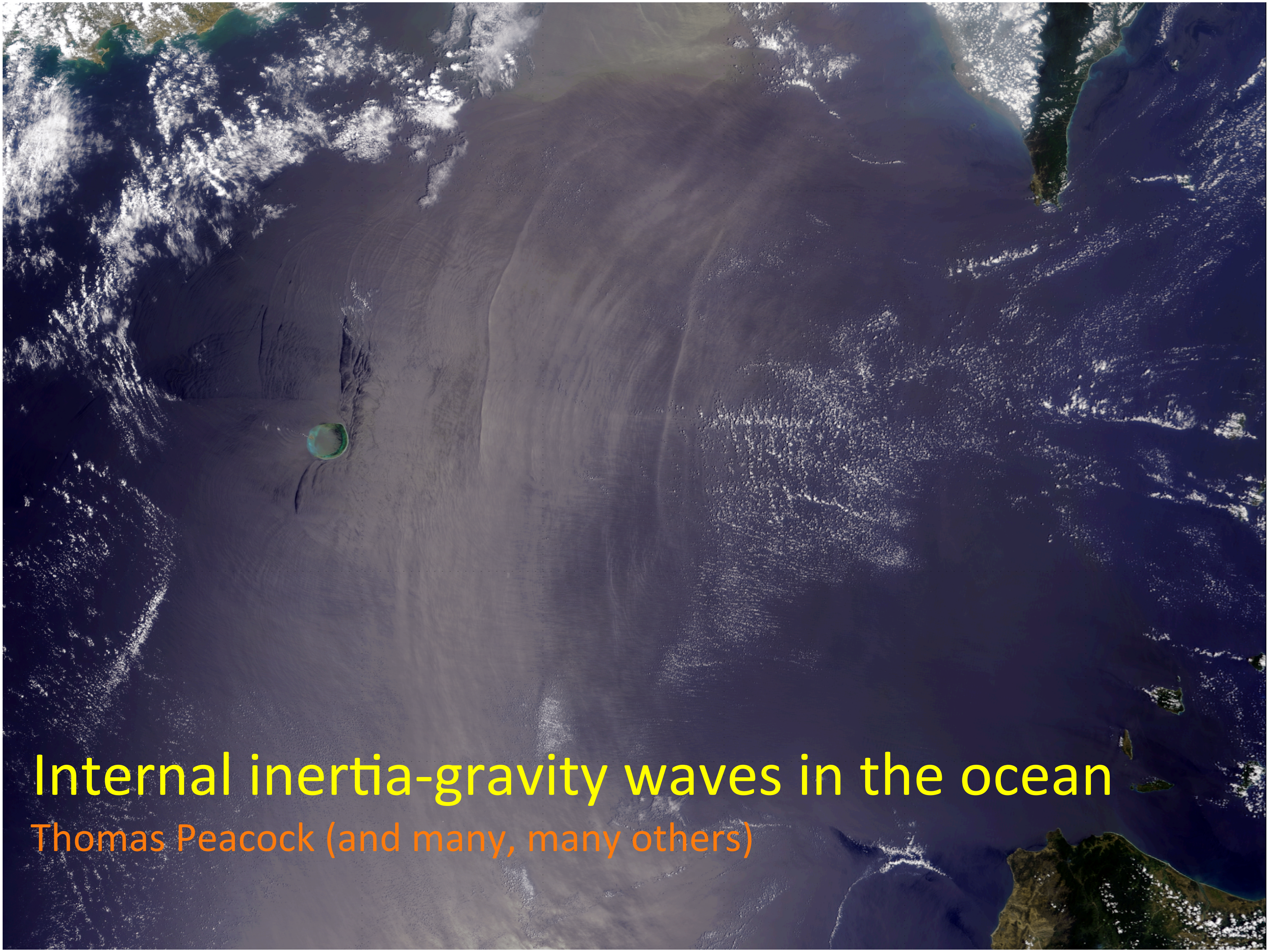
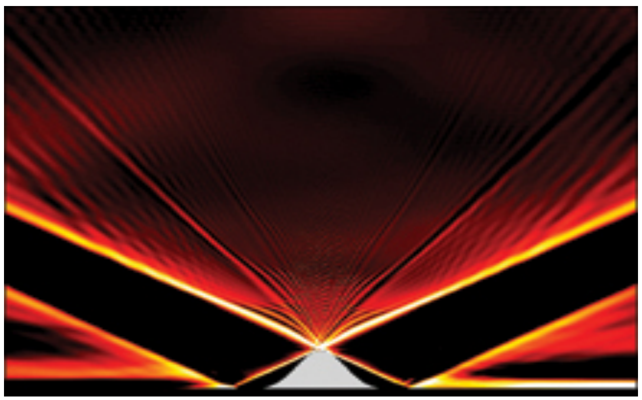

Internal inertia-gravity waves in the ocean
Thomas Peacock (and many, many others)

Generation in the deep ocean

- There are currently understood to be two principle (linear) sources of internal wave energy flux into the deep ocean:

1. Flow past topography



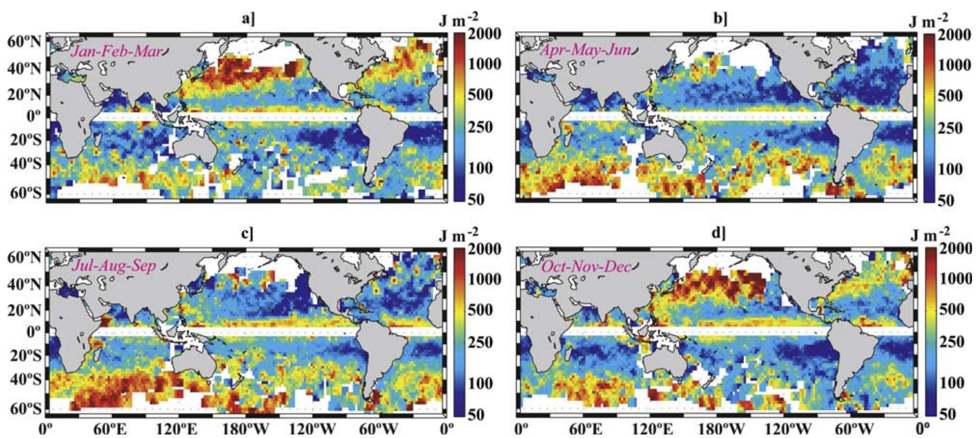
(Image: Sarkar)

2. Ocean surface forcing

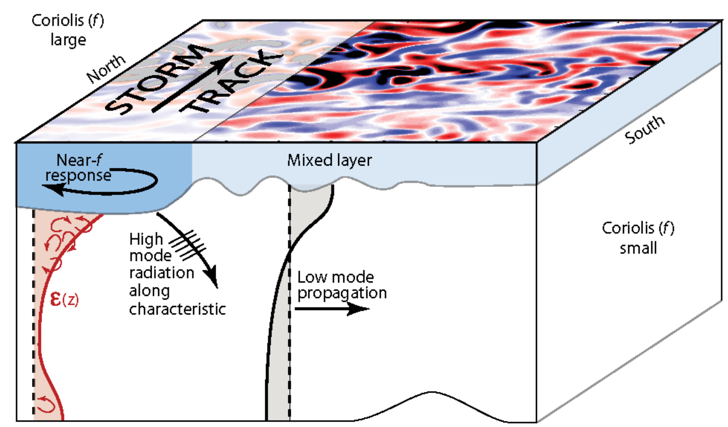
(Image: NASA)

Surface forcing

- The ocean surface is exposed to forcing by surface winds.
- Mid-latitude storms provide the bulk of the energy flux, with much of the remainder coming from hurricanes/typhoons.
- The storm winds excite currents in the surface mixed layer with a resonant coupling at the local inertial frequency $f = 2\Omega \sin\theta$ and scales of order 100km (Pollard & Millard 1970).



(Image: Chaigneau et al 2008)

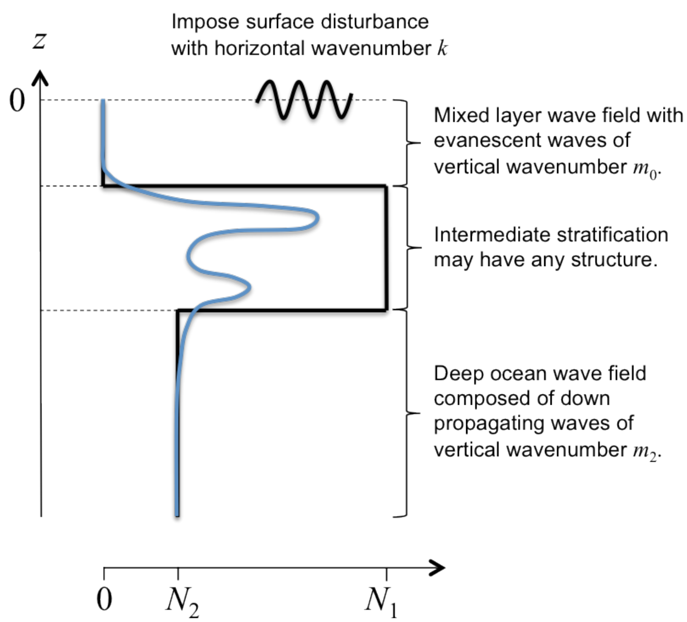


(Image: Simmons & Alford 2012)

- Convergences and divergences in these currents pump vertical motions at the base of the mixed-layer, exciting near-inertial internal gravity waves.
- The waves propagate toward the equator (they cannot propagate toward the poles).

Analytical model

- There are several established models of this process (e.g. Gill 1984, D'Asaro 1989).
- The approach of Nault & Sutherland (2007) can be adapted to handle the downward propagation of inertia-gravity waves excited by forcing of the ocean surface or base of the mixed layer, through arbitrary stratifications with shear.



- The governing internal wave equation (without shear) is:

$$\frac{d^2\phi}{dz^2} + k^2\left(\frac{N^2(z)-\omega^2}{\omega^2 - f^2}\right)\phi = 0,$$

where the stream function for the generated internal wave field is assumed to be of the form:

$$\psi(x,z,t) = \phi(z)e^{i(kx-\omega t)}.$$

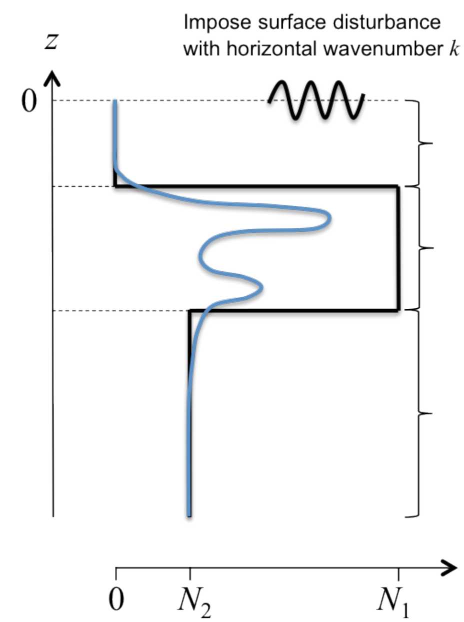
Analytical model

- Consider an imposed disturbance of known horizontal wave number k and frequency ω, and the stratification is known.
- In the deep ocean we require only a downward propagating wave: $\phi(z) = Ce^{im_2 z}$.

- The surface boundary condition is: $w(z) = -\dfrac{\partial \psi}{\partial x} = -ik\phi e^{i(kx-\omega t)} = w_0 e^{i(kx-\omega t)} \rightarrow -ik\phi = w_0$ at $z = 0$.
- We also reconfigure the lower boundary condition:

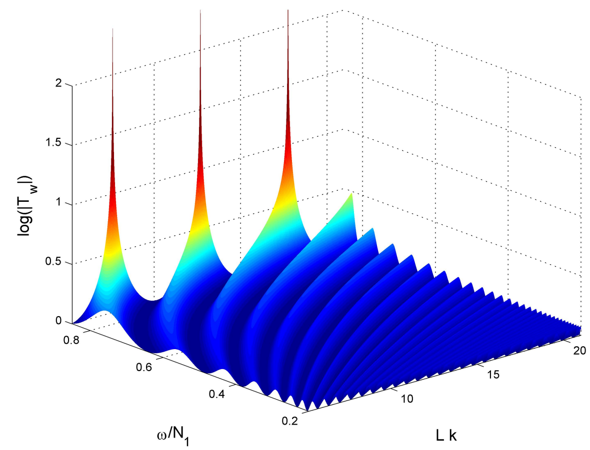
$$\left.\begin{aligned} \phi(z) &= Ce^{im_2 z} \\ \phi'(z) &= im_2 Ce^{im_2 z} \end{aligned}\right\} \rightarrow \phi(-D) + \frac{i\phi'(-D)}{m_2} = 0.$$

- We can then solve as a BVP.



Analytical model

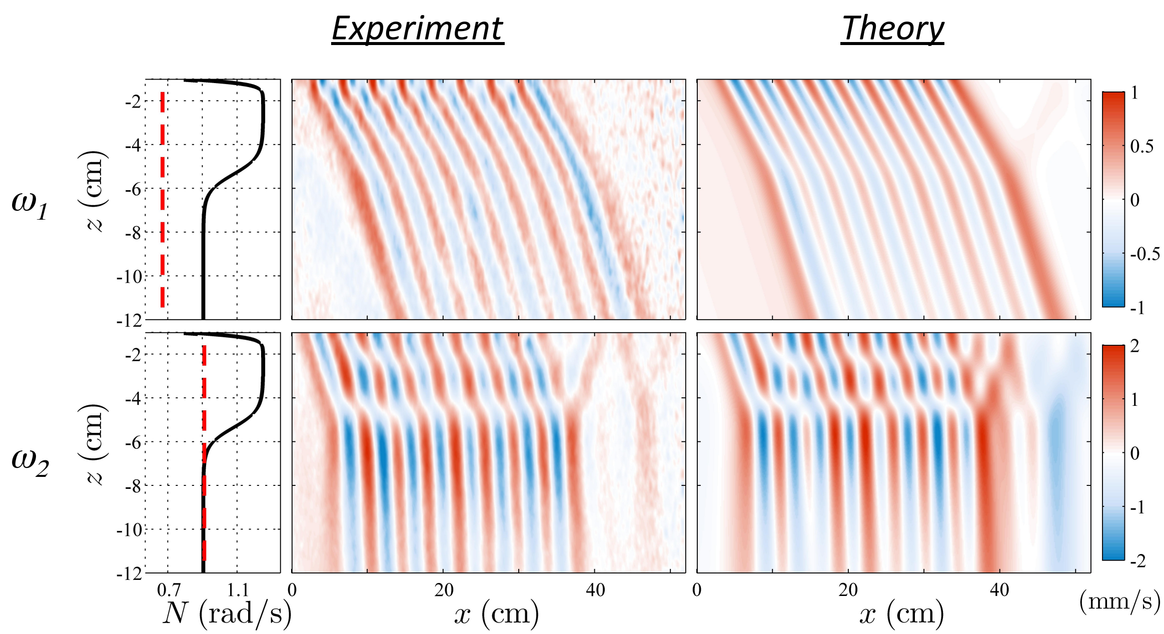
- Even for a two-layer stratification, with upper layer N_1 and deep ocean N_2, there are complex predictions (Ghaemsaidi *et al,* in prep.).



- Laboratory experiments can be performed using a surface-mounted internal wave generator.

video

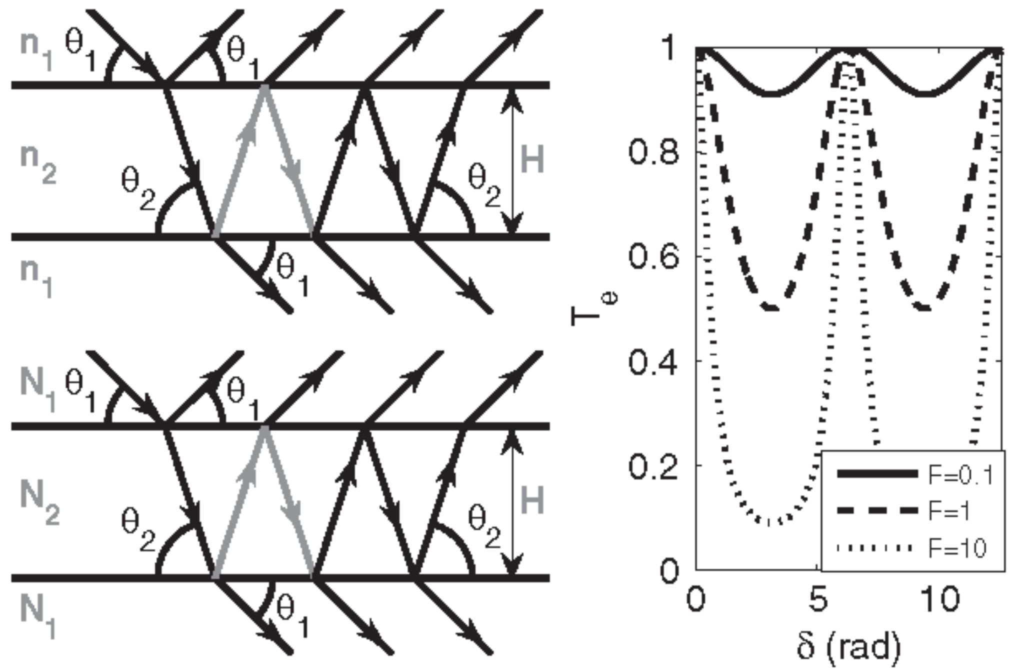
- The first comparisons between laboratory experiments and theory are promising.



Off piste

Interferometer

- There is a nice analogy between internal waves in the ocean and an optical Fabry-Perot interferometer (Mathur & Peacock 2010).



- For the optical problem: $T_e = \left[1 + F\sin^2\left(\frac{\delta}{2}\right)\right]^{-1}, \quad F = \frac{4R}{(1-R)^2}, \delta = \frac{4\pi H}{\lambda} n_2 \sin\theta_2.$

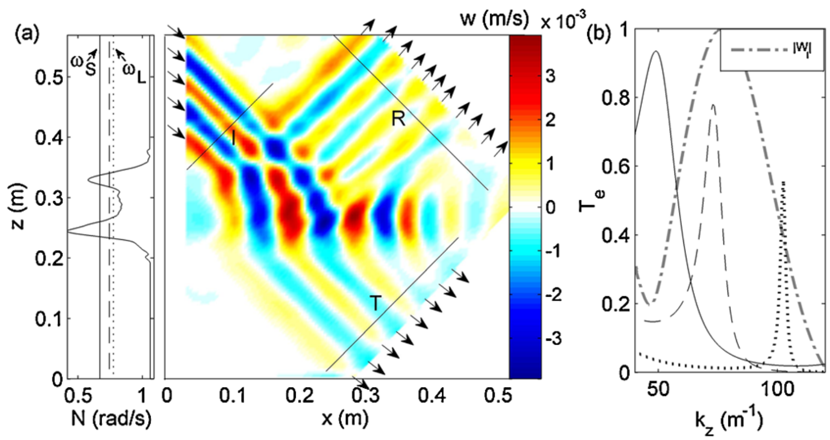
- For the internal wave problem: $T_e = \left[1 + \frac{1}{4}\left(\frac{\cot\theta_1}{\cot\theta_2} - \frac{\cot\theta_2}{\cot\theta_1}\right)^2 \sin^2\left(kH\cot\theta_2\right)\right]^{-1}.$

Interferometer

- Laboratory experiments for a wave beam comprising a range of wavenumbers clearly demonstrate selective transmission (accounting for viscosity).

Wave field snapshot

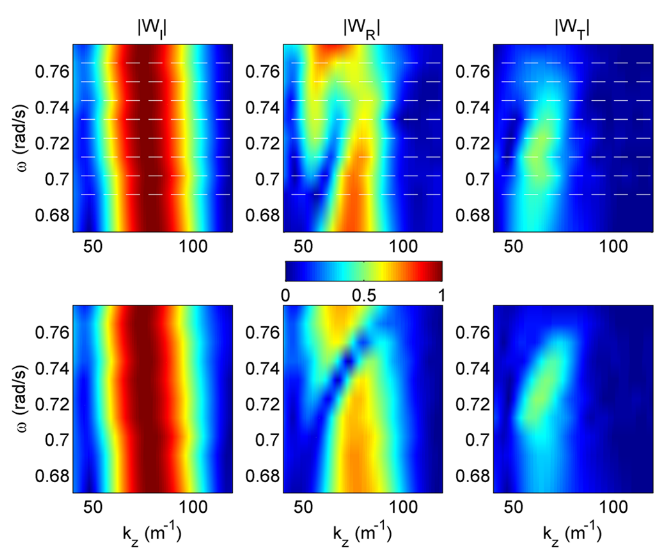
Stratification and forcing frequencies



Wave field and transmission spectra

Experimental results:

Theoretical results:

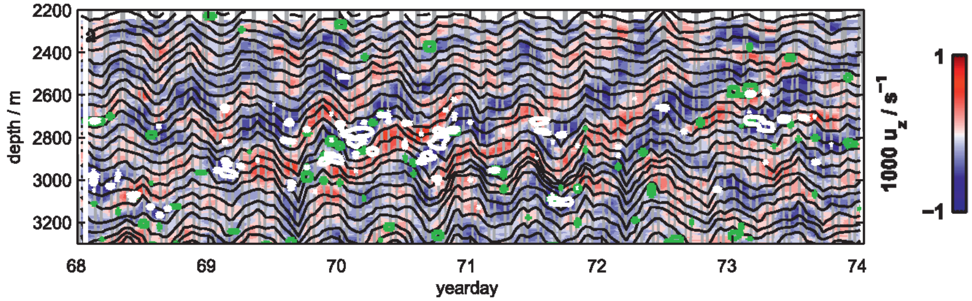


On piste

Field experiments

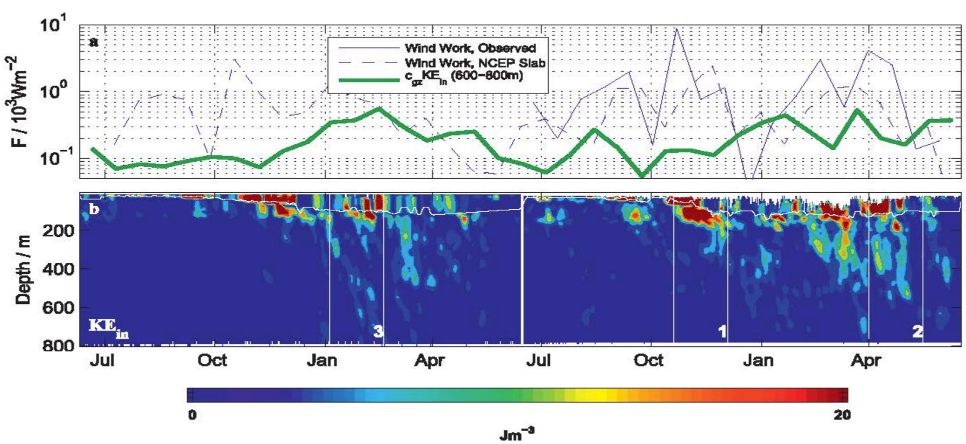
- Near-inertial waves are clearly seen in mooring and ship board ADCP and profiler data.

<u>Mendocino Escarpment</u>



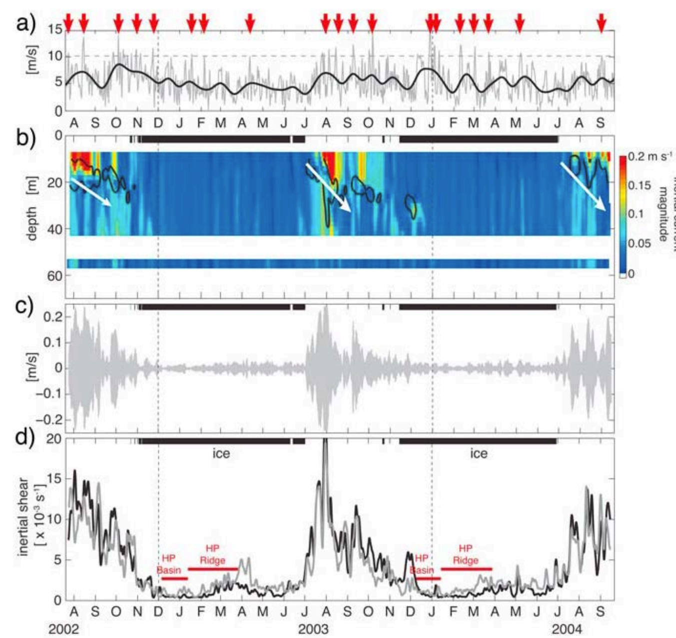
(Image: Alford 2010)

<u>North Pacific</u>



(Image: Alford 2012)

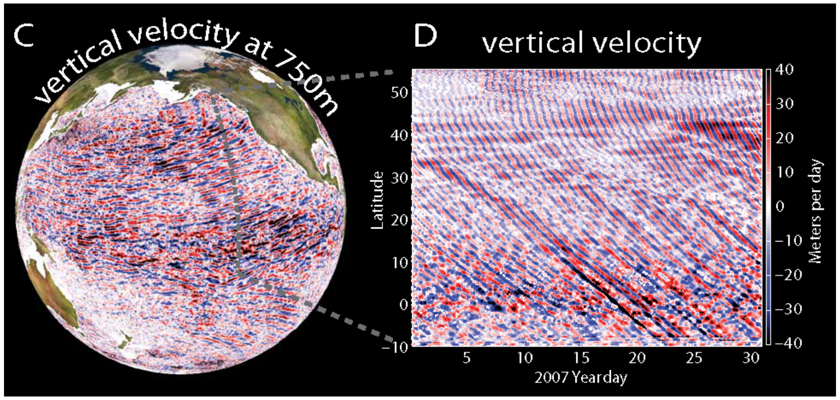
- A pertinent region in which this process needs to be understood is the Arctic Ocean ,where reduced seasonal ice cover exposes the ocean to enhanced forcing and rotation is very strong.

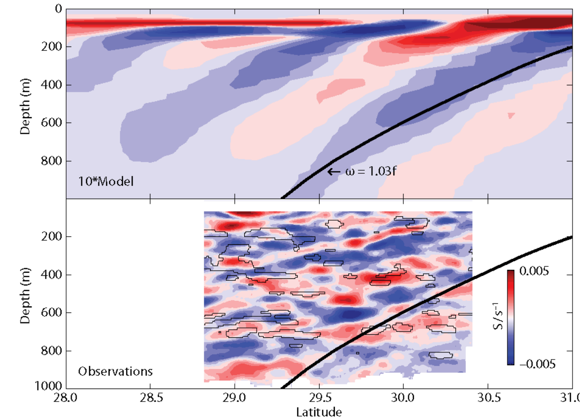


(Image: Rainville & Woodgate 2009)

The global picture

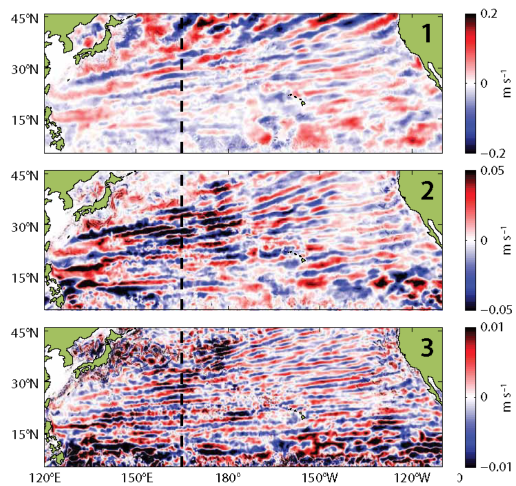
- Near-inertial waves are excited by mid-latitude storms and head equatorward.
- Comparison with ocean measurements north of Hawaii show respectable agreement.

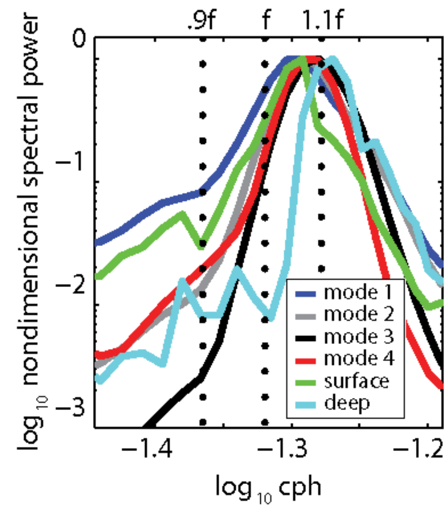




(Images: Simmons & Alford 2011)

- The energy mostly travels as mode-1, and this energy is blue shifted.





- Energy flux of 0.3-1.4TW and contain most of the observed shear and 50% of KE.

The fate of deep ocean internal waves

The fate of deep-ocean internal waves

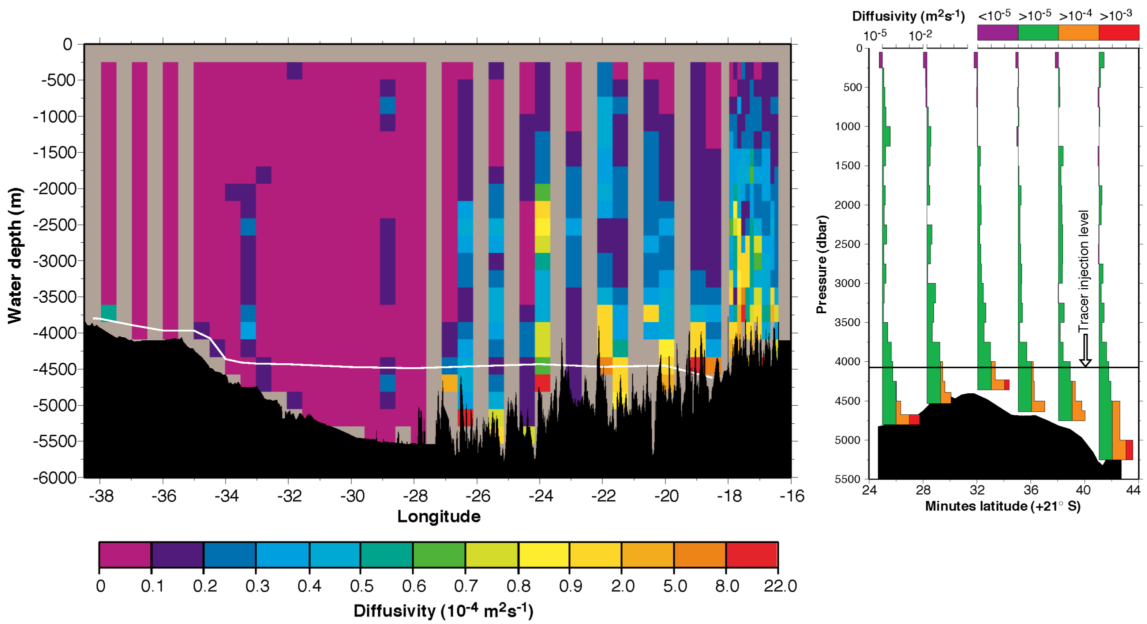
- There are several conduits for the transfer of inertia-gravity wave energy from large to small scales and onward to turbulent mixing via instabilities, including:

    1. Dissipation at topographic generation sites
    2. Scattering by deep-ocean bathymetry
    3. Interaction with pycnocline
    4. Mesoscale interactions
    5. Wave-wave interactions
    6. Solitary waves

1. Dissipation at topographic generation sites

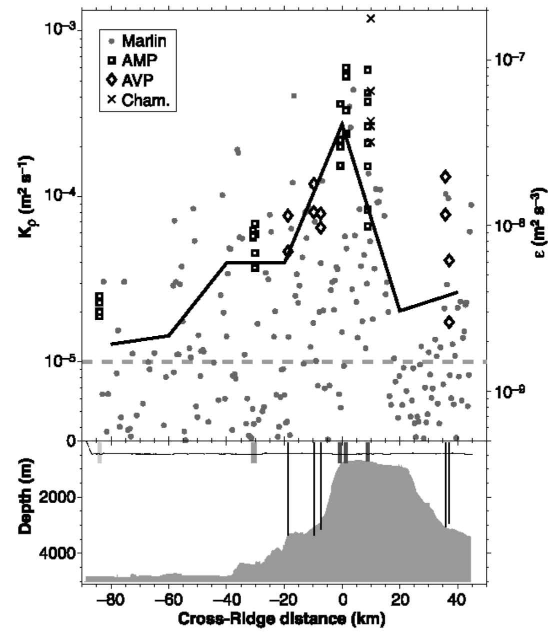
- There is substantial evidence of high turbulent dissipation, and correspondingly high turbulent diffusivity $K_\rho = \gamma \dfrac{\varepsilon}{N^2}$, in the vicinity of internal wave generation sites.

<u>*Brazil Basin*</u>



(Images: Polzin et al 1997)

<u>*French Frigate Shoals (Hawaii)*</u>



(Image: Rudnick et al 2003)

- Detected values of K_ρ between 10^{-3}-10^{-4} m^2s^{-1} are one-to-two orders of magnitude greater than values deduced from the upper ocean interior.
- This activity occurs remote from the bottom and/or is phase locked with the forcing tide, implicating internal wave processes.

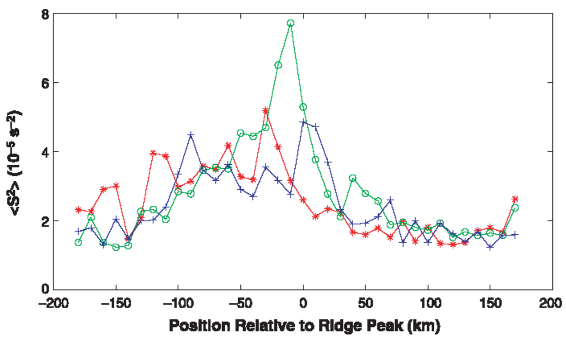
1. Dissipation at topographic generation sites

- Turbulent dissipation attributed to instabilities driven by some combination of shear and unstable buoyancy.

Kauai Channel, Hawaii

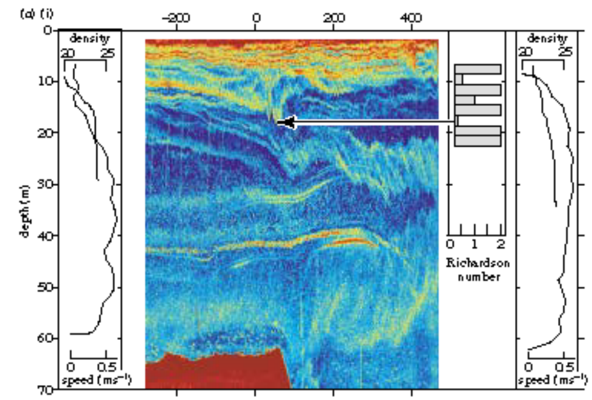
Shear:

$$Ri = \frac{N^2}{(du/dz)^2}$$



(Image: Rudnick et al 2003)

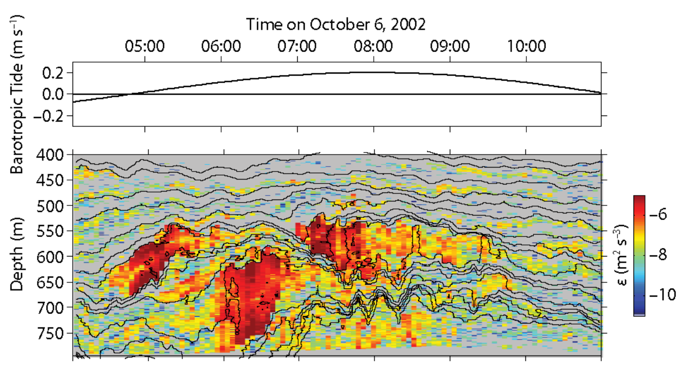
Knight Inlet, British Columbia



(Image: Farmi & Armi 1999)

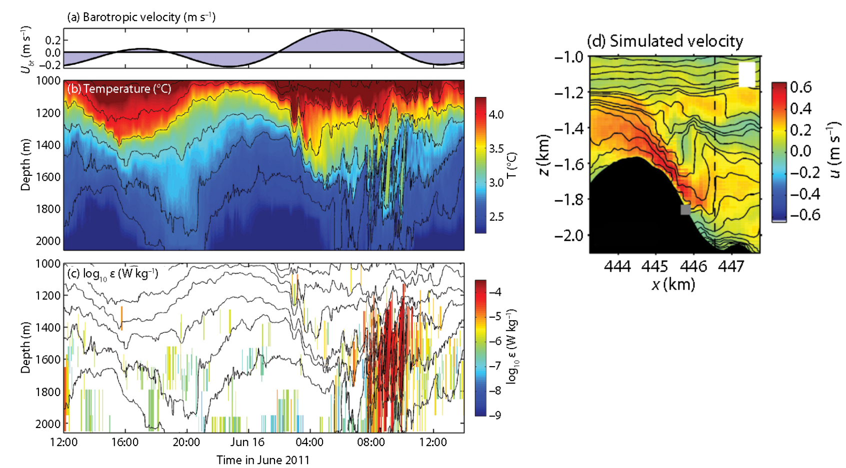
Kauai Channel, Hawaii

Buoyancy:



(Images: Klymak et al 2012)

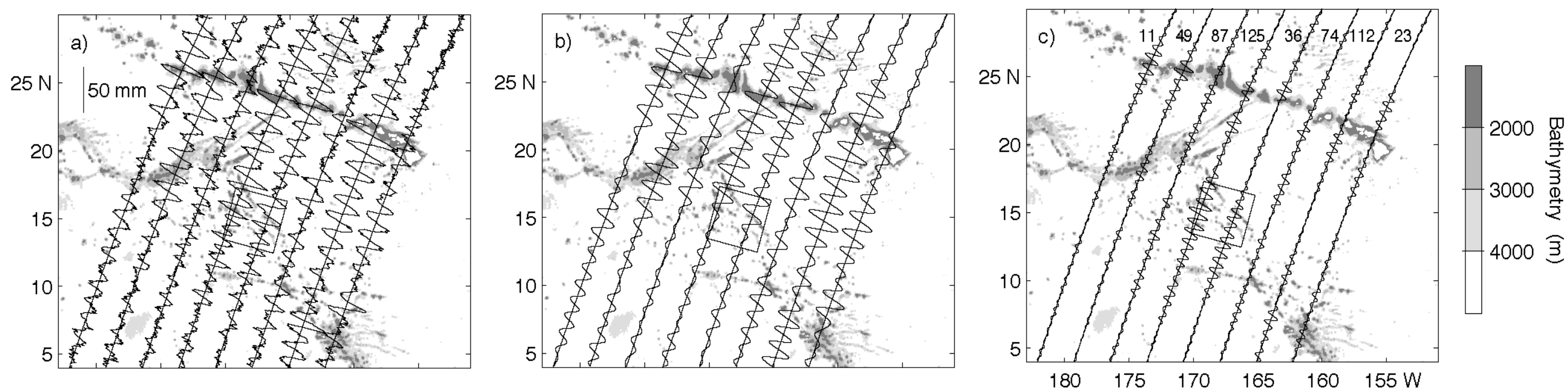
Luzon Strait, South China Sea



- While these events are locally very strong, for large scale topography a majority of the energy radiates away in the form of low modes (e.g. Alford & Zhao 2007).

2. Topographic scattering

- Satellite altimetry suggests that scattering could be significant in regions of strong topography (Johnston et al 2003).
- Scattering provides a means to transfer energy from long to short wavelengths, which are more prone to instability because of higher levels of shear and steepening (Staquet & Sommeria 2002).

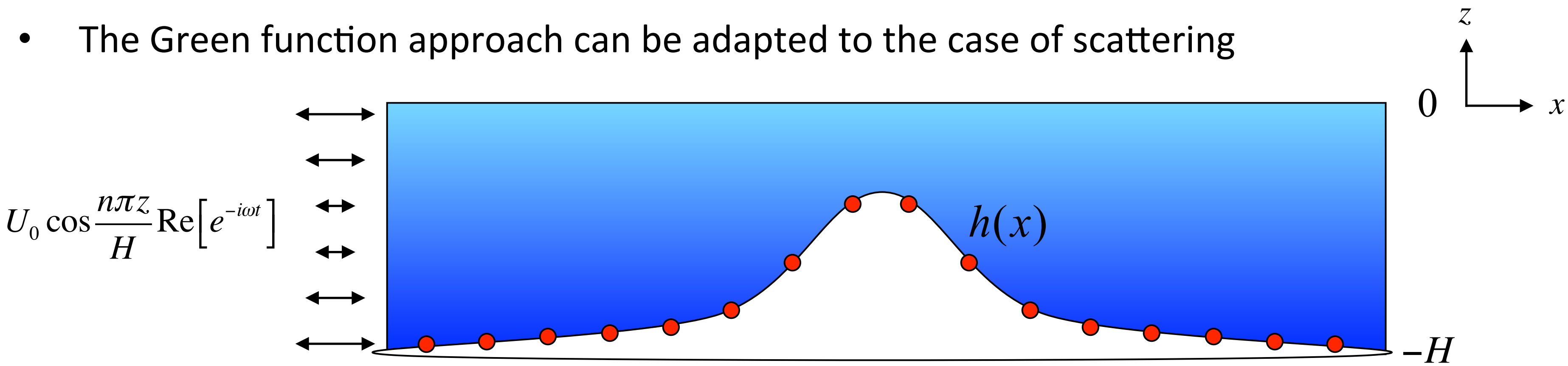


(Image: Johnston et al 2003)

- Previous studies have suggested that topographic scattering is weak (~9% of energy flux), but these analytic models were subject to substantial assumptions.

2. Topographic scattering

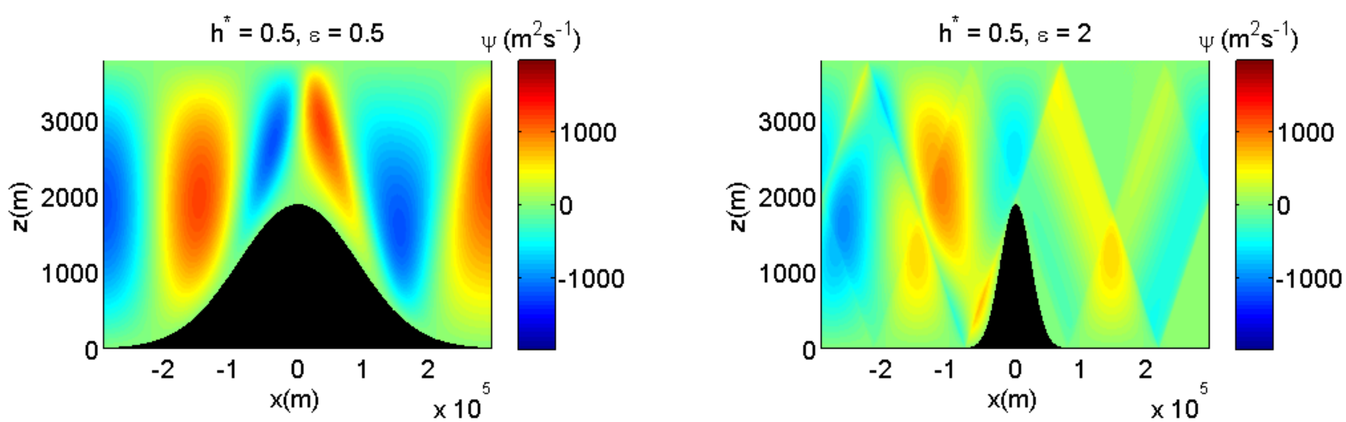
- The Green function approach can be adapted to the case of scattering



- The bottom boundary condition requires no normal flow in the face of an incoming baroclinic mode:

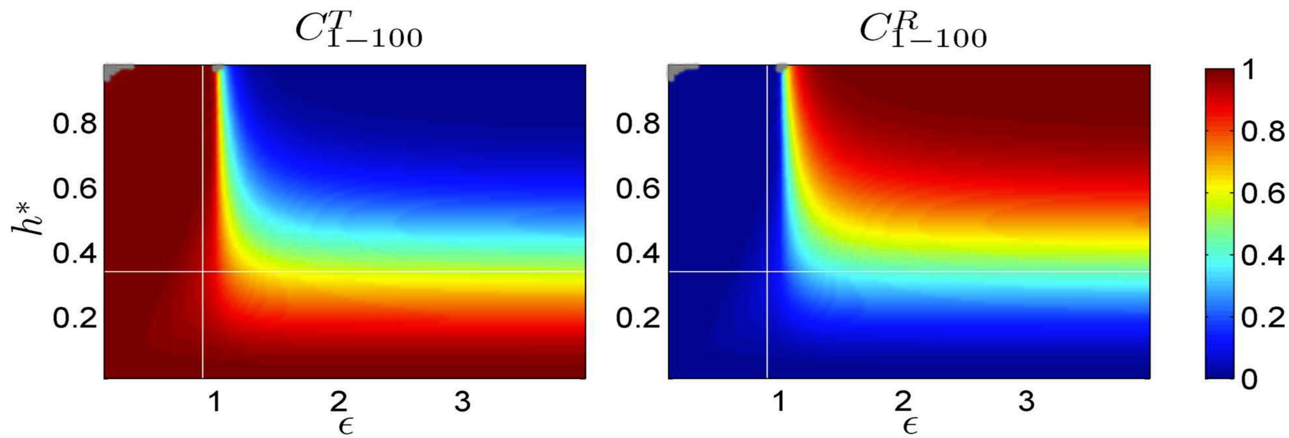
$$\Phi_n(h(x))e^{ik_n x} = \int_{x_1}^{x_2} \gamma(x')G(x,x',h(x),h(x'))dx'.$$

- Scattering by a single ridge in a constant stratification:

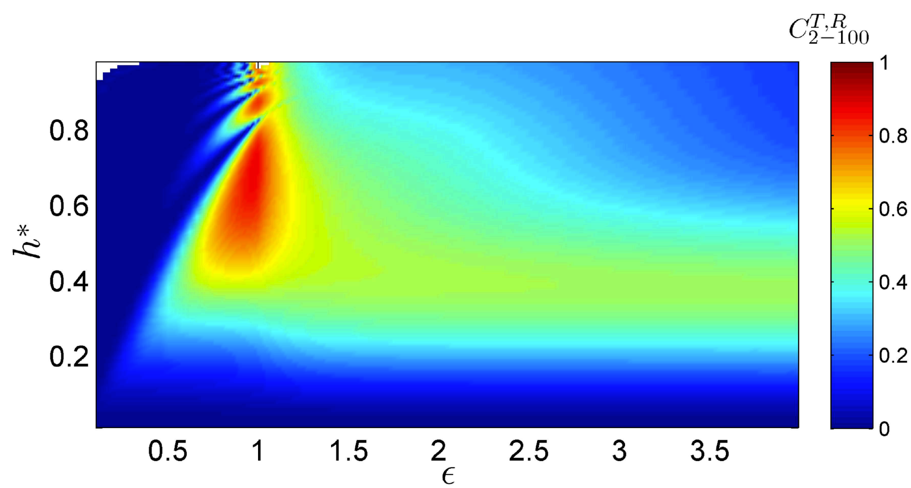


2. Topographic scattering

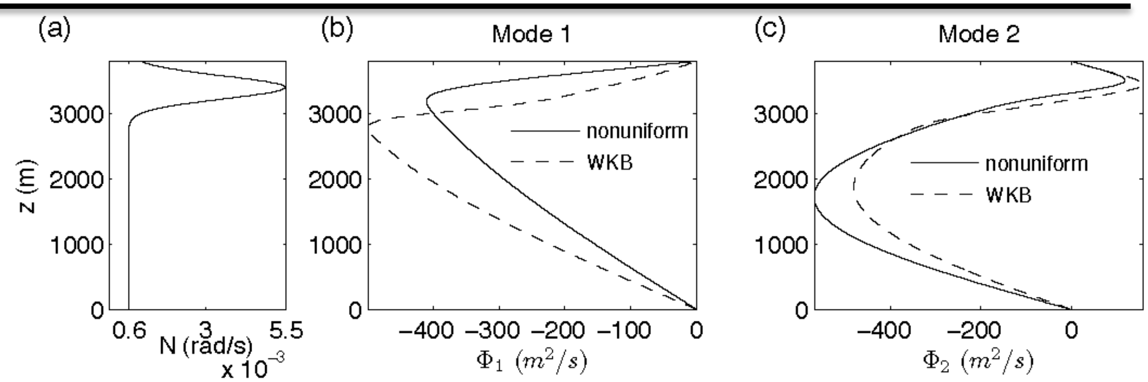
- We calculate the fraction of the energy flux scattered into the transmitted and reflected wave fields as a function of the criticality and the depth ratio.



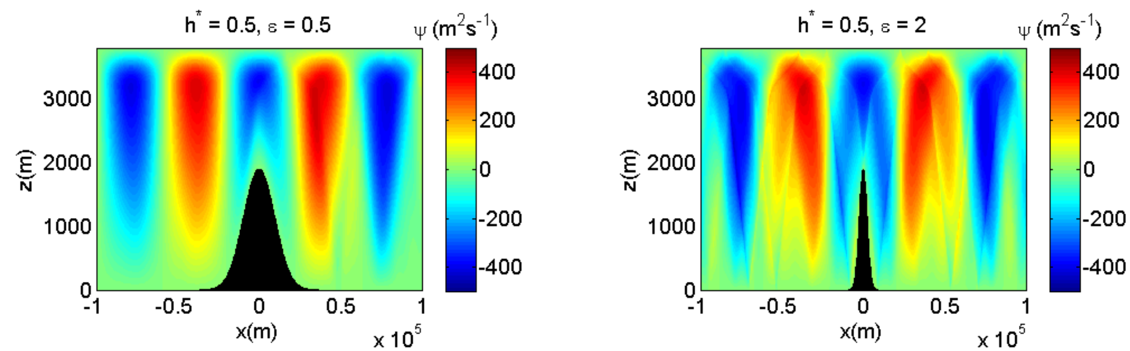
- Critical topographic features are the most efficient at scattering energy to higher modes. Tall critical features can scatter over 80% of the incident energy flux.



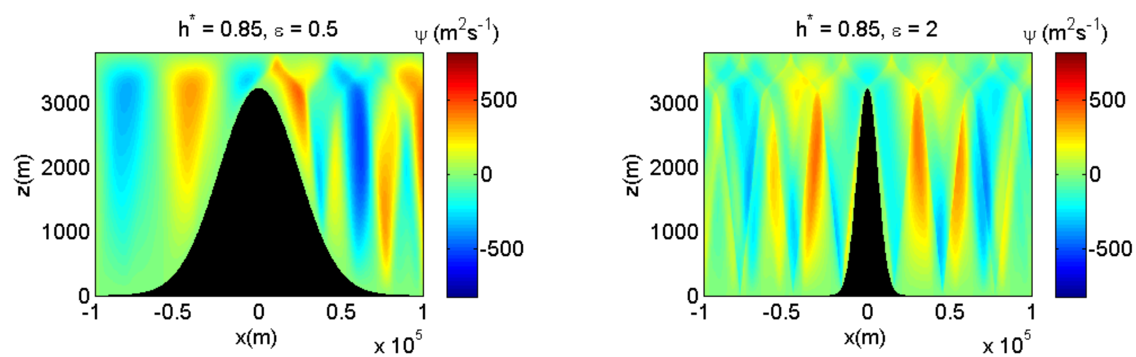
2. Topographic scattering



- Now investigate the same processes for a realistic ocean stratification.



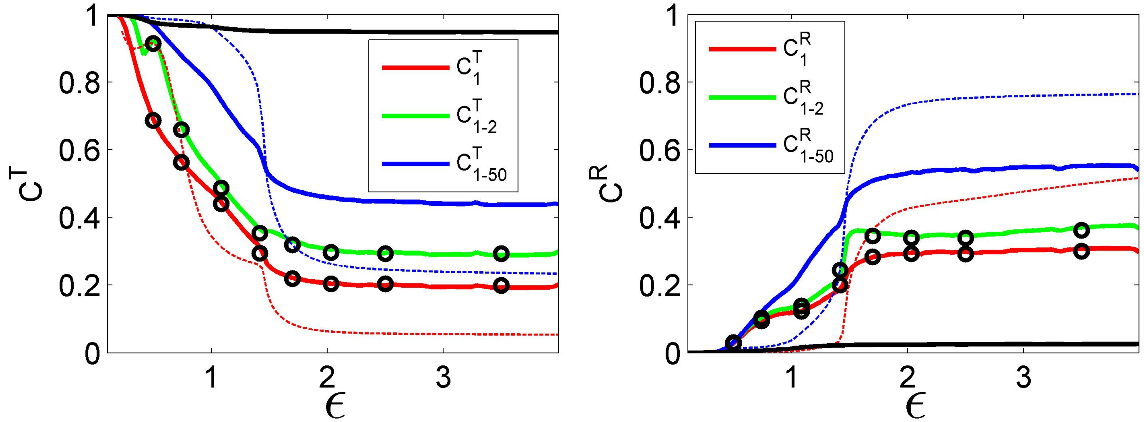
- For topography well below the pycnocline there is very little observable scattering.



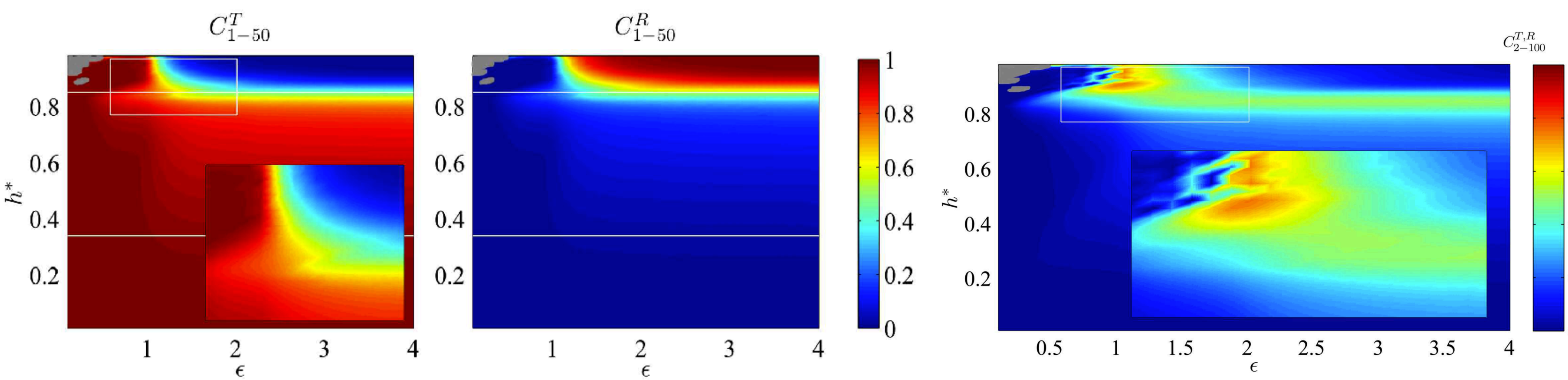
- For topography that impinges on the pycnocline, criticality plays a central role in determining the amount of scattering.

2. Topographic scattering

- There is excellent agreement between the Green function theory and the results of 2D numerical simulations.

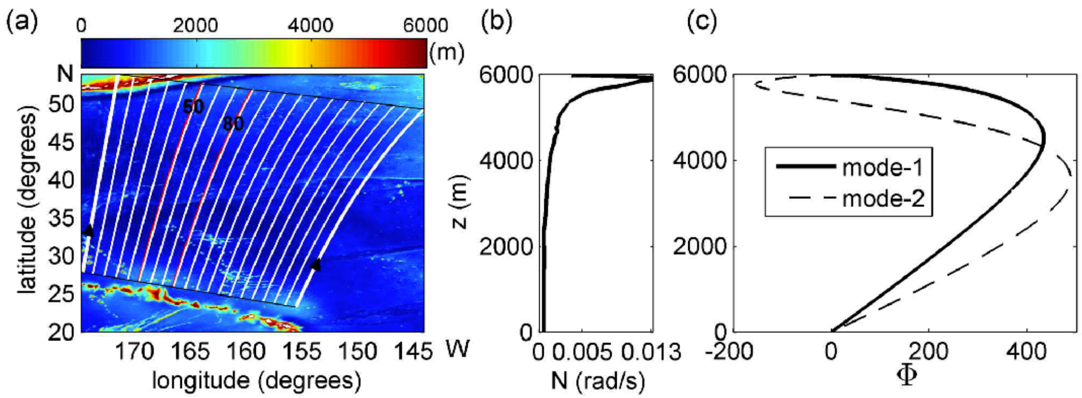


- The fraction of the energy flux scattered into the transmitted and reflected wave fields is a function of the criticality and the depth ratio, with critical topographies being the most efficient at scattering out of mode-1.

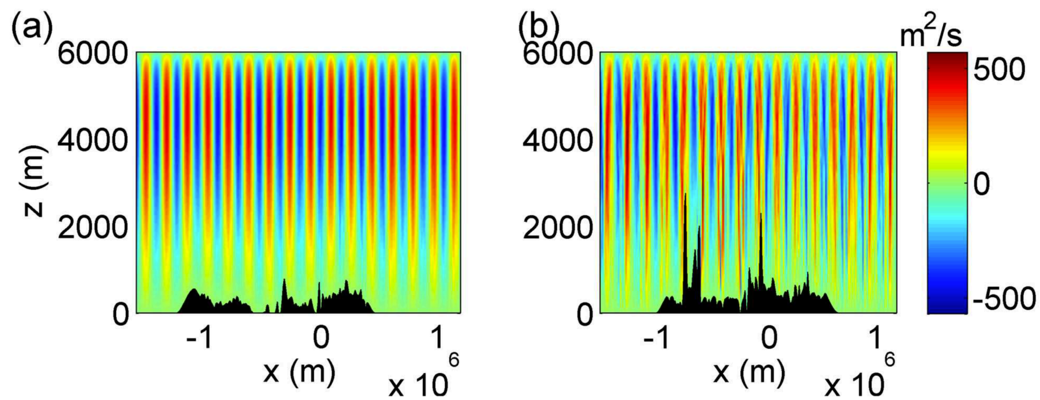


2. Topographic scattering

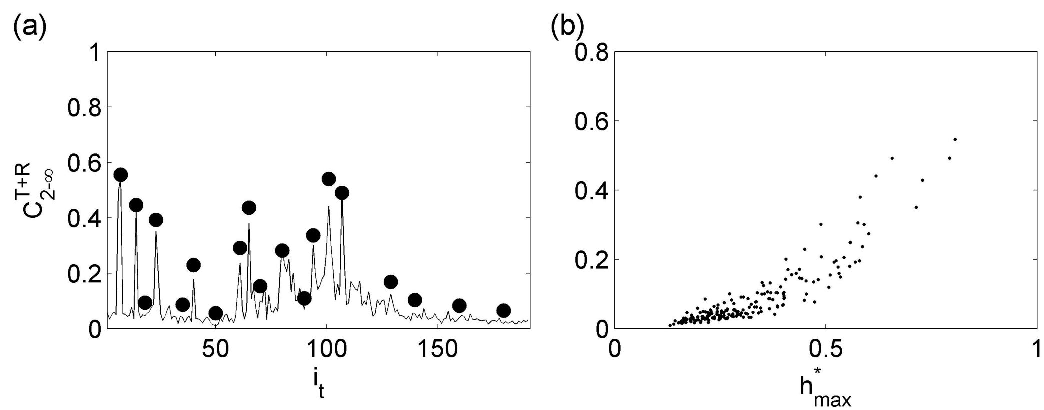
• Consider a mode-1 internal tide propagating north from Hawaii.



• Overall the scattering is very modest, accounting on average for 8% of energy flux.



• There is a very strong correlation between scattering efficiency and existence of tall, supercritical ridges.

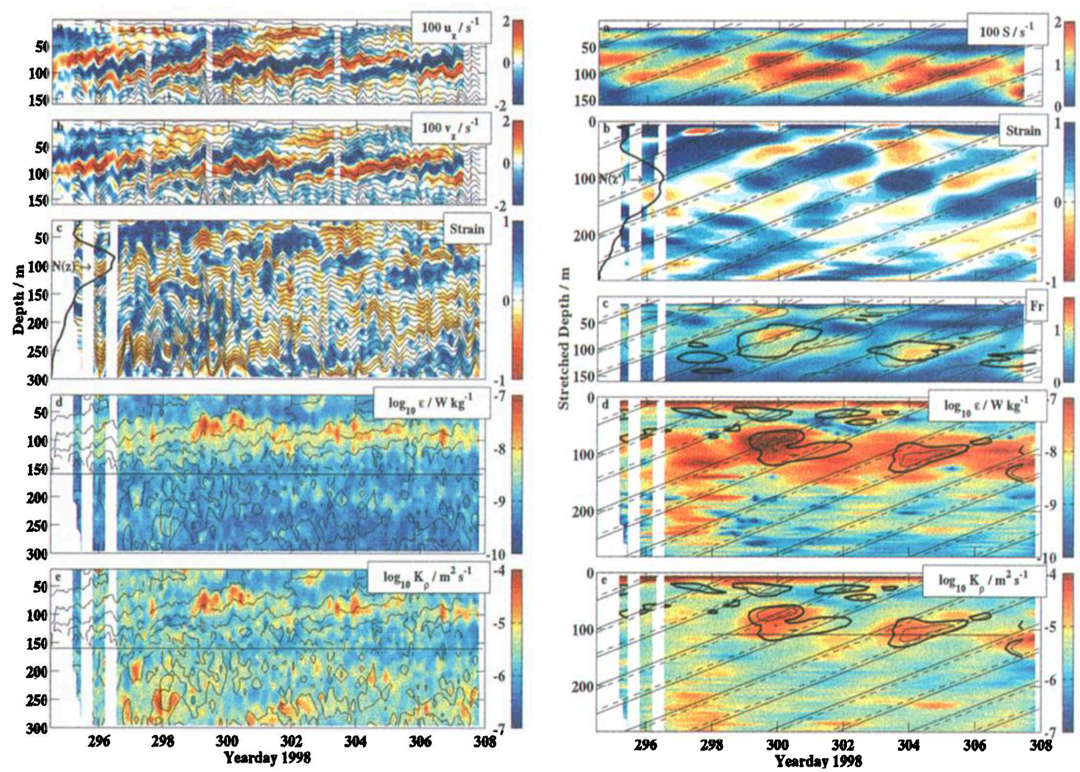


3. Interaction with pycnocline

- For downward and upward propagating inertia-gravity waves, the enhanced stratification near the ocean surface causes regions of enhanced vertical shear and strain that promote instability.

- The lowest Richardson number for a plane wave is expected at the stratification peak, which is where shear and strain of near-inertial waves is greatest, and this is where instability is observed.

<u>*Banda Sea*</u>

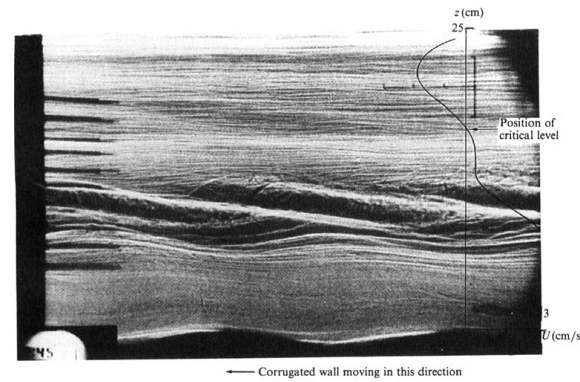


(Image: Alford & Gregg 2001)

4. Critical layers and mesoscale structures

- For a disturbance of frequency ω_{abs}, the Doppler shifted frequency, ω_0, is: $\omega_{abs} = \omega_0 + \vec{k}\cdot\vec{U}$

- The laboratory experiments of Koop (1986) clearly indicate the possibility of instability as an internal wave encounters a critical layer.

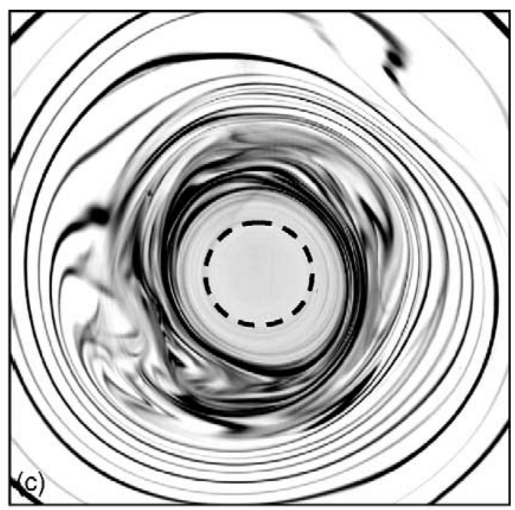


(Image: Koop 1986)

- More generally, in the ocean three-dimensional shear structures exist as mesoscale eddies and the associated shear structure can trap and focus inertia-gravity waves.

- This could be important for lee waves in the ACC.

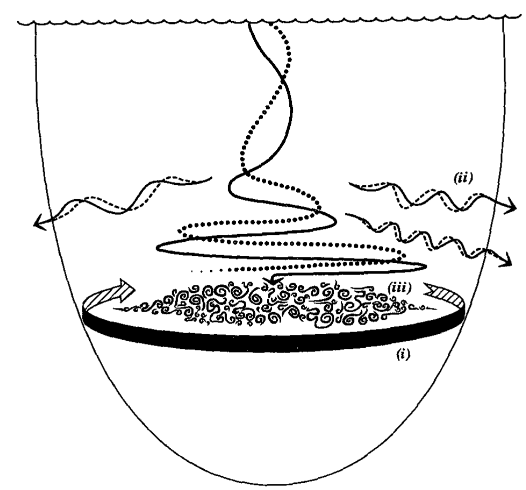
Laboratory Experiments



(Image: Moulin & Flor 2005)

Field studies

WRINCL (Gulf Stream)



(Image: Kunze et al. 1995)

5. Wave-wave interactions

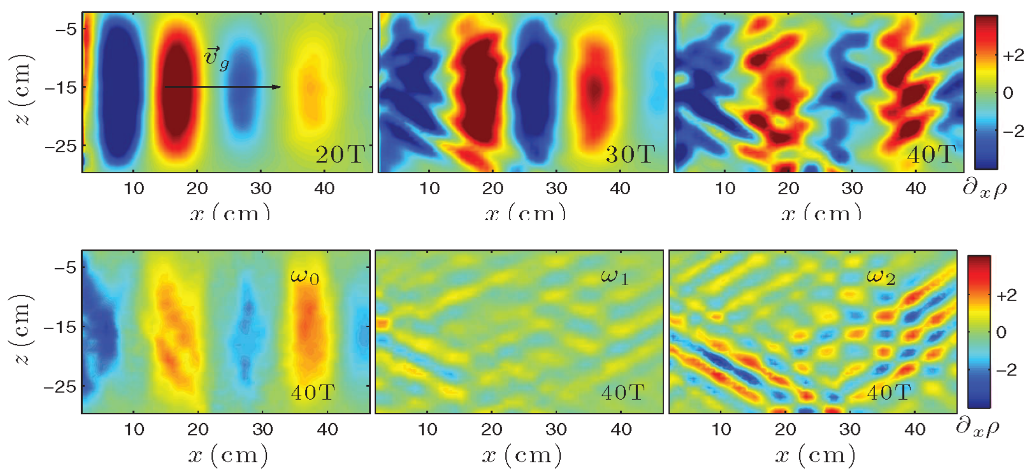
- Consider a wave field composed of several waves: $\vec{u} = \vec{u}_0 e^{i\left[\vec{k}_0\cdot\vec{x}-\omega t\right]} + \vec{u}_1 e^{i\left[\vec{k}_1\cdot\vec{x}-\omega t\right]} + \vec{u}_2 e^{i\left[\vec{k}_2\cdot\vec{x}-\omega t\right]} + \ldots.$

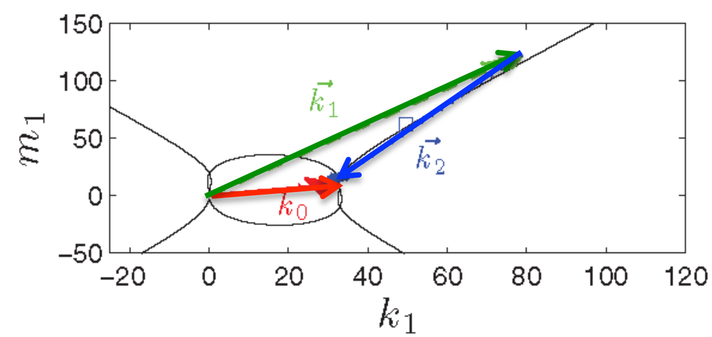
- Weakly nonlinear interaction through quadratic term “slowly” changes wave amplitudes and phases: $$\frac{\partial \vec{u}_0}{\partial t} = -\vec{u}_1.\nabla\vec{u}_2 - \ldots.$$

- Energy exchange is efficient if the following resonance conditions are satisfied: $$\omega_0 = \omega_1 + \omega_2 \quad \text{and} \quad \vec{k}_0 = \vec{k}_1 + \vec{k}_2.$$

- This Parametric Subharmonic Instability (PSI) transfers energy from a large scale to two smaller scale waves at close to half the frequency with lower group velocity.

- PSI is observed in laboratory experiments (e.g. Joubaud *et al* 2012), with good agreement for growth rates.

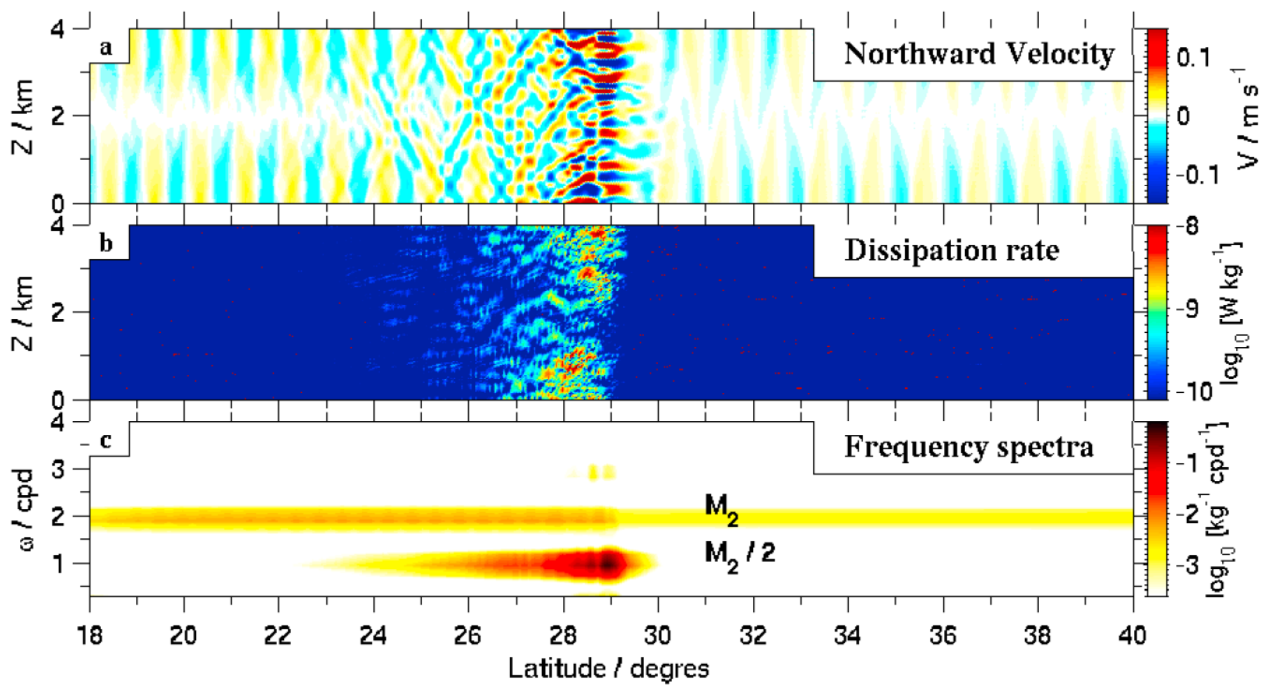




(Images: Joubaud et al 2012)

5. Wave-wave interactions

- Initially believed that PSI is slow (and thus not important) for energy transfer from mode-1.
- Numerical and theoretical studies suggest the process is more efficient at the critical latitude where $\omega_1 \sim \omega_2 \sim f$.

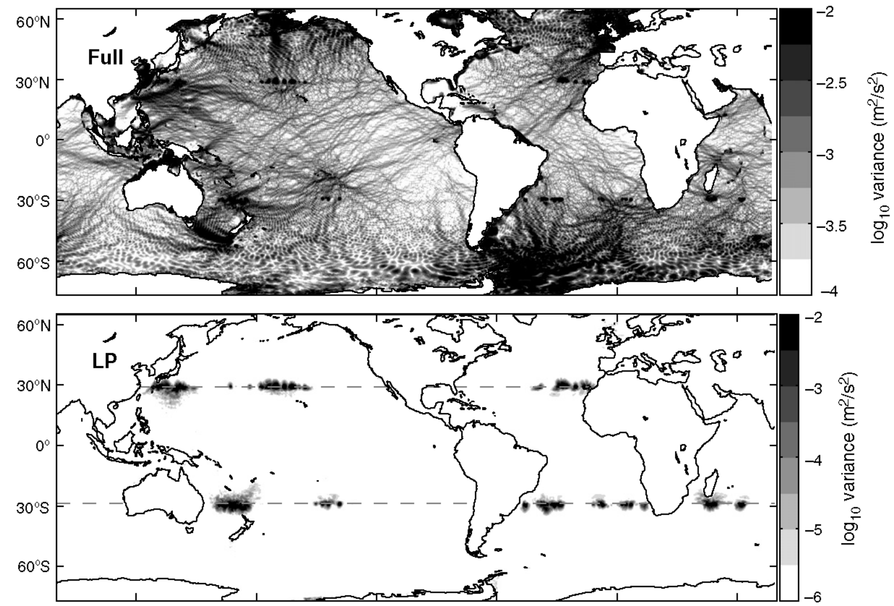


(Image: MacKinnon & Winters 2005)

5. Wave-wave interactions

- Realistic global simulations using M2 only barotropic forcing reveal key generation sites.

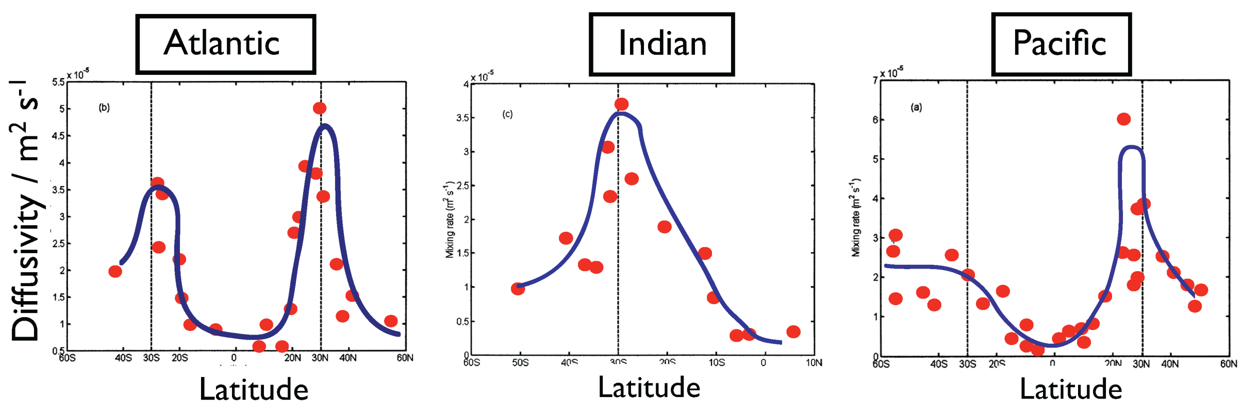
- Where strong internal tides cross the critical latitudes (28.8^o) there is amplification of subharmonics, high vertical wavenumber, high shear (Ri<1/4) and increased diapycnal diffusivity.



(Image: Simmons 2008)

- There is also tantalizing observational evidence, for example from analysis of satellite altimetry data.

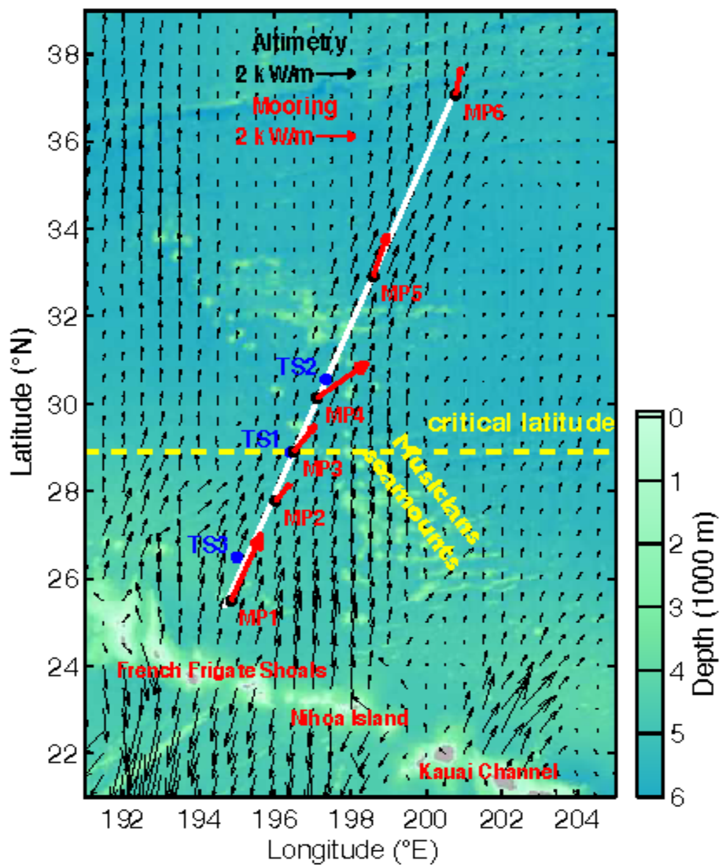
- Plots show latitudinal distribution of mixing rate caused by M2 internal tide.



(Image: Tian et al 2006)

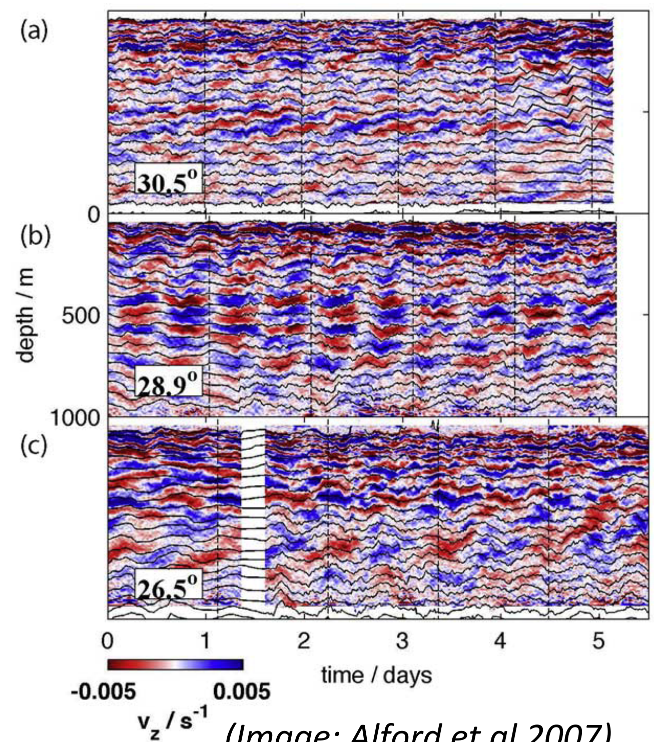
5. Wave-wave interactions

- The Internal Waves Across the Pacific (IWAP) field program set out to observe PSI at the critical latitude.



(Image: MacKinnon et al 2011)

- Observations of near-inertial standing waves consistent with PSI were observed.

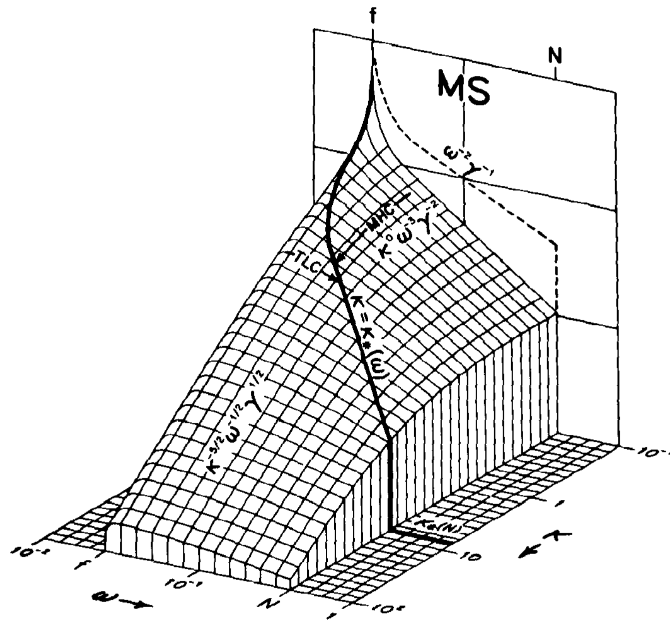


(Image: Alford et al 2007)

- Calculated energy transfer rates from the M2 internal tide, however, were modest, perhaps due to mesoscale structures and spring-neap cycle preventing 'catastrophe'.

5. Wave-wave interactions

- More generally, away from strong generation sites the ocean displays a somewhat universal distribution of energy in wavenumber and frequency space.



- Comprises an integrated energy spectrum in wavenumber that scales as ω^{-2}, which has been detected (e.g. Cairns & Williams 1976)
- The basic picture is that there is a steady cascade of energy from large to small vertical scales, where energy can be dissipated via wave breaking, the cascade being driven by wave-wave interactions (McComas & Bretherton 1977, Staquet & Sommeria 2002).

6. Solitary waves

- As low mode internal waves radiate from their generation site they undergo nonlinear steepening due to a balance between:

  1. Nonlinearity
  2. Non-hydrostatic dispersion
  3. Background rotation

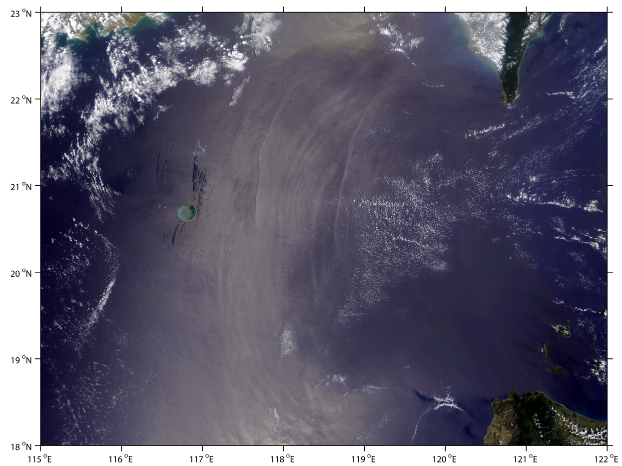
- The governing equation for a two-layer model of this process is the rotationally-modified KdV, or Ostrovsky, equation (Li & Farmer 2011):

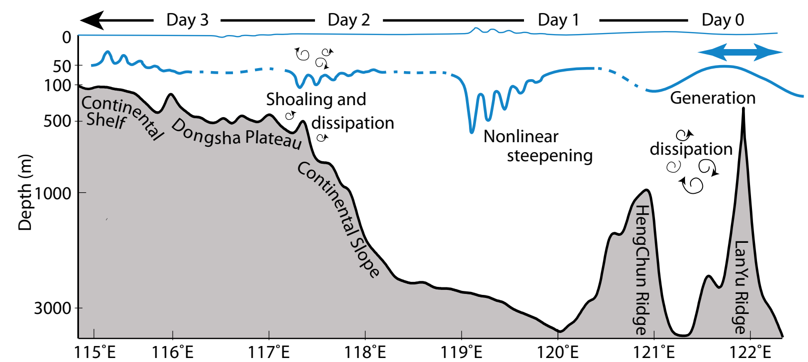
$$\frac{\partial}{\partial x}\left(\frac{\partial \eta}{\partial t}+c_0\frac{\partial \eta}{\partial x}+\alpha\eta\frac{\partial \eta}{\partial x}+\beta\frac{\partial^3 \eta}{\partial x^3}\right)=\frac{f^2}{2c}\eta \quad \text{where} \quad \alpha=\frac{3c_0(h_1-h_2)}{h_1h_2}, \beta=\frac{c_0h_1h_2}{6}.$$

- Both the non-hydrostatic dispersion and the background rotation counteract the nonlinear steepening.
- When the wave is initially a long-wavelength mode-1 wave, rotation dominates over non-hydrostatic dispersion and seeks to prevent steepening.
- Provided that is overcome, then the balance is between nonlinearity and non-hydrostatic dispersion in setting the shape of the wave.
- A fully nonlinear, two-layer model has been developed by Helfrich (2007) to overcome shortcomings of weakly nonlinear analysis.

6. Solitary waves

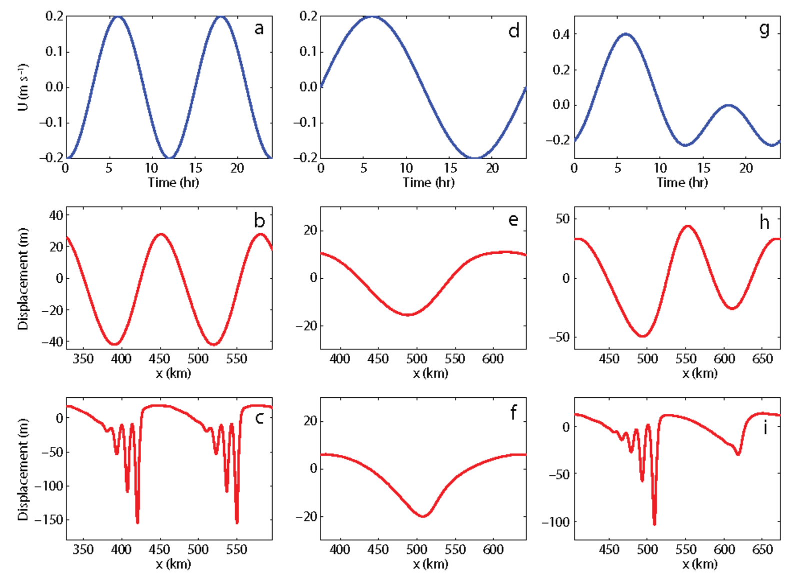
- An excellent example of nonlinear steepening is found in the South China Sea.





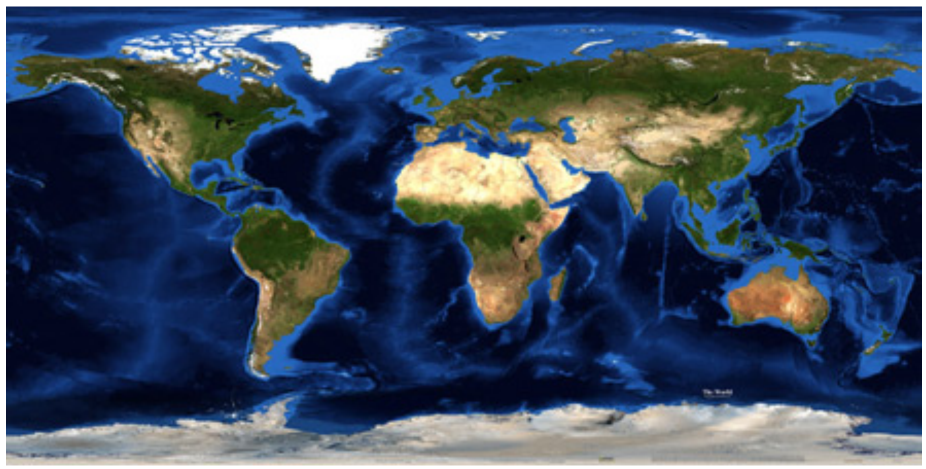
(Images: (left) Jackson (2012). (above) Simmons et al 2001)

- The shorter wavelength mode-1 M2 internal tide can overcome rotational dispersion, leading to the formation of sharp M2 solitary waves.
- The longer wavelength mode-1 K1 internal tide cannot overcome rotational dispersion, and does not significantly steepen.
- The K1 internal tide modulates the M2 internal tide to produce *A* and *B* waves.



(Images: Farmer et al 2011)

Processes at continental shelves



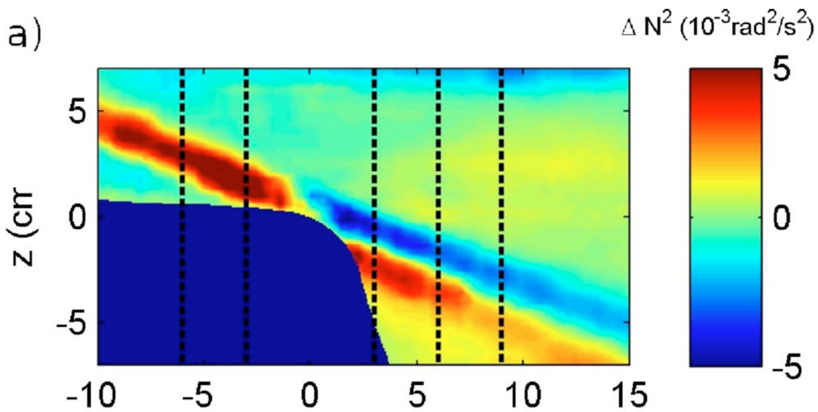
Processes at continental shelves

- There are several key processes at continental shelves, including:

1. Generation
2. Scattering
3. Steepening of solitary waves
4. Wave breaking

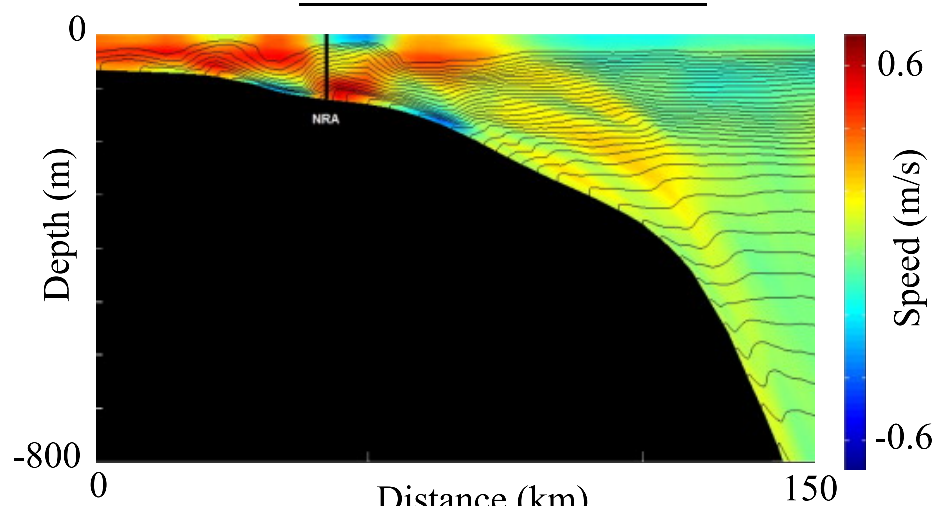
1. Generation

- As well as interacting with deep ocean topography, the barotropic tides interact with continental shelves to generate internal wave fields of a variety of forms.
- Linear internal tide generation can produce modes and internal tidal beams at continental shelves.



(Image: Gostiaux & Dauxois 2007)

North West Australia

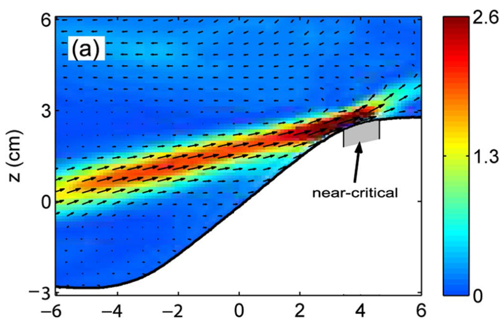


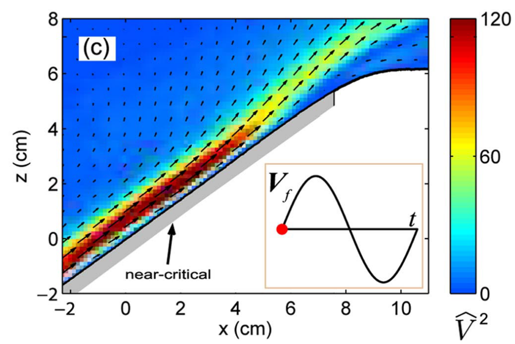
(Image: Van Gastel et al. 2009)

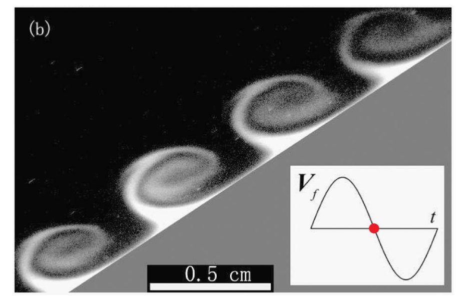
- Overall, tidal conversion at continental shelves is considered to be weak compared to the deep ocean because the "across-ridge" velocities are weaker.

1. Generation

- The interaction of barotropic tides with continental shelves can lead to a variety of local nonlinear processes.
- Extended regions of critical slope are locally strong generators of potentially unstable internal wave fields.

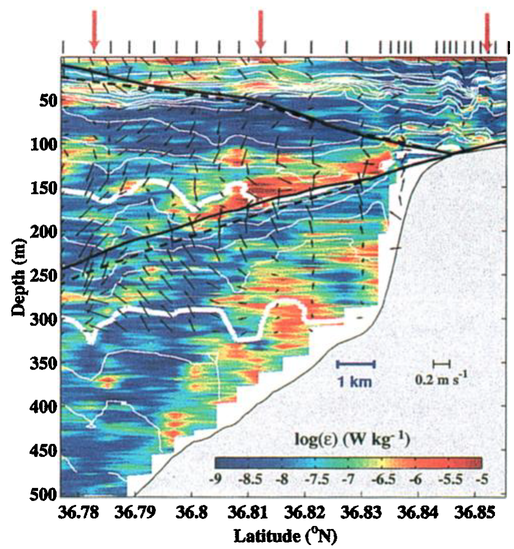




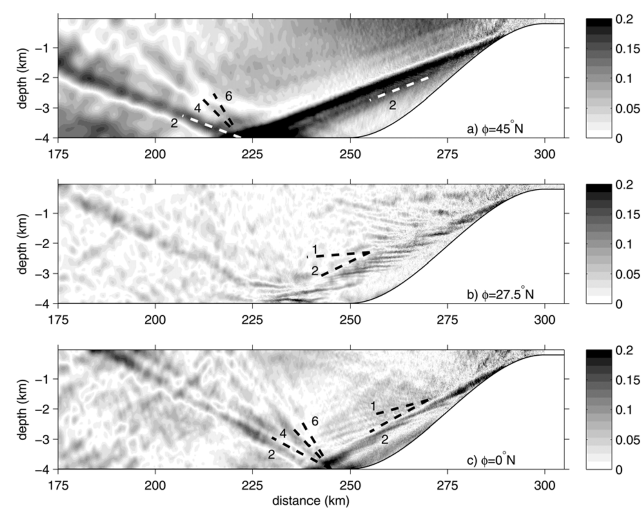


(Images: Zhang & Swinney 2008)

- The radiated wave fields can be unstable to all the aforementioned mechanisms such as wave-wave interactions and shear.



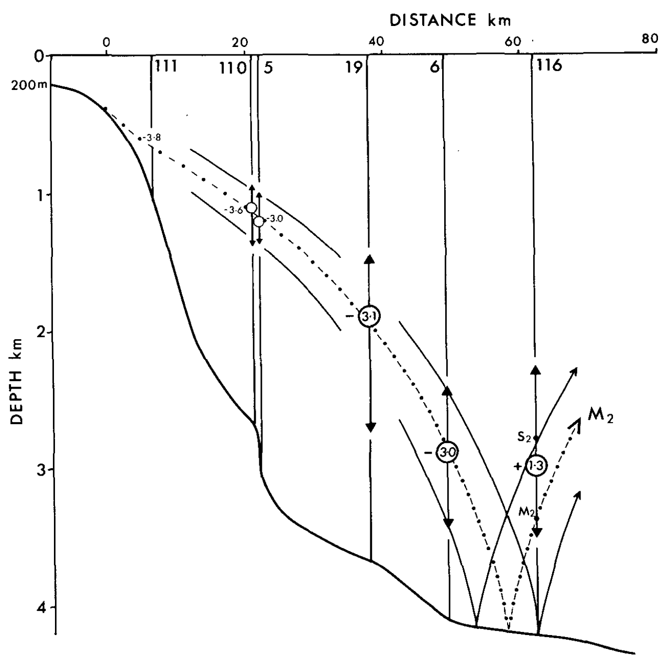
(Image: Lien & Gregg 2001)



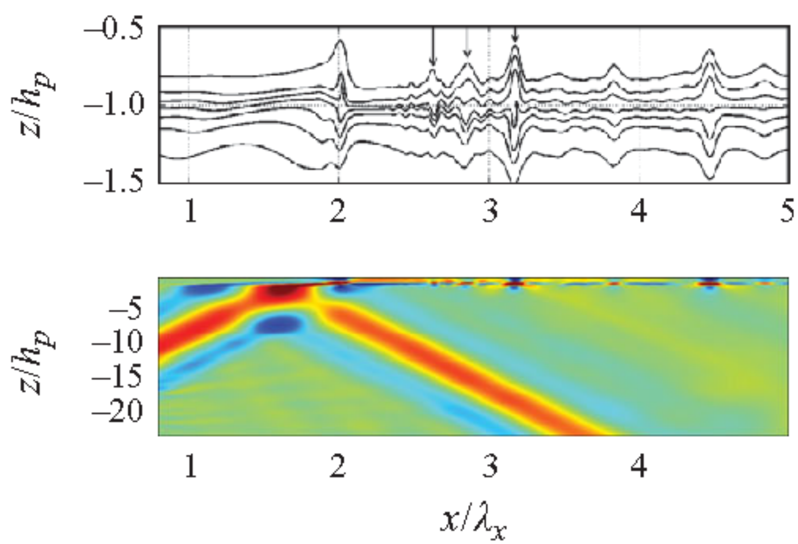
(Image: Gerkema, Staquet & Bouret-Aubertot 2006)

1. Generation

- An interesting consequence of internal wave beams that was first reported in the vicinity of continental shelves occurs when a wave beam strikes a pycnocline.
- Evidence of this process was reported by Pingree & New (1991) in the Bay of Biscay.
- Trains of nonlinear internal waves were observed to arise somewhat suddenly in mid-ocean, well away from any topography.
- The process has been studied analytically (Gerkema 2001, Akylas *et al* 2007), numerically (Grisouard, Staquet & Gerkema 2011) and experimentally (Mercier *et al* 2012) ⟶ phase speed of excited solitary waves matches horizontal phase speed of incident wave beam.



(Image: Pingree & New 1991)

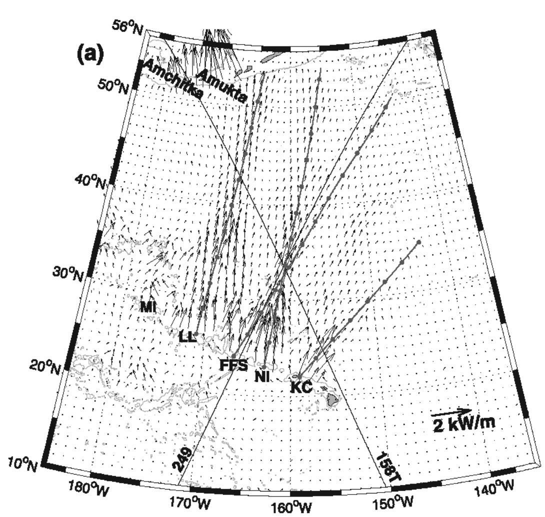


(Image: Grisouard, Staquet & Gerkema 2011)

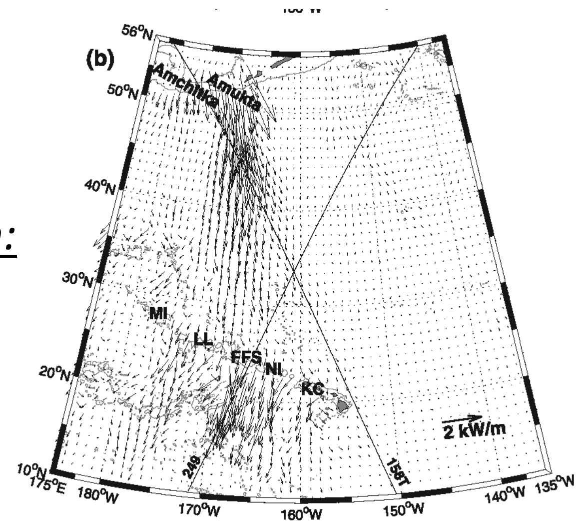
2. Scattering

- Low-mode internal tides have been observed to propagate thousands of kilometers from their generation sites.

North:



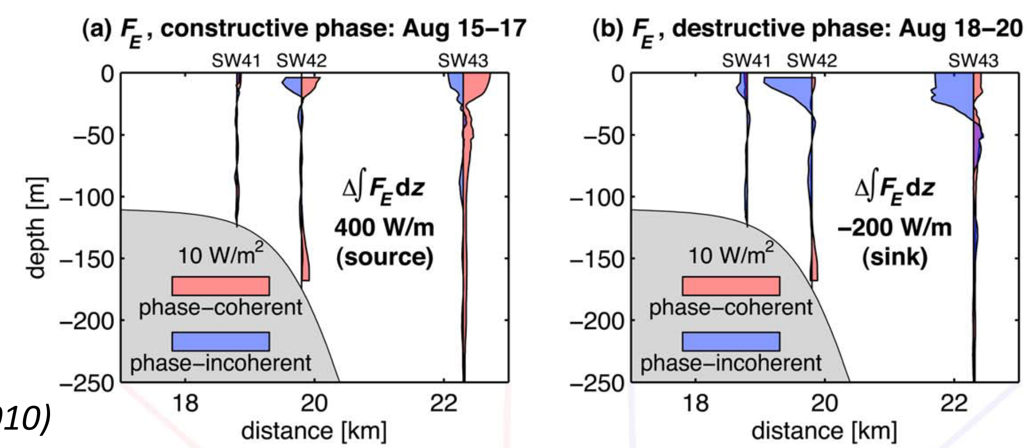
South:



(Images: Zhao & Alford 2009)

- Altimetry can only detect temporally coherent signals, which maybe disrupted by propagation through mesoscale eddy fields, for example (Rainville & Pinkel 2006).

- There is clear evidence of the arrival of internal tides from the deep ocean at continental shelves.

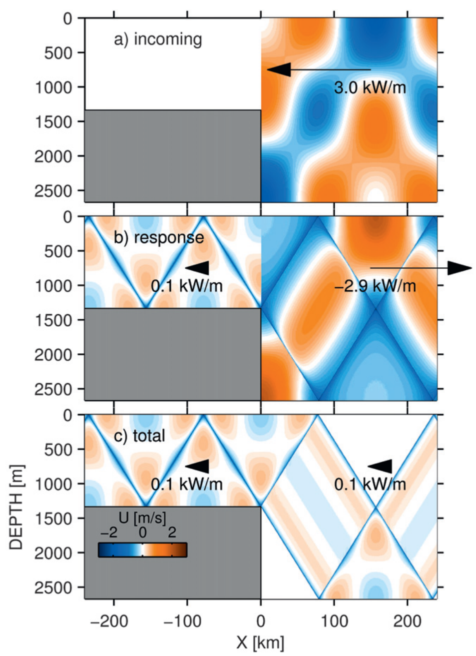


(Images: Kelly & Nash 2010)

2. Scattering

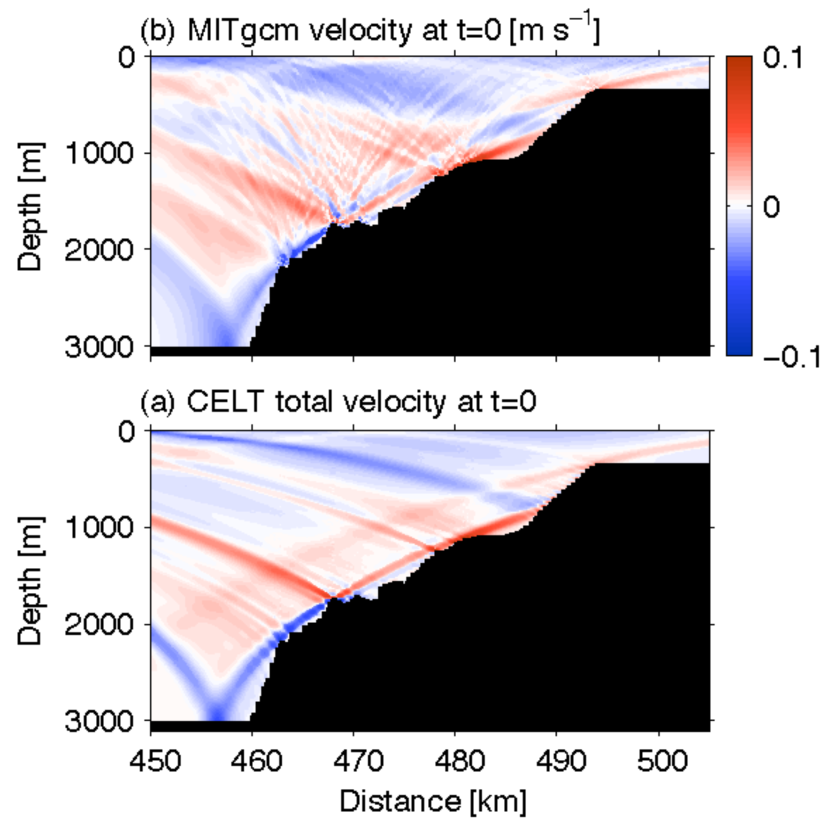
- For an internal tide impinging on a continental shelf, one needs to consider the scattering of this internal tidal energy.

- A simple, linear model of scattering is obtained by considering a 'step' and using vertical modes to enforce continuity across the step.
- The response turns out to be highly sensitive to the phasing of the incoming modes.



(Image: Klymak et al. 2011)

- A more sophisticated model allows investigation of more realistic topographic features.

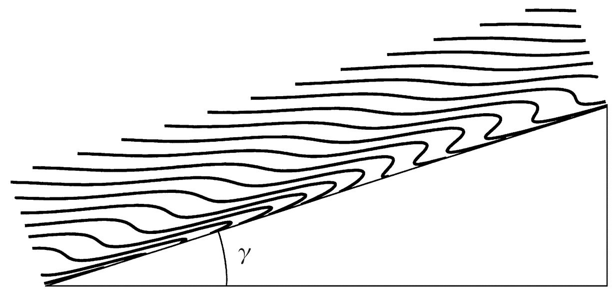


(Image: Kelly, Jones & Nash, submitted)

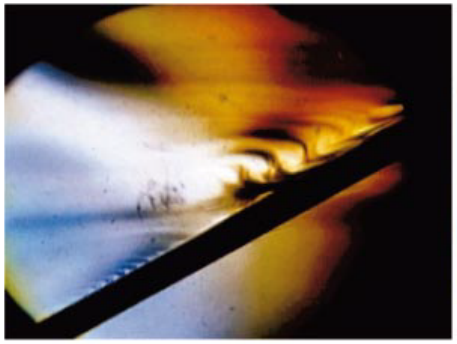
2. Scattering

- The interaction of an internal tide and a critical slope can give rise to intense internal wave activity.

- Theoretical and laboratory studies reveal the onset of buoyancy driven instability at critical slopes.

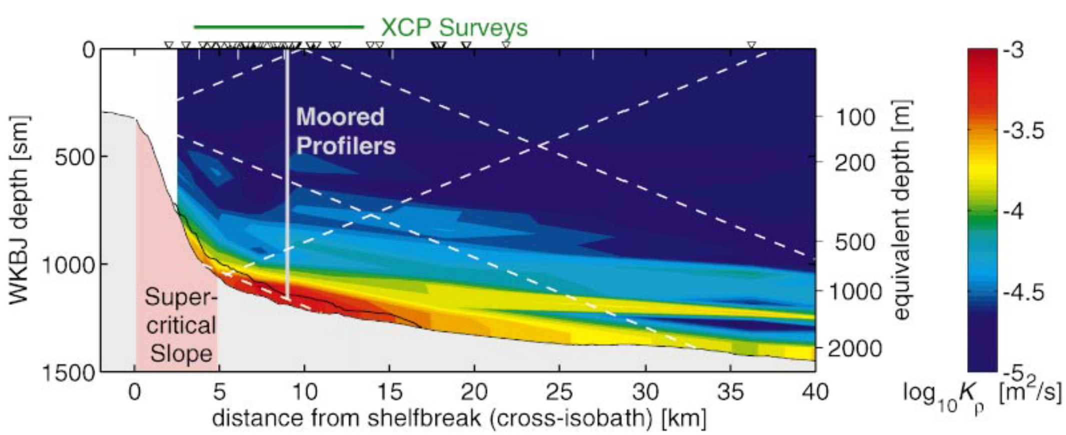


(Image: Dauxois & Young 1999)



(Image: Dauxois et al 2004)

- Field observations of a remotely generated M2 internal tide reveal greatly enhanced mixing in the region of critical slopes.



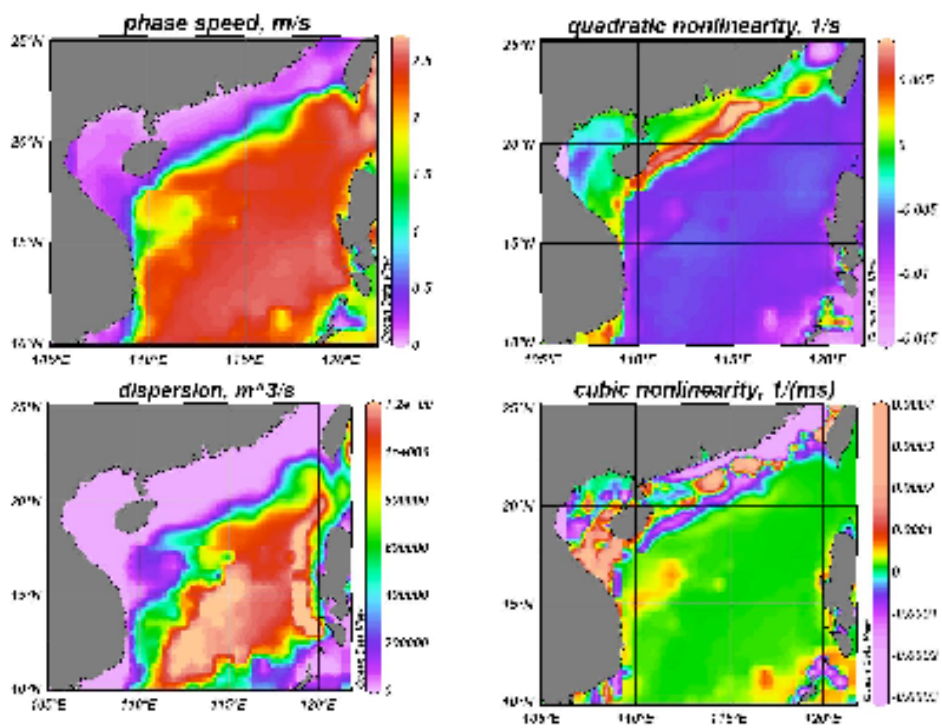
(Image: Nash et al 2006)

3. Steepening of solitary waves

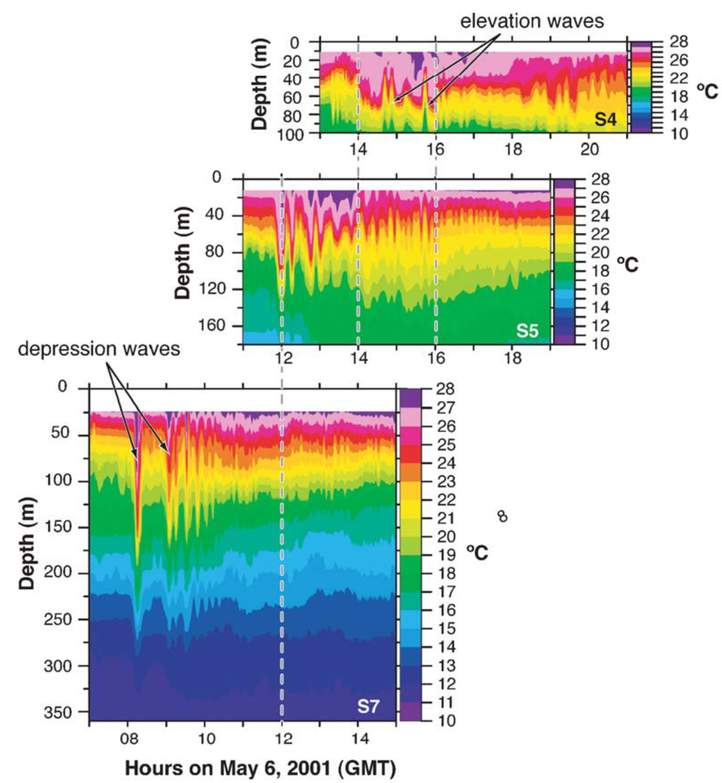
- When the background bathymetry and stratification vary (slowly) the extended, variable KdV (evKdV) equation can be used (Grimshaw *et al.* 2010):

$$\frac{\partial \eta}{\partial t} + c_0(x)\frac{\partial \eta}{\partial x} - c_0(x)\frac{Q_x}{Q}\eta + \alpha_0(x)\eta\frac{\partial \eta}{\partial x} + \alpha_1(x)\eta^2\frac{\partial \eta}{\partial x} + \beta(x)\frac{\partial^3 \eta}{\partial x^3} = 0.$$

- Internal waves can steepen substantially and even encounter a turning point (a change of sign of $\alpha_0(x)$), where waves of depression transform into waves of elevation.



(Image: Grimshaw et al. 2010)



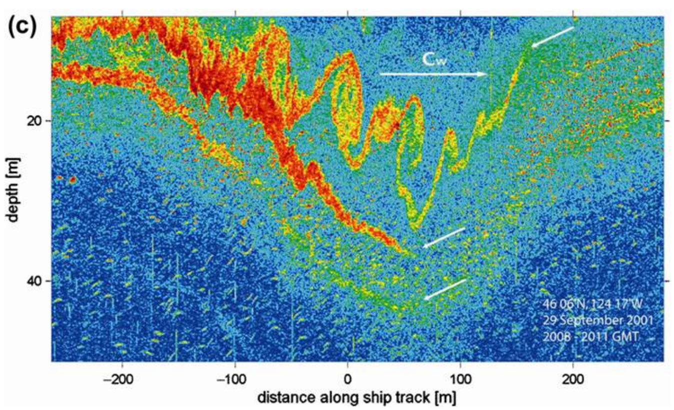
(Image: Lynch et al. 2004)

3. Steepening of solitary waves

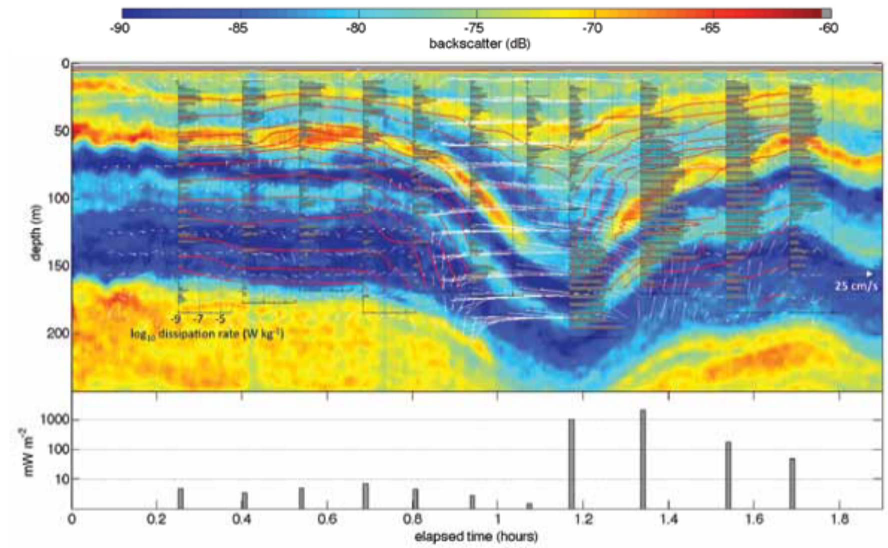
- When solitary waves approach continental shelves, dissipation can occur through a variety of means, including: (i) radiation damping, (ii) boundary layer, (iii) shear instability and (iv) buoyancy instability.

South China Sea

Oregon Shelf

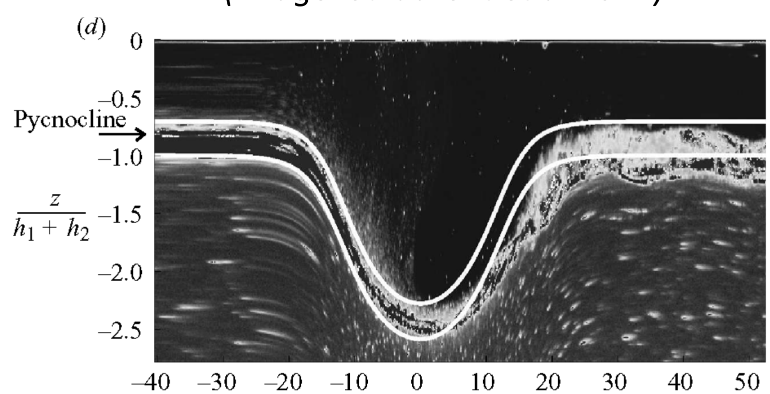


(Image: Moum et al 2003)



(Image: StLaurent et al 2011)

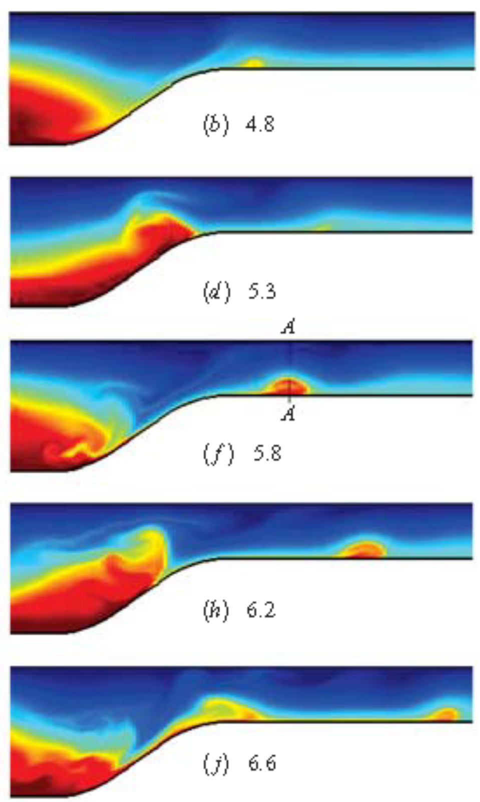
- This process of instability is observable in laboratory experiments.
- A Richardson number criterion is generally used to identify the onset of instability, but this isn't the complete story.



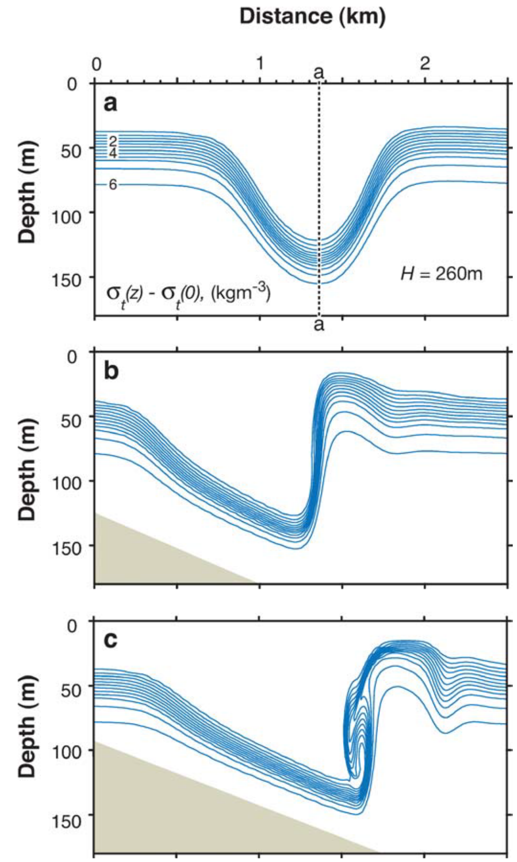
(Image: Fructus et al 2009)

4. Wave breaking

- When baroclinic tides or large amplitude solitary waves propagate onto continental shelves, wave breaking can occur.
- Laboratory experiments find mixing efficiencies of up to 25% (Michallet & Ivey 1999).



(Image: Venyamgamoorthy & Fringer 2007)



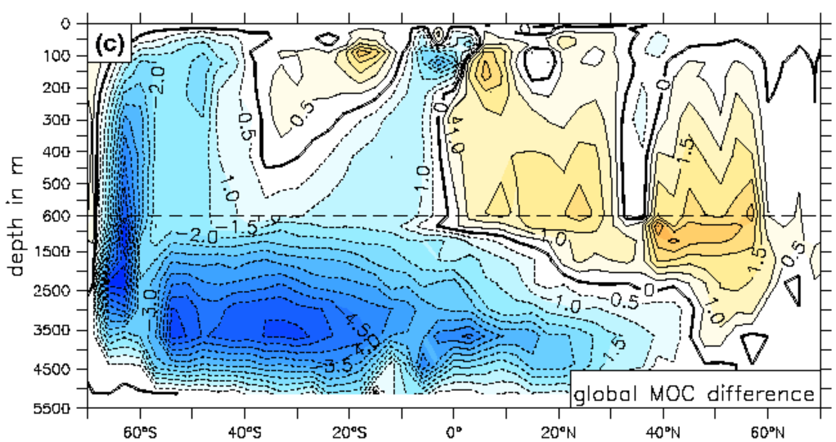
(Image: Vlasenko & Hutter 2002)

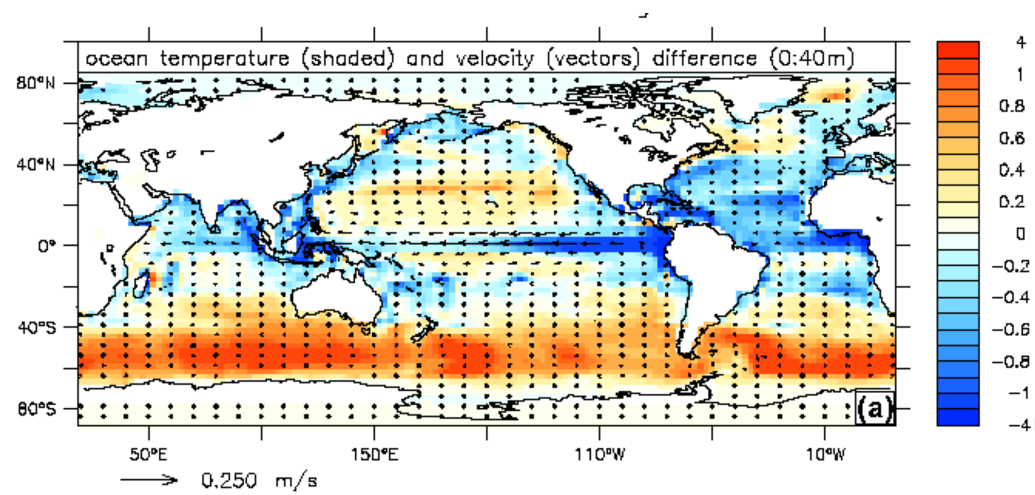
- In many cases boluses of fluid to propagate up on to the shelf.

Global picture

Global picture

- There are significant energy fluxes, on the order of 1TW, being input into the ocean by topographically generated internal tides and surface generated near-inertial waves.
- This 1TW+1TW=2TW of energy is believed to be important to maintaining the large scale circulation of the ocean by virtue of its impact on vertical diffusivity.
- The energy flux is dissipated somewhere and somehow, but there are a range of processes in the deep-ocean and at continental shelves that are possible sinks.
- At present there is no one process or location at which dissipation appears to be dominant, leading to the notion of ‘patchy mixing’.
- This makes things challenging because the large scale behavior of ocean models is sensitive to the distribution of mixing.





(Images: Friedrich et al 2011)

Outstanding questions

- There are many outstanding questions to address to obtain a more complete picture on the role if internal waves in the ocean, including:

    1. Can we more clearly define the energy flux associated with near-inertial waves?
    2. Are there other sources of energy flux that still need to be seriously considered (e.g. lee-waves in the ACC, loss of balance, high frequency waves generated by the mixed layer)?
    3. Is anything really going on at the ‘critical’ latitudes?
    4. What fraction of low-mode energy flux makes it all the way to continental shelves?
    5. Are there one or two dominant dissipation processes or do low-mode internal waves suffer death by a thousand cuts?
    6. Can this knowledge be incorporated into large scale numerical models?
    7. What will be the impact of enhanced internal wave activity in the Arctic Ocean?


The End
WB

On piste