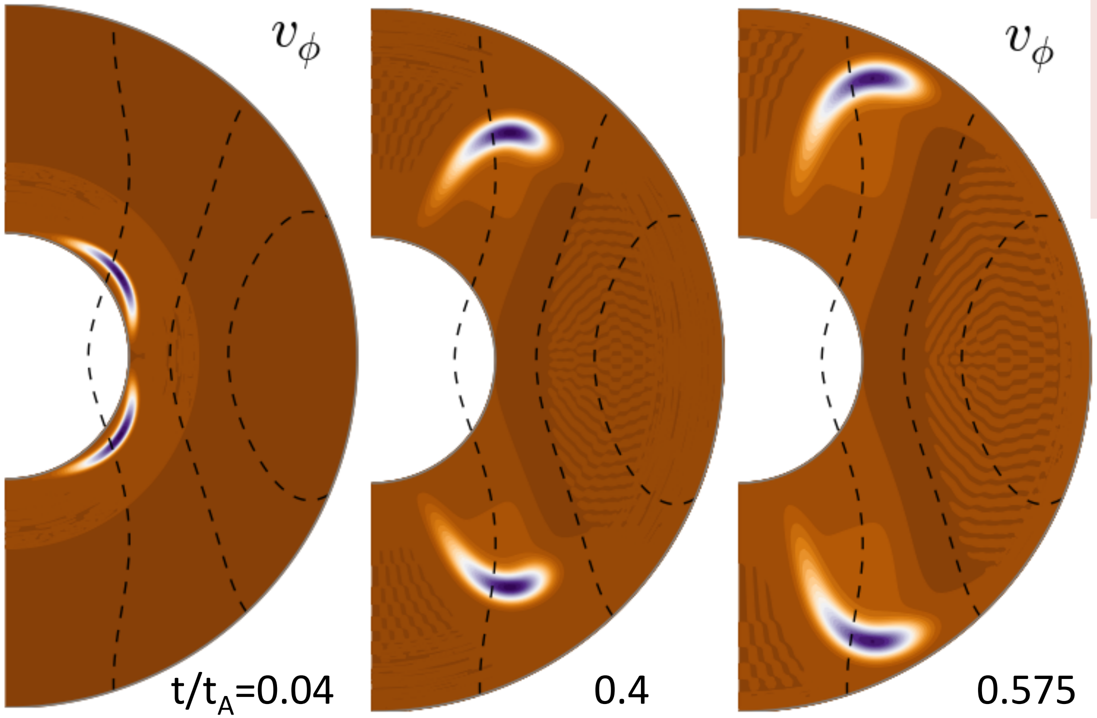




Jameson, 1961



Axisymmetric Alfvén wave generated by a jerk of the inner sphere



- Imposed magnetic field (field lines drawn)
- No rotation
- P_m = 1 (magnetic Prandtl)
- Lu = 1000 (Lundquist)

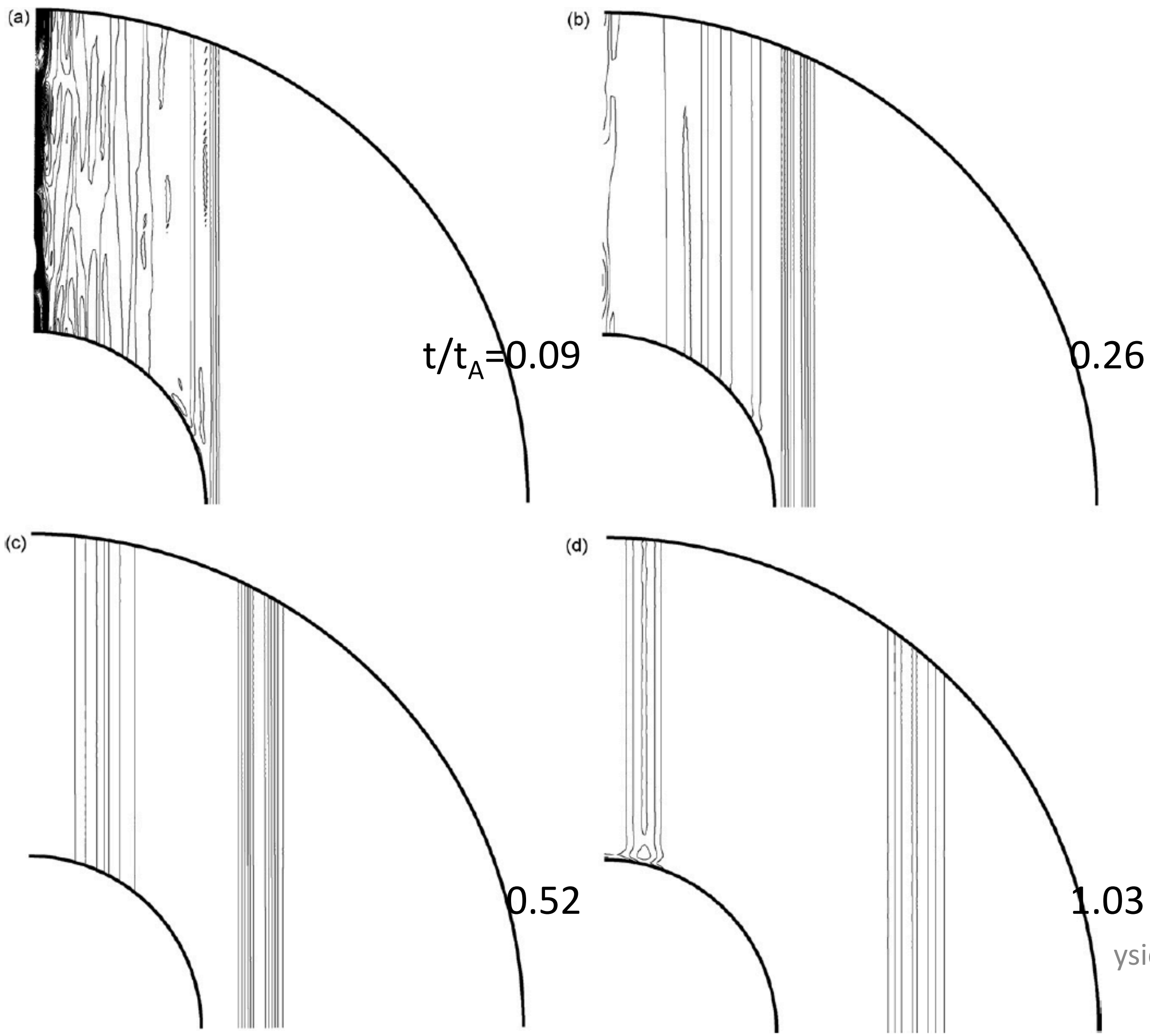
$$Lu = \frac{R_{oc} B_0}{\eta \sqrt{\rho \mu_0}} = \frac{t_\eta}{t_{Alfvén}}$$

Drouart, Schaeffer & Plunian, 2011

Alfvén waves in the Earth’s core

Nicolas Gillet, Dominique Jault, Elisabeth Canet & Alexandre Fournier,
*Nature, **465**, 74-77, 2010*

Geostrophic torsional wave generated by a jerk of the inner sphere



- Same imposed magnetic field
- Global rotation:
  - λ = 1.7 x 10^{-4} (Lehnert)
  - Λ = 0.52 (Elsasser)
- P_m = 1
- Lu = 1500

$$\lambda = \frac{t_\Omega}{t_{Alfvén}} = \frac{B_0}{R_{oc}\Omega\sqrt{\rho\mu_0}}$$

Lehnert number

Jault, 2008

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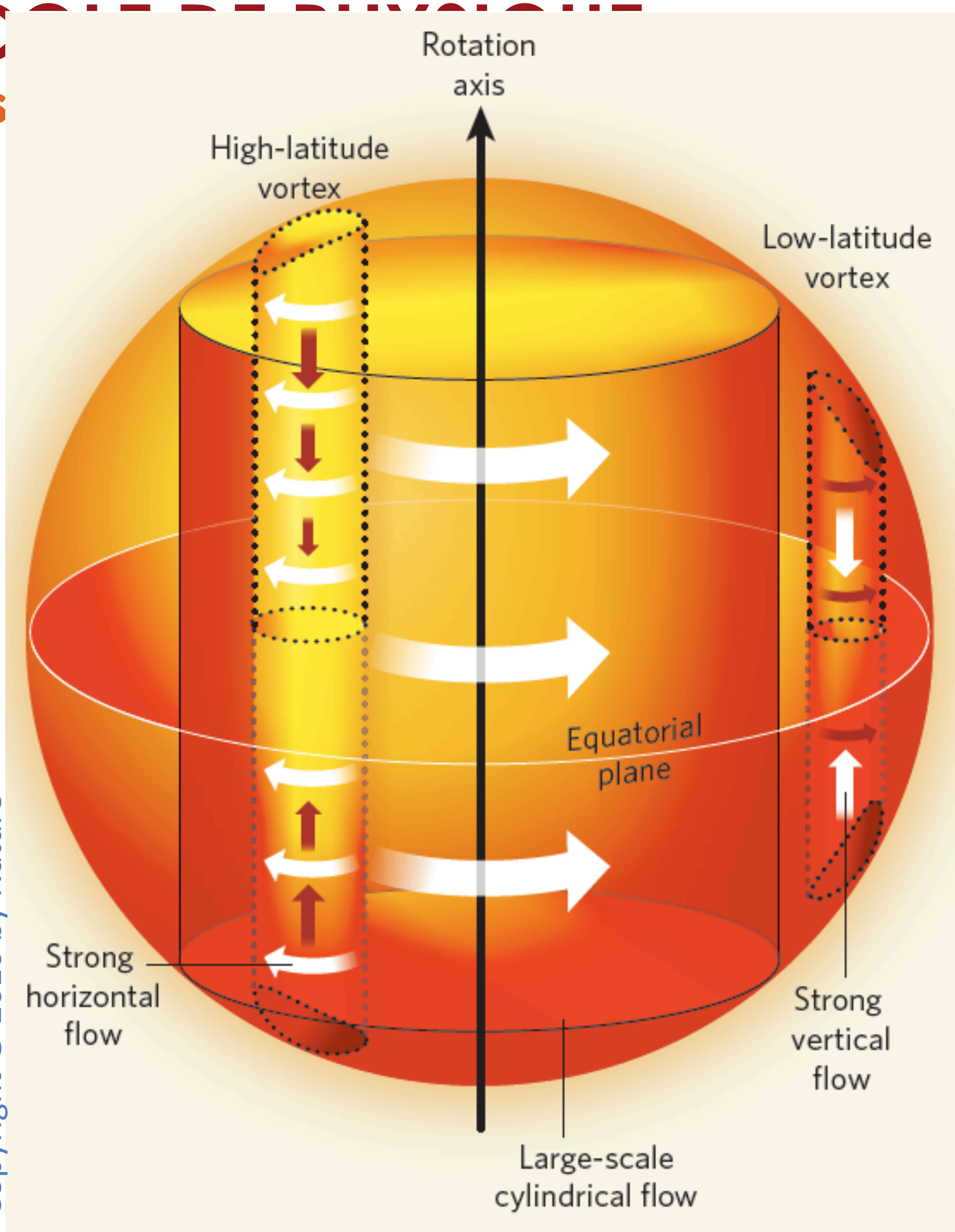
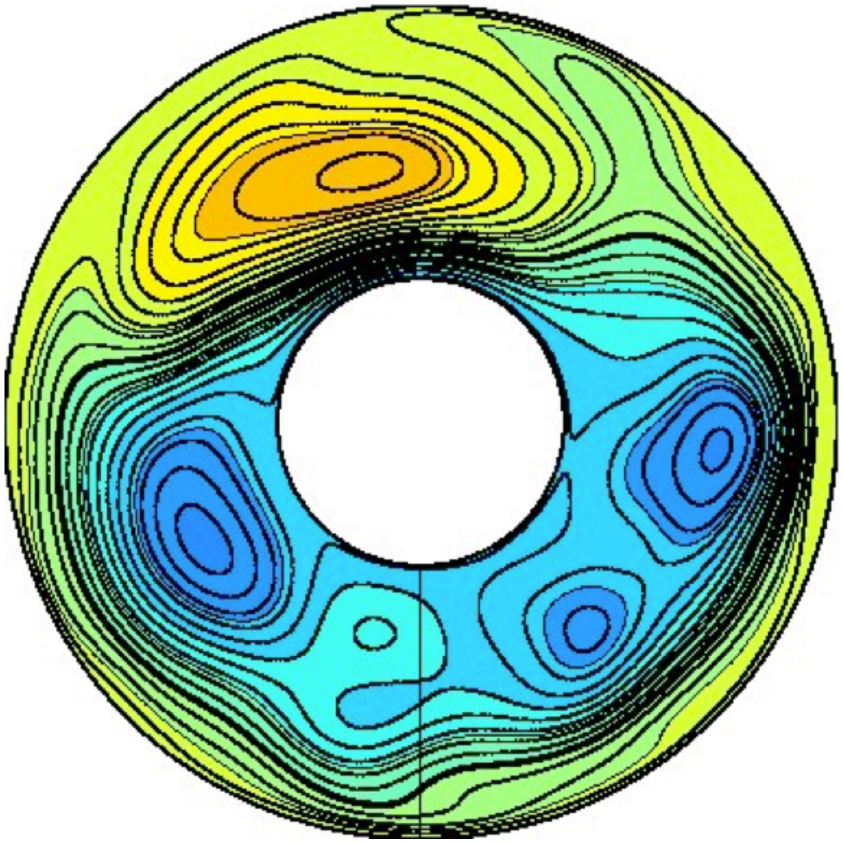


Jackson, news & views, Nature, 2010

- At "short" time scales, the Earth's rapid rotation imposes its law: quasi-geostrophic flow.

Jault, 2008
Gillet, Schaeffer & Jault, 2012
Schaeffer & Pais, 2011

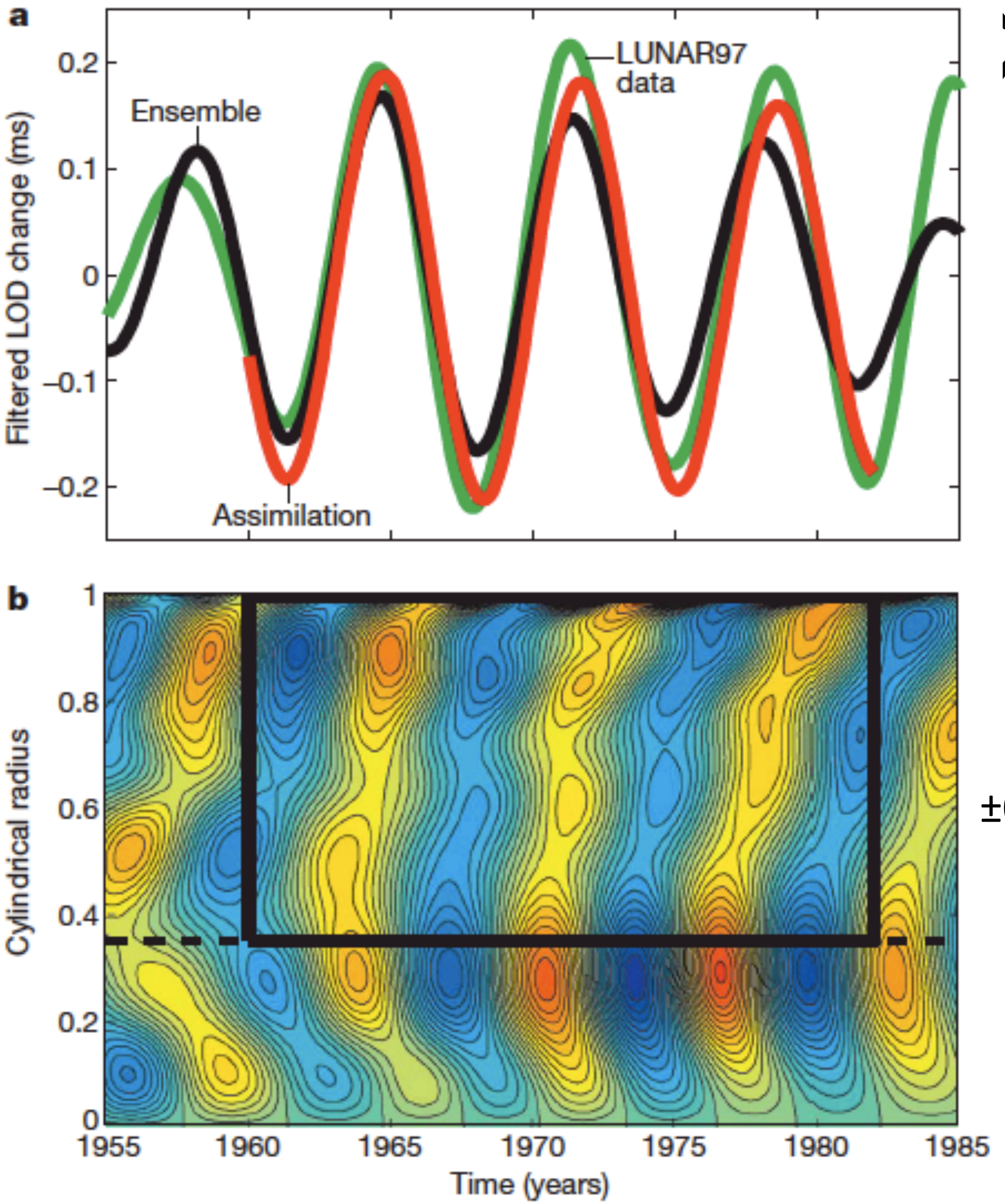
Discovering the flow inside the core: a large non-axial anti-cyclone



Equatorial map of the stream function, from data of year 2000

$t_{SV} \approx 1000$ years

Pais & Jault, 2008



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- and Alfvén waves that jerk the Earth !

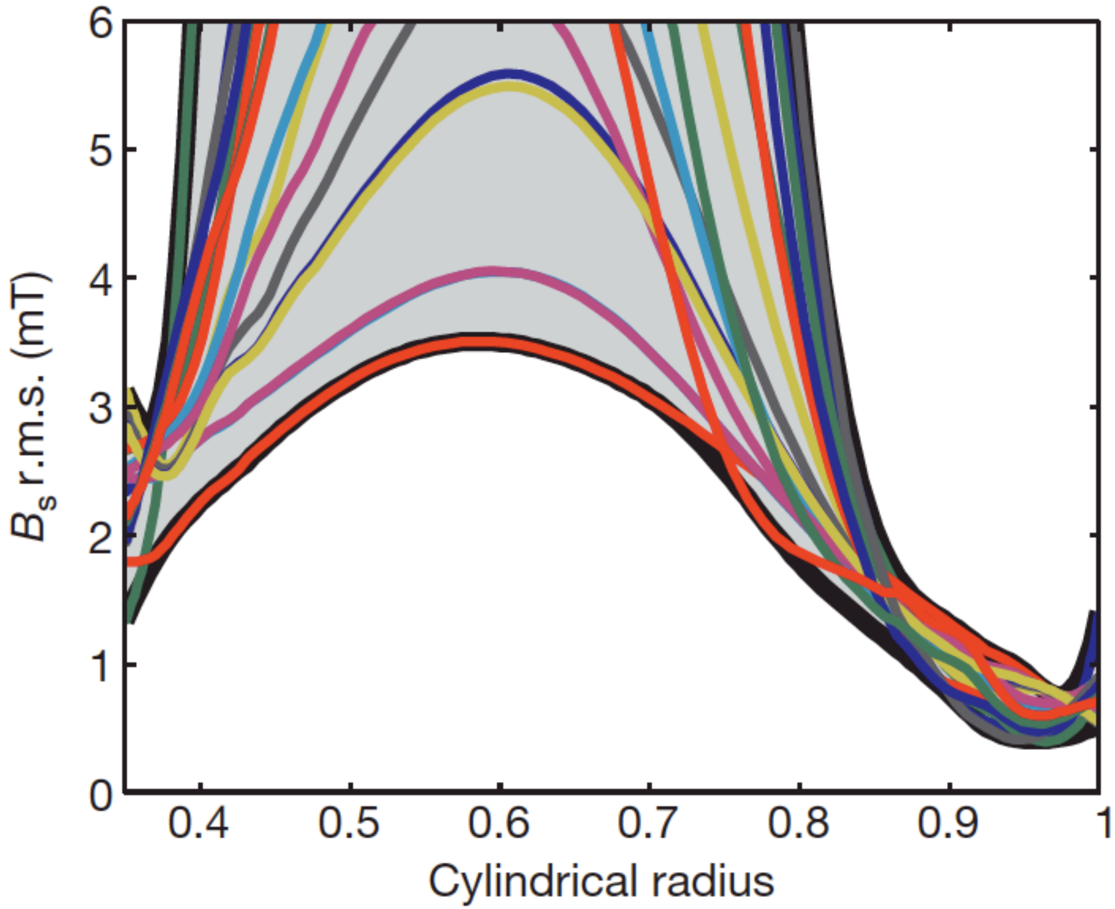
±0.4km/year

$$V_A = \frac{B}{\sqrt{\rho \mu_0}}$$

$t_{Alfvén} \approx 4.3$ years

Gillet et al, Nature, 2010

- Deducing the first profile of magnetic field inside the core.
- In agreement with Lorentz forces needed to sustain the non-axial anticyclone



Gillet et al, Nature, 2010

What turbulence in the Earth's core?

- And how much dissipation?

  → Introducing "NS regime diagrams"

"Turbulence in the core", H-C. Nataf and N. Schaeffer, in *Treatise on Geophysics*, volume 9 "the Core" 2nd edition, Ed P. Olson, Elsevier, to appear in 2013.

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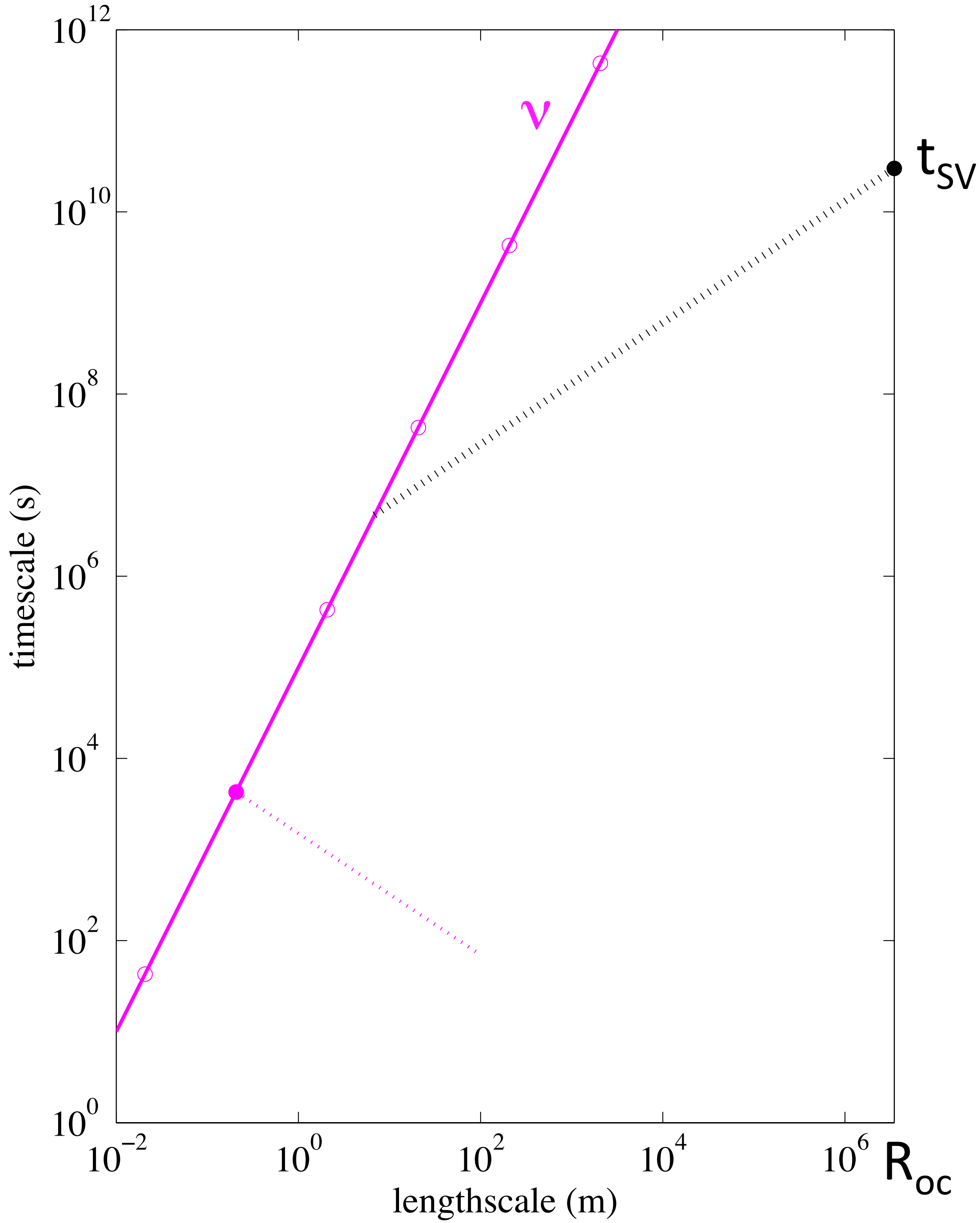
Turbulence => range of scales

→ Plot time-scale versus length-scale (log-log)

→ Try to infer a path from the known large scales to the small scales



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$$t_\nu(\ell) = \frac{\ell^2}{\nu}$$

$$u_\ell^2 = E(k)k$$

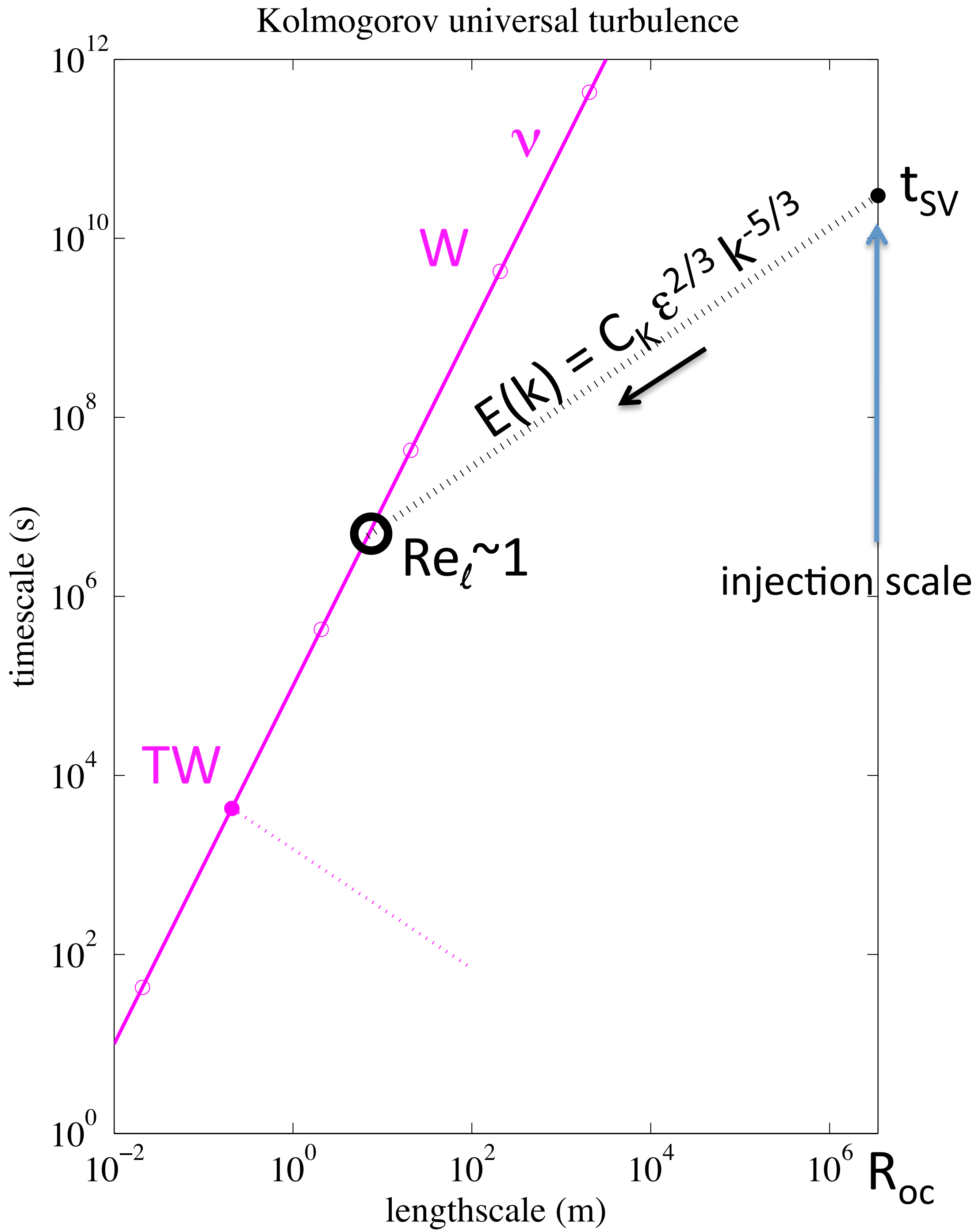
$$u_\ell = \frac{\ell}{t_u(\ell)}$$

$$t_u(\ell) = \varepsilon^{-1/3}\ell^{2/3}$$

$$\mathrm{Re}_\ell = \frac{u_\ell \ell}{\nu}$$

$$P = M_{oc}\varepsilon = \frac{M_{oc}\nu}{t_\nu^2}$$

t_{SV} = 1000 years
R_{oc} = 3480 km
M_{oc} = 1.835 10^{24} kg
$\nu = 10^{-5}$ m^2/s



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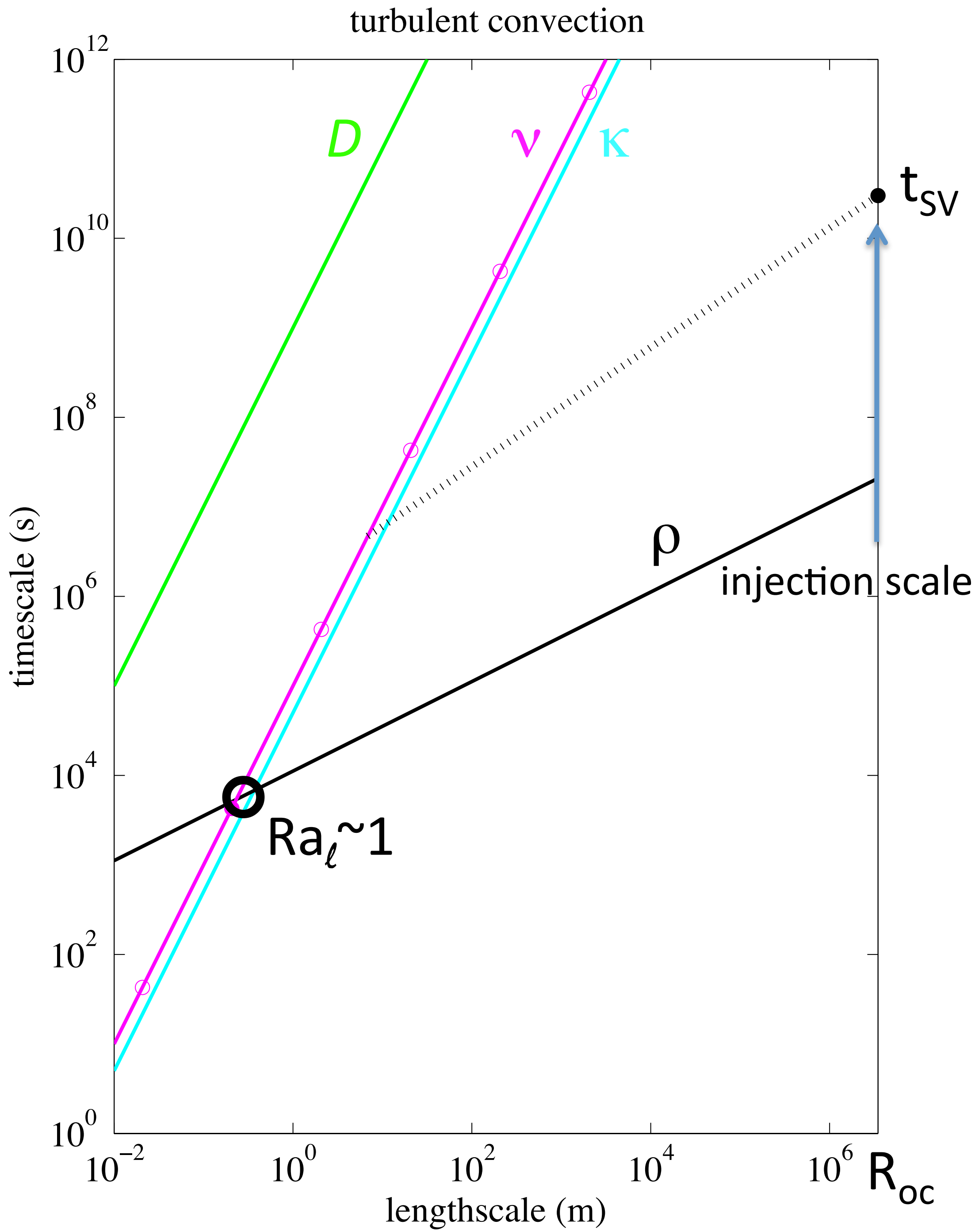
$$t_\rho(\ell) = \sqrt{\frac{\ell}{g}\frac{\rho}{\Delta\rho}}$$

$$Ra_\ell = \frac{t_\kappa(\ell)\, t_\nu(\ell)}{t_\rho^2(\ell)}$$



$\kappa = 2 \times 10^{-5}\ m^2/s$
$D = 10^{-9}\ m^2/s$
$\Delta\rho/\rho = 10^{-9}$
$g = 8\ m/s^2$

Controversial ideas

1. The injection scale for convection is the integral scale (the scale height for compressible convection).

1. Kolmogorov-Obukhov-Corrsin versus Bolgiano scaling:

Kolmogorov and Bolgiano scaling in thermal convection: the case of Rayleigh-Taylor turbulence, G. Boffetta, F. De Lillo, A. Mazzino, S. Musacchi, 2011.

Turbulent convection at very high Rayleigh numbers, J. J. Niemela, L. Skrbek, K. R. Sreenivasan & R. J. Donnelly, *Nature*, 2000.

NB: report $Nu \propto Ra^{0.309}$ for $10^6 < Ra < 10^{17}$.

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$$\delta_E = \sqrt{\frac{\nu}{\Omega}}$$

$$E = \frac{\nu}{\Omega R_{oc}^2}$$

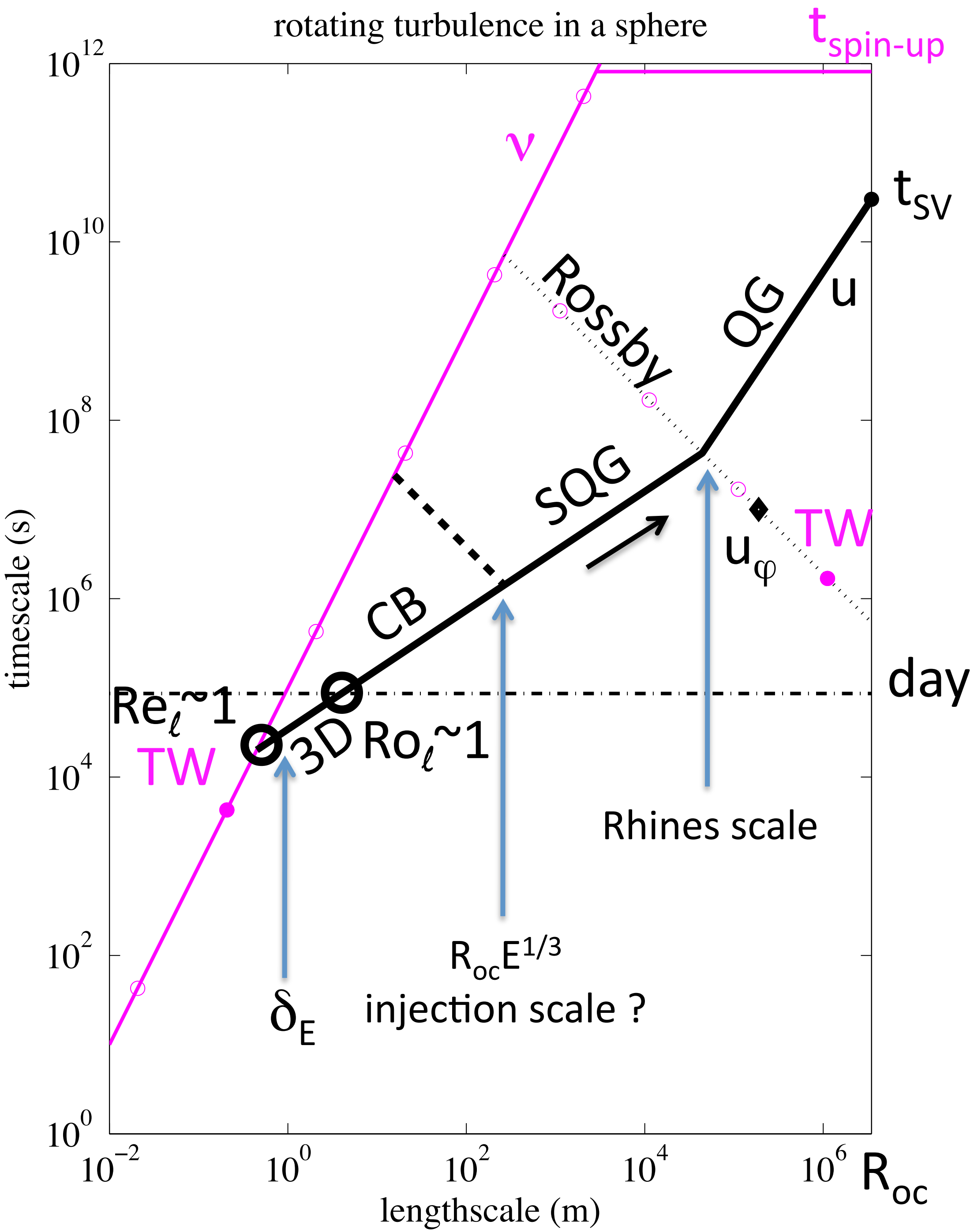
$$t_{Rossby} = \frac{R_{oc}}{\Omega \ell}$$

$$Ro_\ell = \frac{u_\ell}{\Omega \ell}$$

$$t_{spin-up} = \frac{R_{oc}}{\sqrt{\nu \Omega}}$$

$$P_{QG} = M_{oc}\varepsilon_{QG} = \frac{M_{oc} R_{oc} \delta_E}{t_{Rossby}^4 \Omega}$$

$2\pi\Omega$ = day^{-1}
sphere



Controversial ideas

2. Quasi-geostrophic approximation is more limited than we thought for deep layer dynamics.
3. Enstrophy cascade is irrelevant for rotating turbulence (unless layer is shallow).
4. We don't know how to predict the amplitude of zonal jets (and hence their Rhines scale).

2. Time-scale for QG columns:

$$t = \frac{L_z}{\Omega \ell}$$

On the evolution of eddies in a rapidly rotating system, P.A. Davidson, P.J. Staplehurst & S.B. Dalziel, *JFM*, 2006.

3. 'Why anisotropic turbulence is neither weak nor two-dimensional'

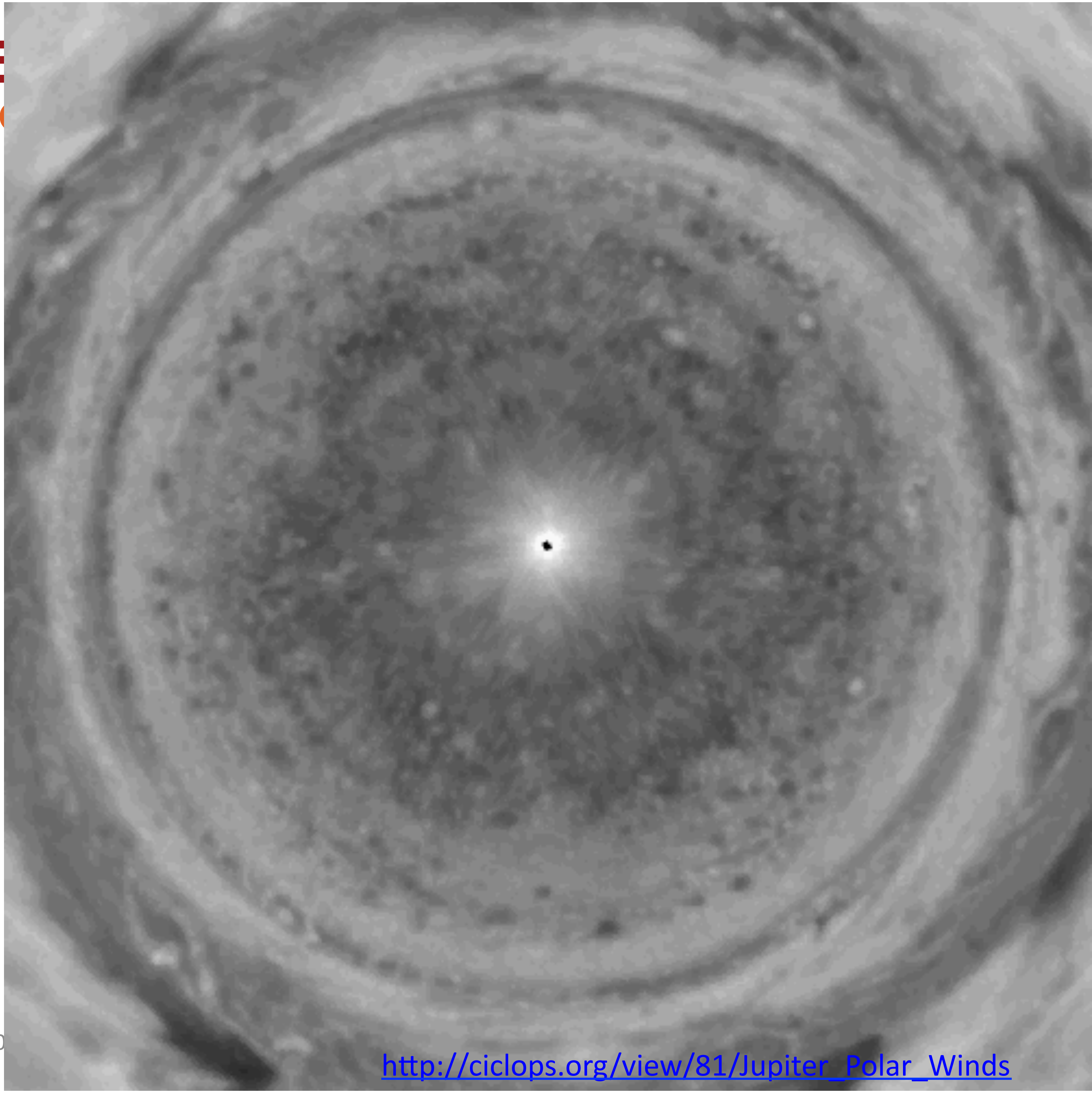
Critical balance in magnetohydrodynamic, rotating and stratified turbulence: towards a universal scaling conjecture, S.V. Nazarenko & A.A. Schekochihin, *JFM*, 2011.

4. Rhines scale for zonal jets:

$$\frac{u_\varphi}{\ell} \approx \beta = \frac{2\Omega \cos\lambda}{R_{oc}}$$

Motion in the Interiors and Atmospheres of Jupiter and Saturn: Scale Analysis, Anelastic Equations, Barotropic Stability Criterion, A.P. Ingersoll & D. Pollard, *Icarus*, 1982.

Cassini,
2001.
cliché NASA


http://ciclops.org/view/81/Jupiter_Polar_Winds

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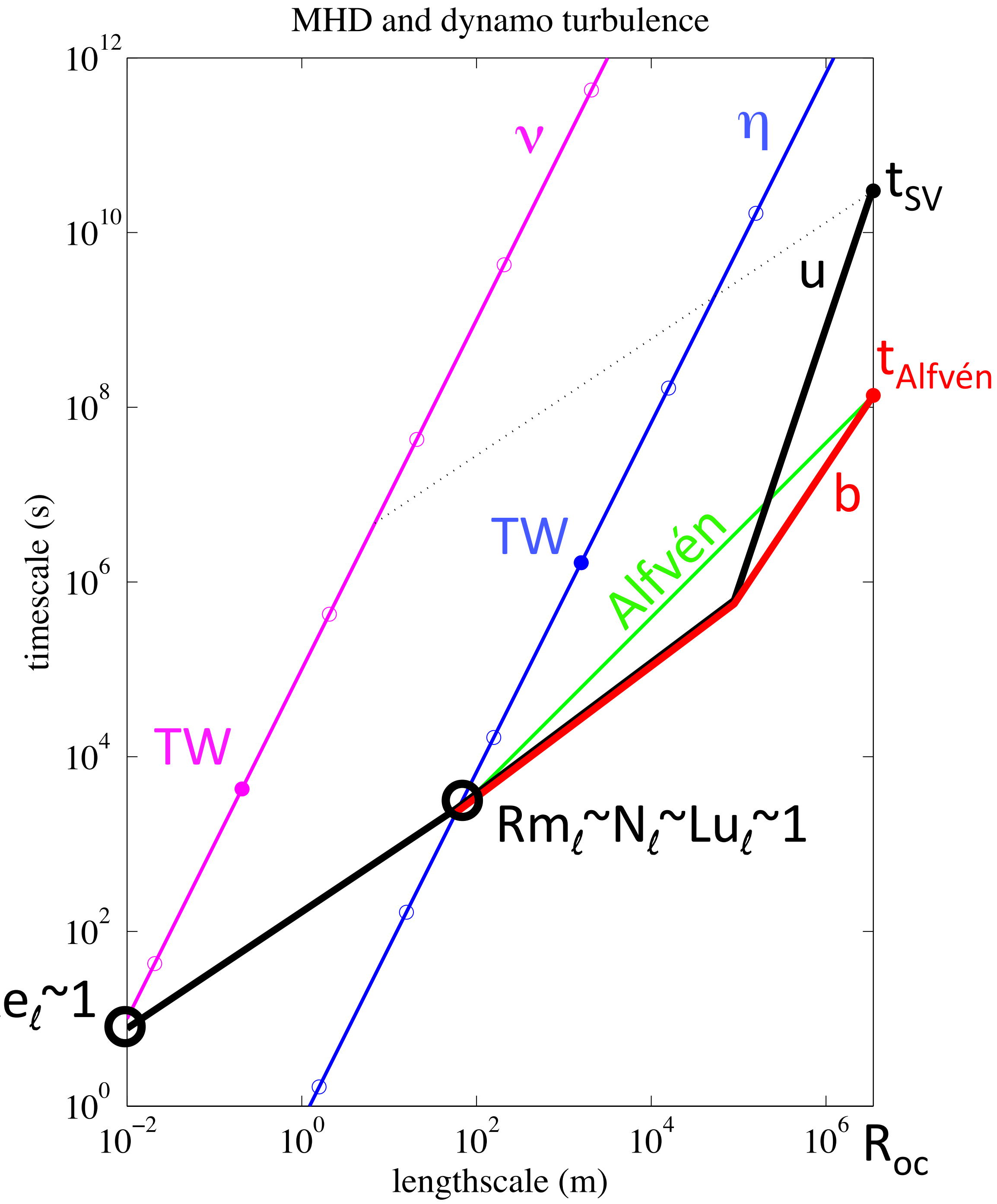
$$t_{Alfvén} = \frac{\ell}{V_{Alfvén}} = \frac{\ell\sqrt{\rho\mu_0}}{B_0}$$

$$Rm_\ell = \frac{u_\ell \ell}{\eta} = \frac{t_\eta(\ell)}{t_u(\ell)}$$

$$Lu_\ell = \frac{t_\eta(\ell)}{t_{Alfvén}(\ell)}$$

$$N_\ell = \frac{B_0 \nabla b}{u \nabla u} = \frac{t_u^2(\ell)}{t_b(l)\, t_{Alfvén}(\ell)}$$



$t_{Alfvén}$ = 4.3 years
η = 1.5 m^2/s

Controversial ideas

5. Alfvén wave collision-based MHD implies energy equipartition down to the diffusion scale => $Rm_\ell \sim N_\ell \sim Lu_\ell \sim 1$.

5. MHD turbulence by collisions of Alfvén waves:

Tobias S., Boldyrev & Cattaneo, in *10 Chapters on Turbulence*, Eds Davidson, 2012.

if equipartition continues down to the dissipation scale, then:

$$B_0 \nabla u = \eta \nabla^2 b$$

implies that this happens when:

$$\frac{t_\eta(\ell)}{t_{Alfvén}(\ell)} \approx 1$$

and at that point, one also has $Rm_\ell \approx N_\ell \approx 1$

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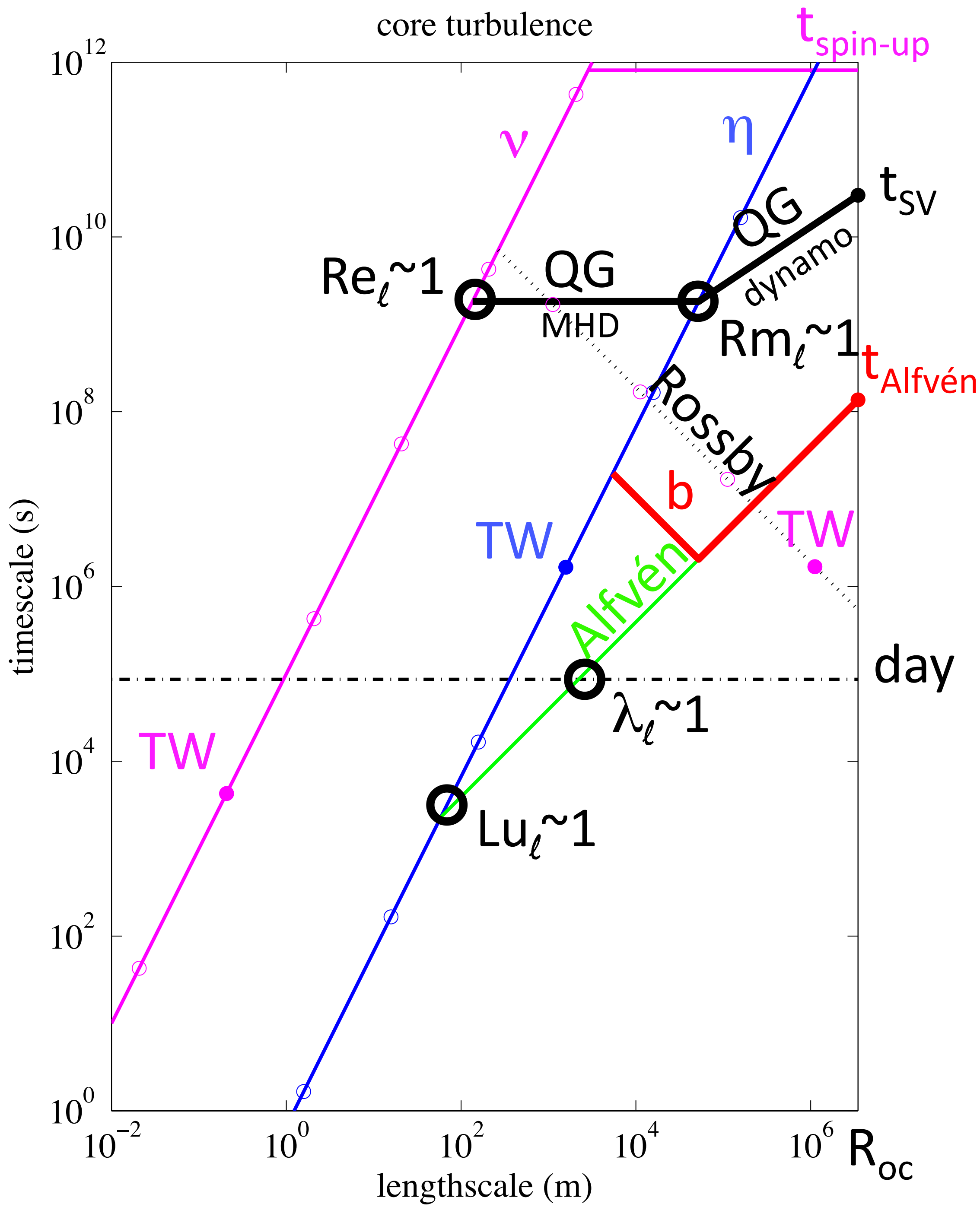
$$\lambda_\ell = \frac{\Omega^{-1}}{t_{Alfvén}(\ell)}$$



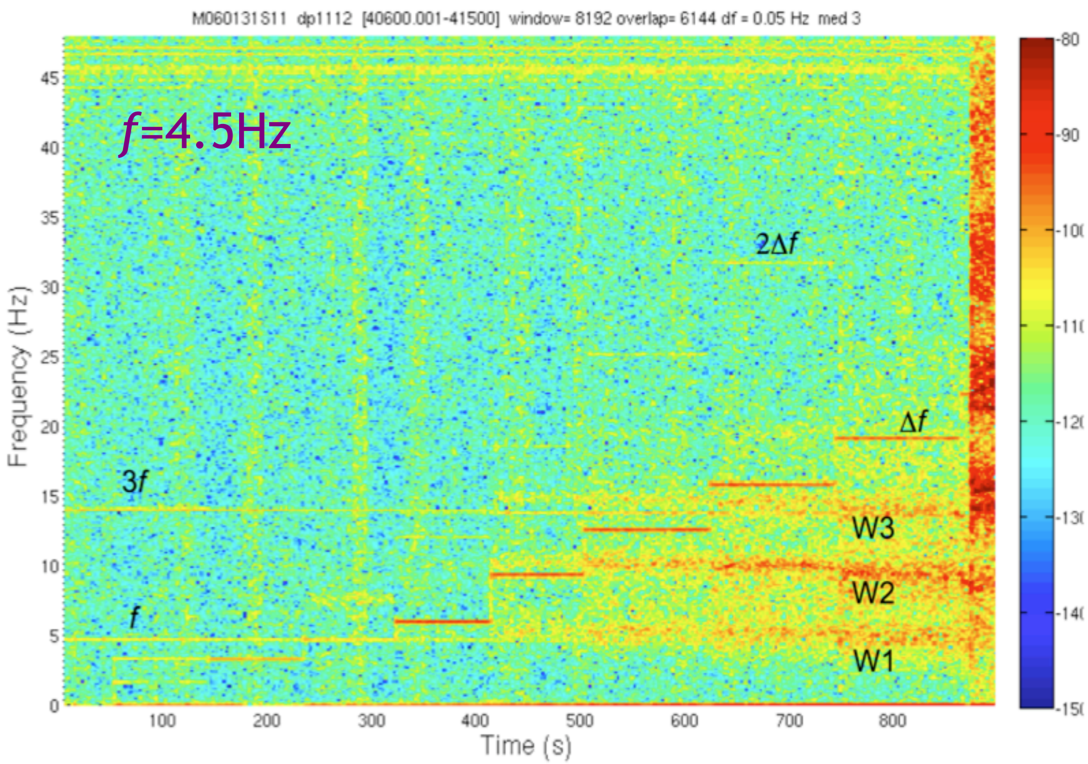
Controversial ideas

6. Almost no turbulence is possible when both Coriolis and Lorentz forces dominate. Rotation prohibits Alfvén waves.

6. Almost no fluctuations in our DTS experiment with global rotation

Schmitt D., T. Alboussière, D. Brito, P. Cardin, N. Gagnière, D. Jault, and H-C. Nataf, Rotating spherical Couette flow in a dipolar magnetic field: experimental study of magneto-inertial waves, *JFM*, 604, 175-197, 2008.

Nataf H-C. and N. Gagnière, On the peculiar nature of turbulence in planetary dynamos, *C.R. Physique*, 9, 702-710, 2008.



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A detailed analysis of a dynamo mechanism in a rapidly rotating spherical shell, Takahashi F. & H. Shimizu, *JFM*, 2012.

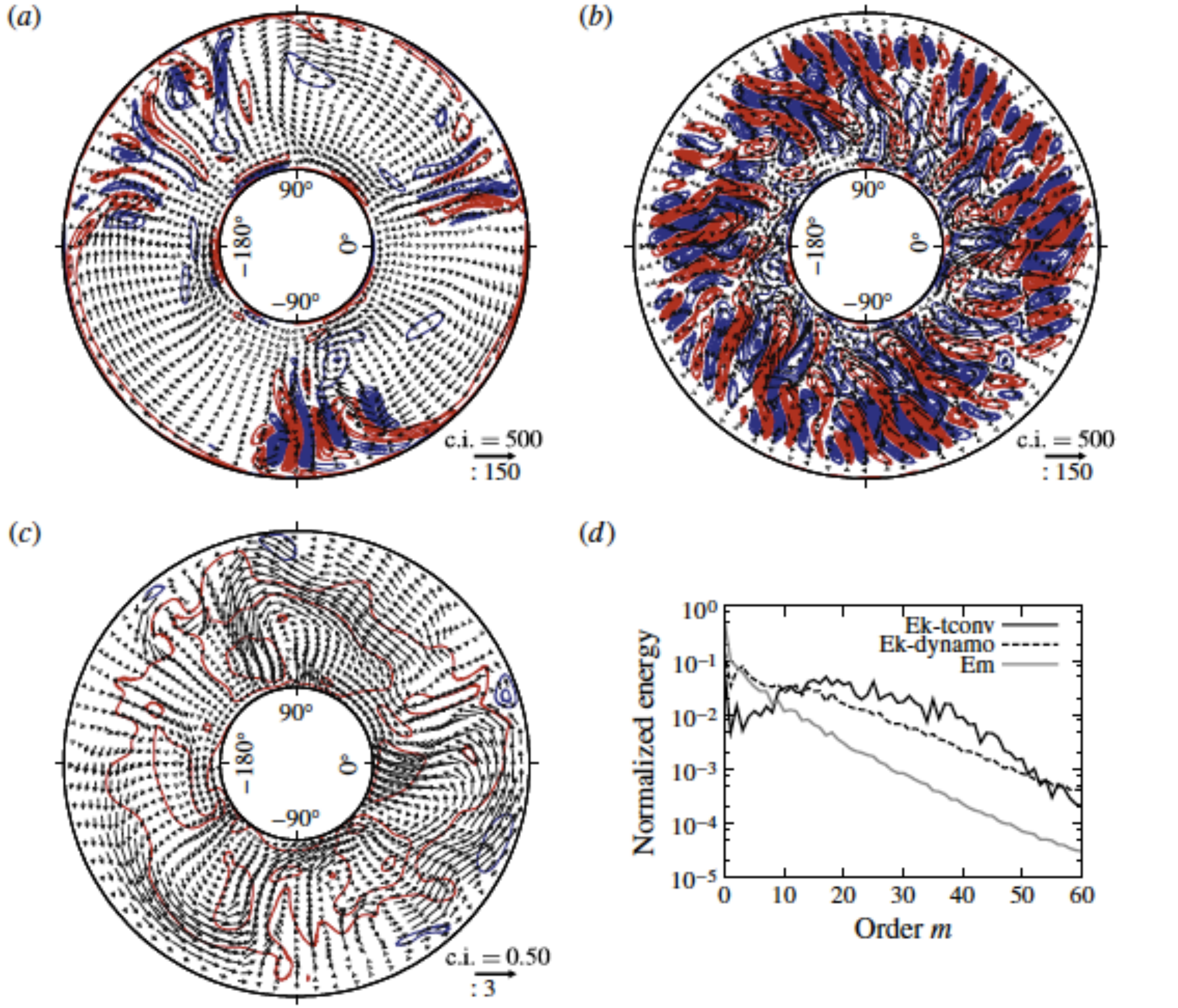

FIGURE 2. (Colour online available at journals.cambridge.org/flm) Snapshots of (*a*,*b*) convection and (*c*) magnetic field on the plane at $z = 0.2$ viewed from the north ($z > 0$). Cases for dynamo (*a*,*c*) and purely thermal convection (*b*) are shown. The axial components of the vorticity (ω_z) and the magnetic field (B_z) are drawn by contour lines. Red (blue) lines represent positive (negative) values. The horizontal components of convection (u_h) and magnetic field (B_h) are drawn by arrows. Here $t = 1.803$ for the dynamo case. (*d*) Time-averaged energy spectra as a function of spherical harmonic order, *m*. Solid, dashed and grey solid lines represent kinetic energy spectra in purely thermal convection, kinetic and magnetic energy spectra in dynamo, respectively. Each spectrum is normalized by the time-average.

conclusion

- Rotation could play a major role in limiting the *dissipation* of the geodynamo.
- The *magnetic field itself* could help in limiting the dissipation (!!!)

Controversial ideas

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