Instabilities and mixing in stellar interiors

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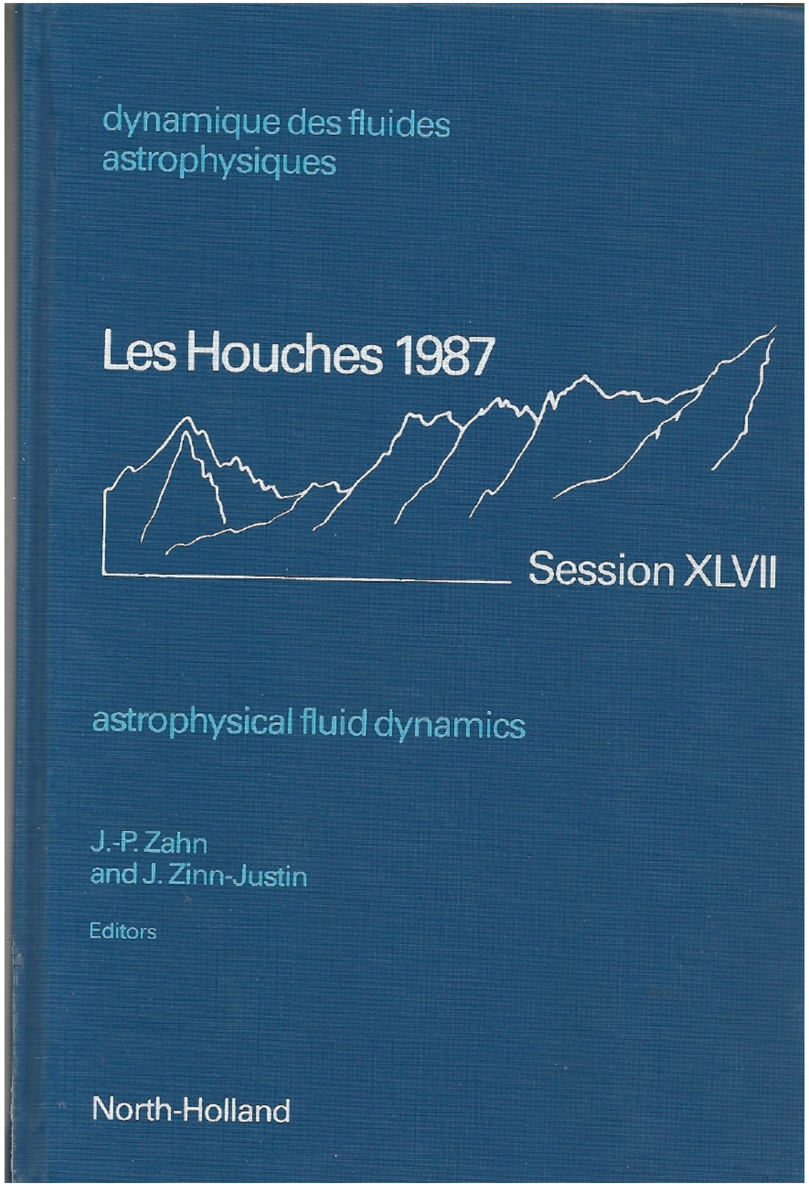


First school held in France on astrophysical fluid dynamics

Lecturers

D.O. Gough
M. Lesieur
A. Pouquet
P.H. Roberts
E.A. Spiegel
O. Thual
J. Toomre
J.-P. Zahn

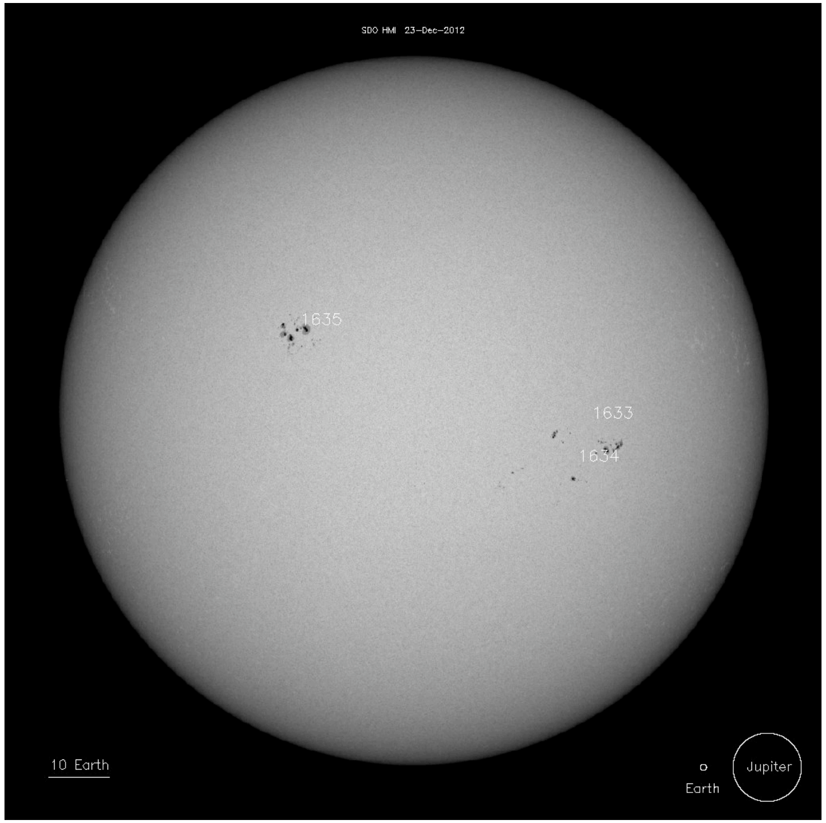
Participants

A. Brandenburg
C. Catala
P. Drossart
B. Dubrulle
A. Fowler
L.N. Howard
V. Karas
J.B. Keller
W. Kley
N. Lebovitz
R. Lehoucq
J. Léorat
H. Muthsam
M. Rieutord
C. Rosenthal
S. Zaleski
etc.

The Sun :
our nearest star

To first approximation :
self-gravitating sphere
in hydrostatic equilibrium

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2} \qquad \Rightarrow \frac{P_c}{R} \approx \rho_c \frac{GM}{R^2}$$



→ at surface (through spectroscopy) :

ionized gas, T ≈ 6000 °K

composition H 73.3% He 24.9% [O, C, Fe, Ne, N, …] 1.8%

If perfect gas $P = \frac{\Re \rho T}{\mu}$ central temperature $T_c \approx \frac{\mu}{\Re} \frac{GM}{R} \approx 15\, 10^6 \, °K$

Heat source: nuclear fusion H → He

Solar granulation

Center of granules:
hot, rising

Size $\approx$ 1,000 km (1.3 arcsec)

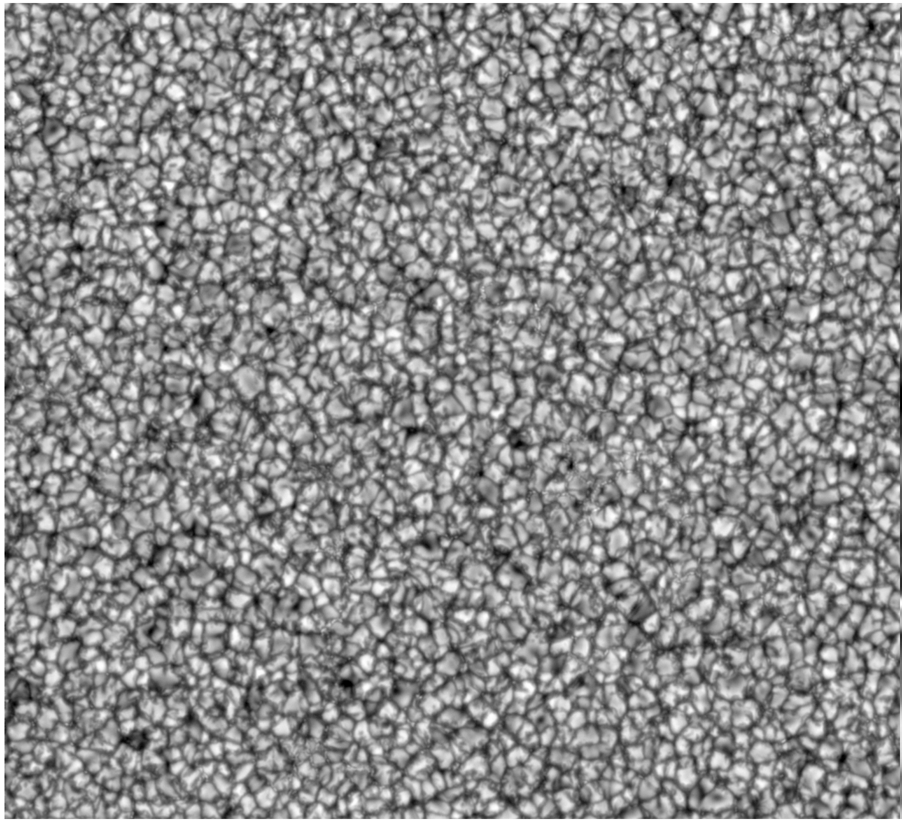
Intergranular lanes:
cool, descending

Life time $\approx$ 10 mn

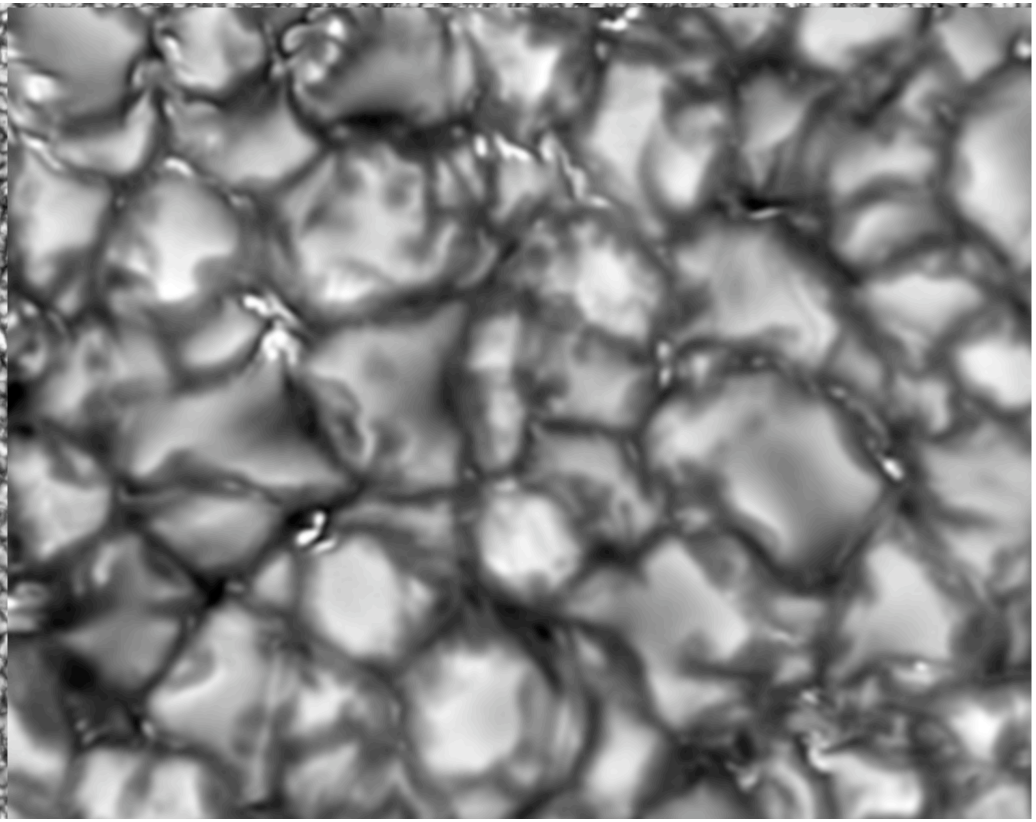
Speed: a few km/s

$\Rightarrow$ thermal convection

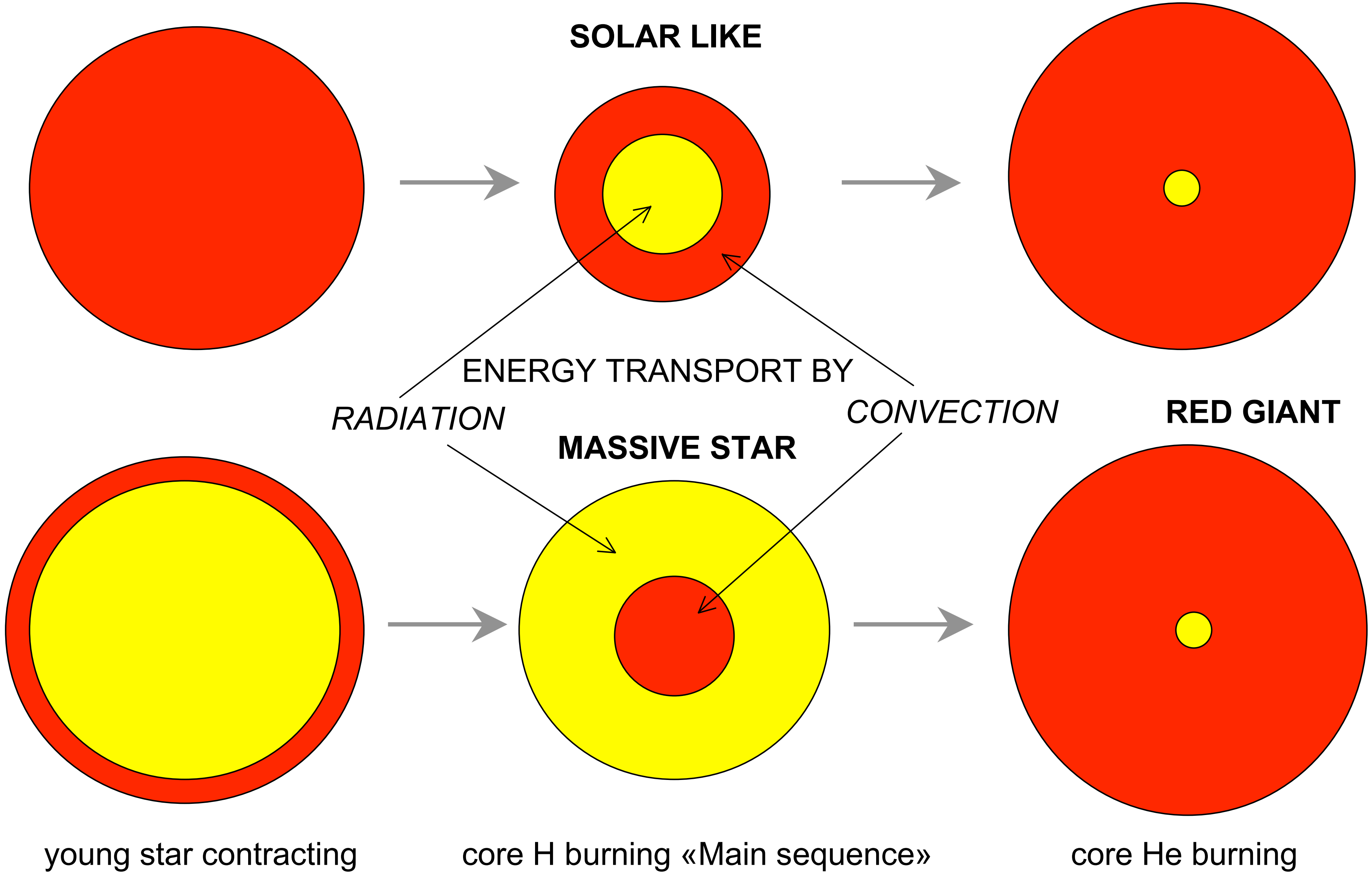
HINODE/SOT



SST



Stellar structure



How such models are built
Mixing-length treatment of convection

Instability when $\frac{d\ln T}{d\ln P} > \left(\frac{\partial \ln T}{\partial \ln P}\right)_{ad}$ $\nabla > \nabla_{ad}$

Boehm-Vitense 53, 58

Convective heat transport (assuming turbulent diffusion)

$$F_{conv} = -K_{conv}\rho T \frac{dS}{dr} = \frac{K_{conv}\rho C_P T}{H_P}[\nabla - \nabla_{ad}]$$

Convective velocity - a crude estimate:
buoyancy work over mean free path Λ (mixing length)

$$\frac{w^2}{2} \approx -\int_0^\Lambda g\frac{\delta\rho}{\rho}dr \approx \int_0^\Lambda g\varphi\frac{\delta T}{T}dr \approx \frac{g\varphi}{H_P}[\nabla - \nabla_{ad}]\frac{\Lambda^2}{2}$$

$$\varphi = -(\partial\ln\rho/\partial\ln T)_P$$

Convective diffusivity $K_{conv} = w\,\Lambda = \left(\frac{g\varphi}{H_P}\right)^{1/2}[\nabla - \nabla_{ad}]^{1/2}\Lambda^2$

Usual recipe : $\Lambda = \alpha H_P$ $H_P = P/\rho g$ α calibrated through observations

Scaling $F_{conv} \propto [\nabla - \nabla_{ad}]^{3/2}$

Crude treatment
but still much in use

1D stellar models
with mixing-length treatment of convection

$$\boxed{\begin{aligned}\frac{d\ln P}{dr} &= -\frac{\rho}{P}\frac{GM_r}{r^2}\\ \frac{dM_r}{dr} &= 4\pi r^2\rho\\ \frac{dL_r}{dr} &= 4\pi r^2\rho(\varepsilon+\varepsilon_g)\\ \frac{d\ln T}{dr} &= \frac{d\ln P}{dr}\nabla\end{aligned}}$$

Stellar structure equations

$$\varepsilon_g = -T\frac{\partial s}{\partial t} \qquad P(\rho,T) \quad \kappa(\rho,T) \quad \varepsilon(\rho,T)$$

$$K_{rad}(\nabla_{rad}-\nabla) = K_{conv}(\nabla-\nabla_{ad})$$

$$\nabla_{rad} = \frac{3}{64\pi\sigma}\frac{P\kappa}{T^4}\frac{L_r}{GM_r} \qquad K_{rad} = \frac{16\sigma}{3\rho\kappa}\frac{T^3}{\rho C_P} \qquad K_{conv} = w\,\Lambda$$

$$\mathrm{Pe} = K_{conv}/K_{rad} = w\,\Lambda/K_{rad}$$

Pe $\gg 1$ efficient convection $\nabla \rightarrow \nabla_{\mathrm{ad}}$

Pe $\ll 1$ inefficient convection $\nabla \rightarrow \nabla_{\mathrm{rad}}$

Why the Mixing-Length treatment is so successful

- simple prescription, easy to implement
- requires modest computer ressources
- provides all what is needed to build a model of stellar interior:
  the specific entropy profile of the convection zone

$\Rightarrow$ explains why it is still used

and why one still tries to improve it

"There appears to be no better convection theory emerging
that might be applicable to stars in the foreseeable future;
the mixing length is likely to stay with us for some time."

D. Gough 1976, IAU coll.39, Problems of stellar convection

Shortcomings of M-L models

- involves free adjustable parameter(s)

  efforts to remedy this: Canuto & Mazzitelli 1991

  calibration by hydro calculations Ludwig et al. 1997

- local prescription, unable to capture overshoot

  remedies: Maeder 1975, Roxburgh 1978, Zahn 1991
  Canuto, Goldman & Mazzitelli 1996

- very crude description of turbulence,
  difficult to use to describe coupling
  with rotation, pulsation, magnetic field

- needs additional parameters to predict spectral lines
  (micro & macroturbulence)

- lines of different formation depth require different α

- α a function of depth ?

How to improve the modeling of CZ?

Convection is genuinely a 3-dimensional phenomenon

⇒ it should be treated by solving the full HD or MHD equations

<u>Difficulties</u>

- strong stratification (14 H_P - x 250 in T)
- vast range of temporal scales (mn → 10 yrs)
- vast range of spatial scales (m → Gm)

<u>Remedies</u>

- filter out sound waves:
  anelastic approximation (Gough 1969)
- enhanced viscosity (DNS)
- numerical hyperviscosity (LES)
- subgridscale turbulence

Early 1970's - the dawn of solving the full equations…

thanks to the supercomputers

1.5D Boussinesq (Gough, Spiegel, Toomre)
1.5D anelastic (Latour, Spiegel, Toomre, JPZ)
2D Boussinesq (Weiss, Galloway)
2D compressible (Nordlund)
3D Boussinesq global (Busse)
3D compressible (Graham 1975)

IBM 360/95 - the supercomputer of the 70's

Only 2 were built by IBM especially for NASA:
one was at Greenbelt, Md
the other at GISS in NYC

IBM 2250 monitor
with its lightpen

"over 330 millions of 14-digit multiplications in one minute!"

i.e. 100 times slower than a current laptop - 10^8 times slower than a petaflops computer

First step toward 3-D models

Modal treatment to mimic 3 D

Modal expansion

$$T'(x,y,z,t) = \sum f_k(x,y)\; T_k(z,t)$$

$f_k(x,y)$: horizontal pattern, periodic in x and y

characterized by wavenumber a_k $\qquad \nabla^2 f_k(x,y) = -a_k^2 f_k(x,y)$

and n.l. interactions by coupling constants $C^{klm} = \frac{1}{2} < f_k f_l f_m >$

keep all non-linear terms, which depend on a_k and C^{klm}

first applied to laboratory convection (Boussinesq)

Gough, Spiegel & Toomre, 1975; Toomre, Gough & Spiegel 1977

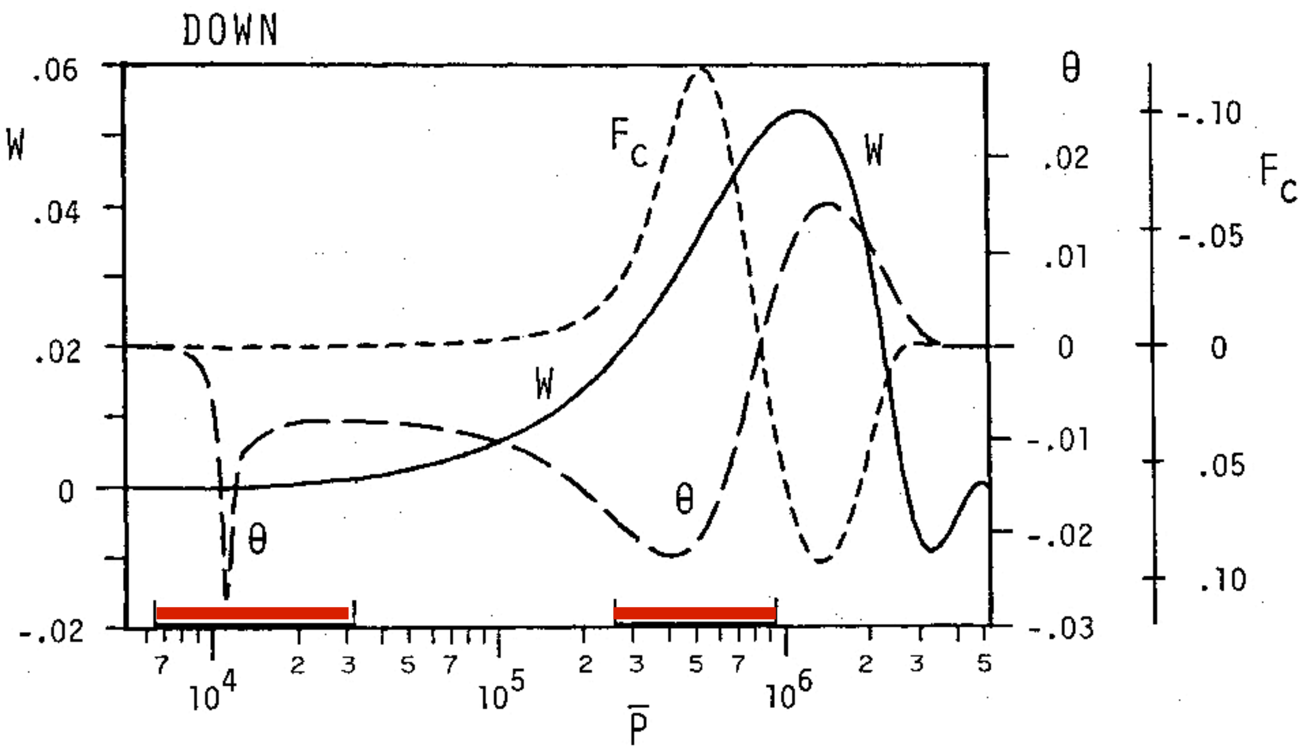
then to stellar convection Latour, Spiegel, Toomre & JPZ 1976, 1981

Modal treatment of stellar convection

Application to A-type stars - anelastic approximation

why? mild stratification, inefficient convection (low Péclet number)

⇒ the 2 convection zones are linked by overshoot (downdrafts)

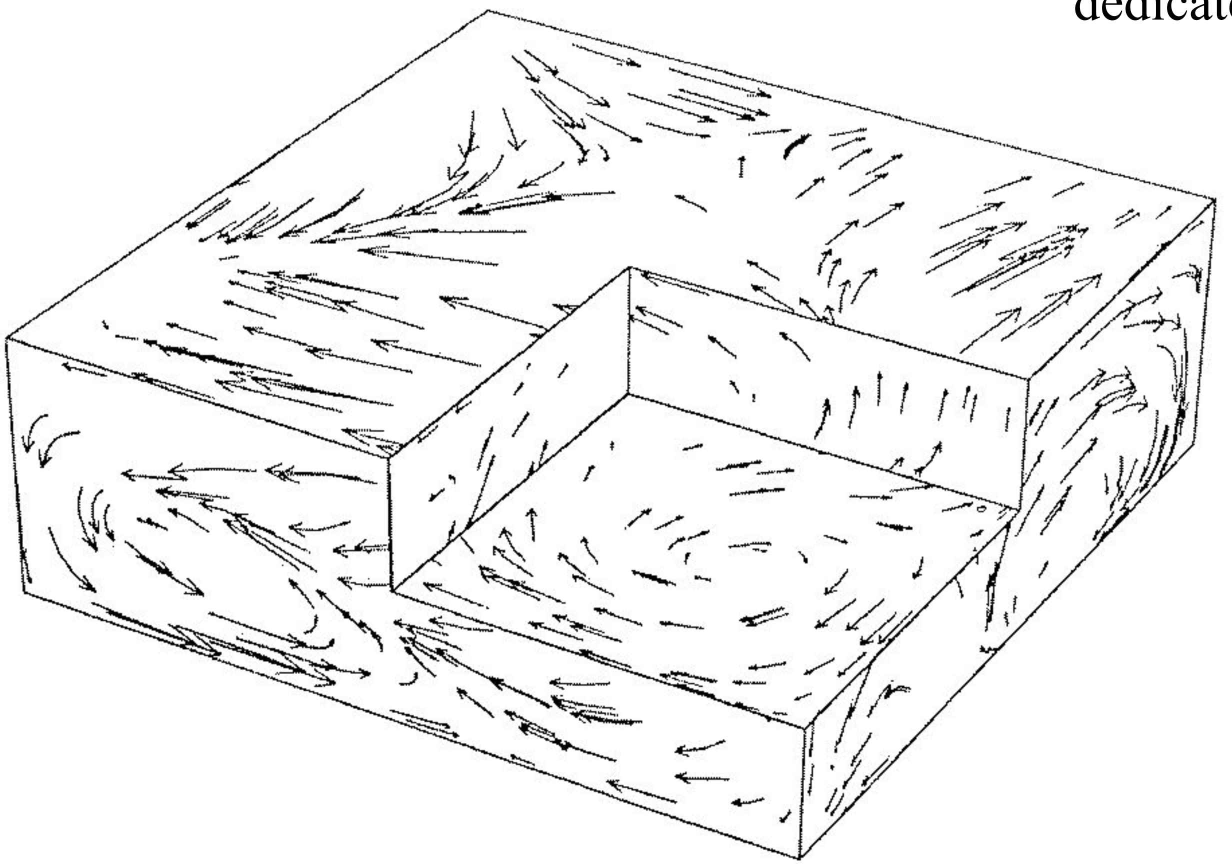


Toomre, JPZ, Latour & Spiegel 1976; Latour, Toomre & JPZ 1981

First 3D simulation of convection (fully compressible)

Graham 1975

displayed at IAU Coll. 38
dedicated to stellar convection
Nice 1976



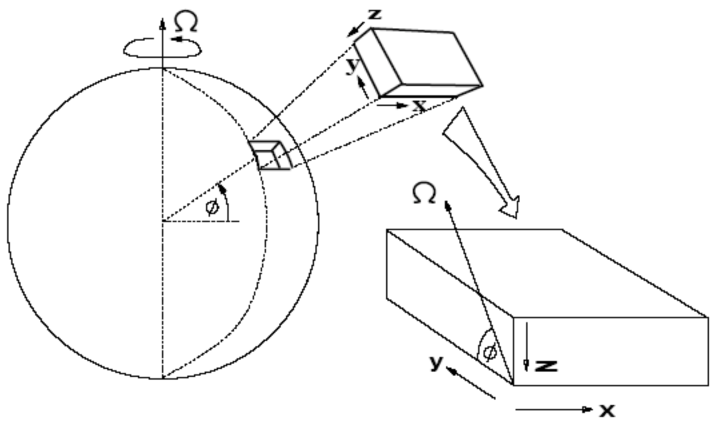
$T_{bottom} = 2\ T_{top}$

$Ray = 10\ R_{crit}$

2 decades later : high resolution 3D simulations

In Cartesian geometry (f-plane):

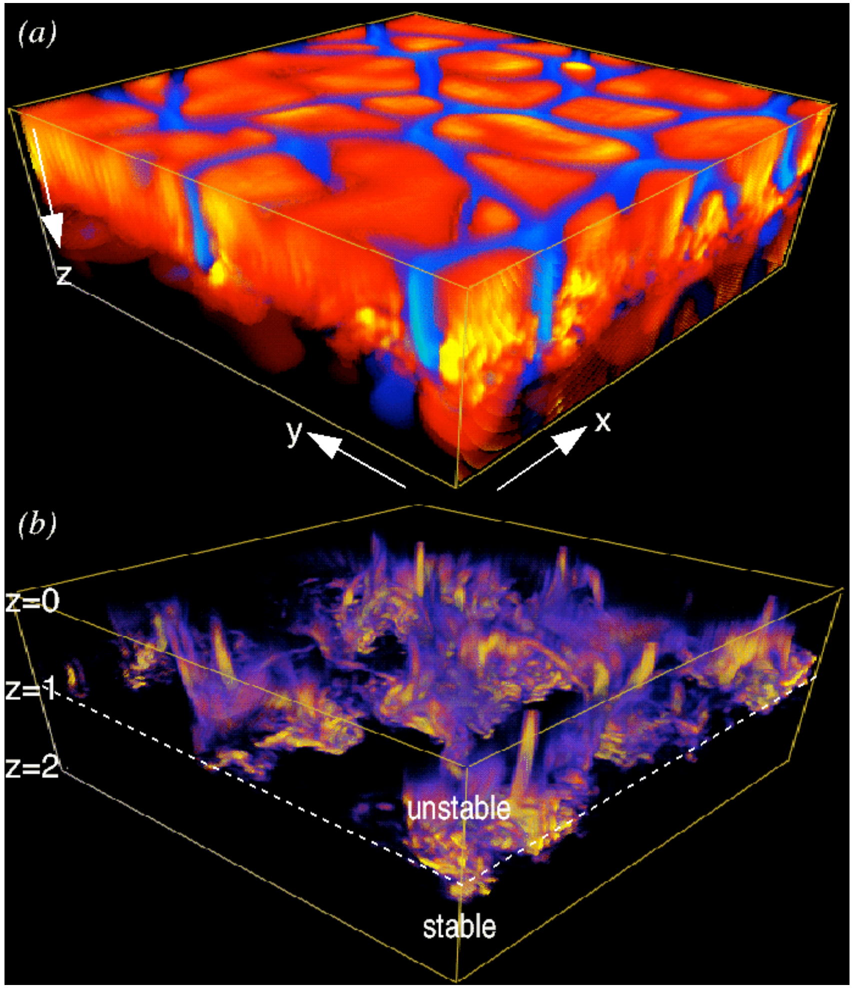
penetration, effect of rotation (Brummell)



Deep bulk convection - Cartesian geometry

512x512x576

Cattaneo, Hulburt & Toomre 1990
Muthsam et al. 1995
Chan & colleagues 1990's
Brummell, Clune & Toomre 2002 →



all aimed at better understanding
turbulent convection,
overshoot, etc.

crude treatment of radiation transfer
(diffusion approx.)

→ cannot be used
to predict emergent spectrum

Models aimed at better rendering of atmosphere

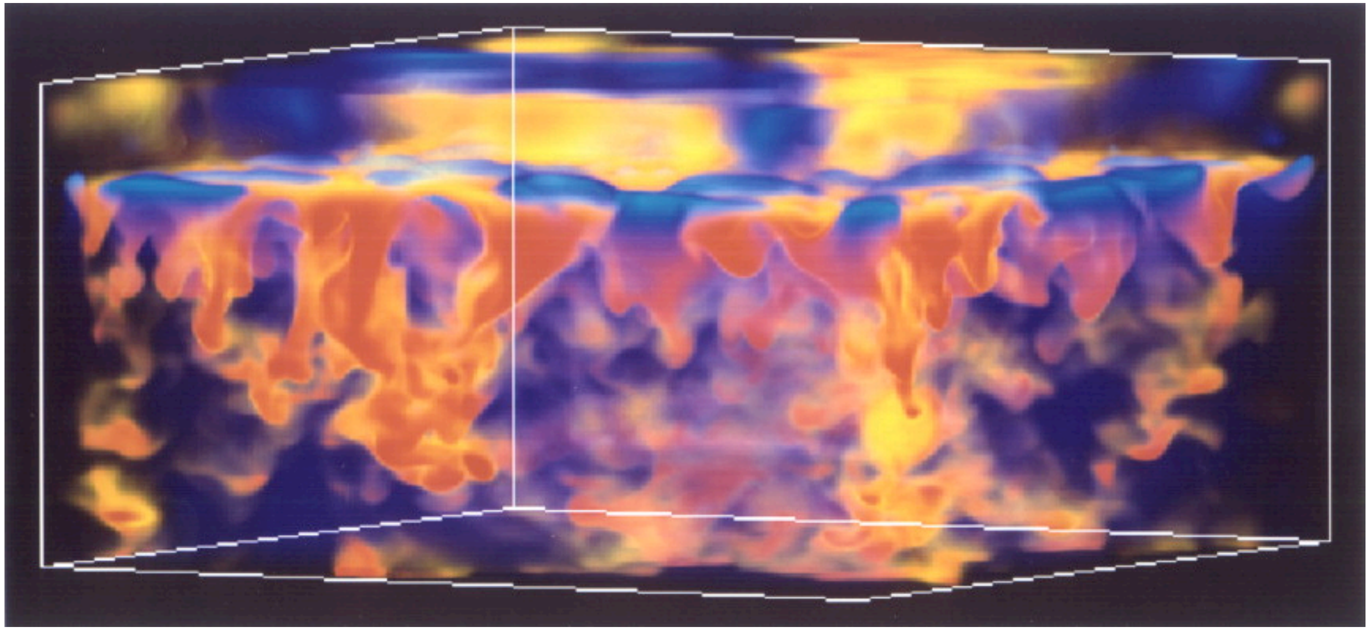
3D compressible

Nordlund 1982, 1985
Stein & Nordlund 1998

- Large Eddy Simulations with hyperviscosity
- realistic radiative transfer

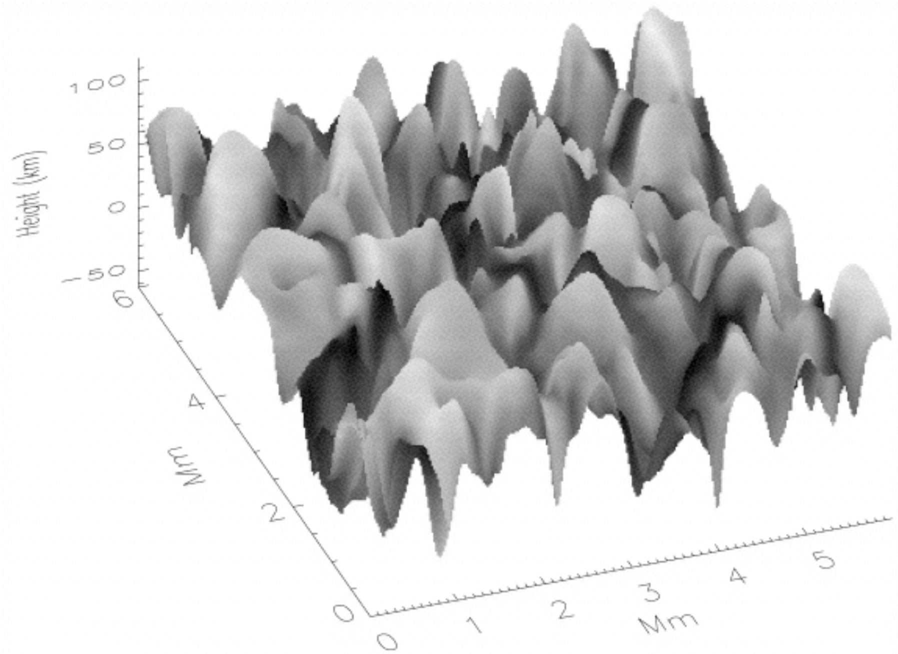
6 x 6 Mm
- 0.5 to 2.5 Mm

253 x 253 x 163



3D with realistic radiation transfer

τ = 1 surface

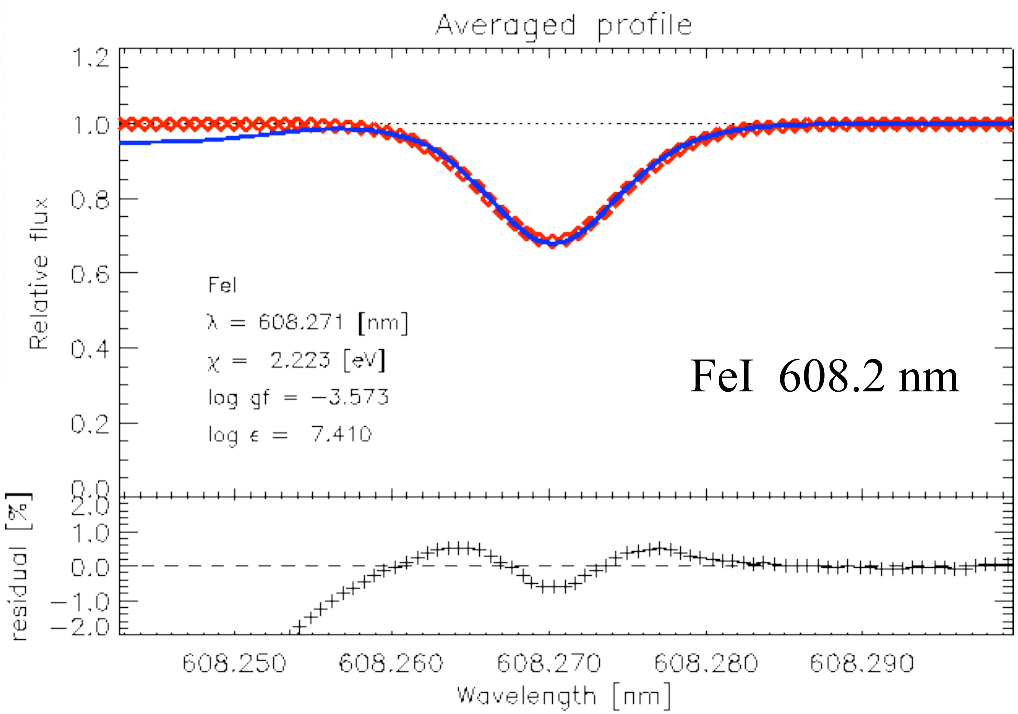


Stein & Nordlund 1998

Disk-integrated lines

compared with Sun
⇒ excellent line profiles

Asplund et al 2000



3D simulations - deep convection

In cartesian geometry:

penetration, effect of rotation (Brummell)

In spherical geometry:

Sun, with rotation (Brun, Miesch, Toomre)
convective core of A-type star (Brun, Bowning & Toomre)
SN progenitors - shell burning (Meakin, Arnett)
Red giants (Freytag; Palacios & Brun; Smilianic)
Giant planets (Evonuk, Chan)

3D simulations

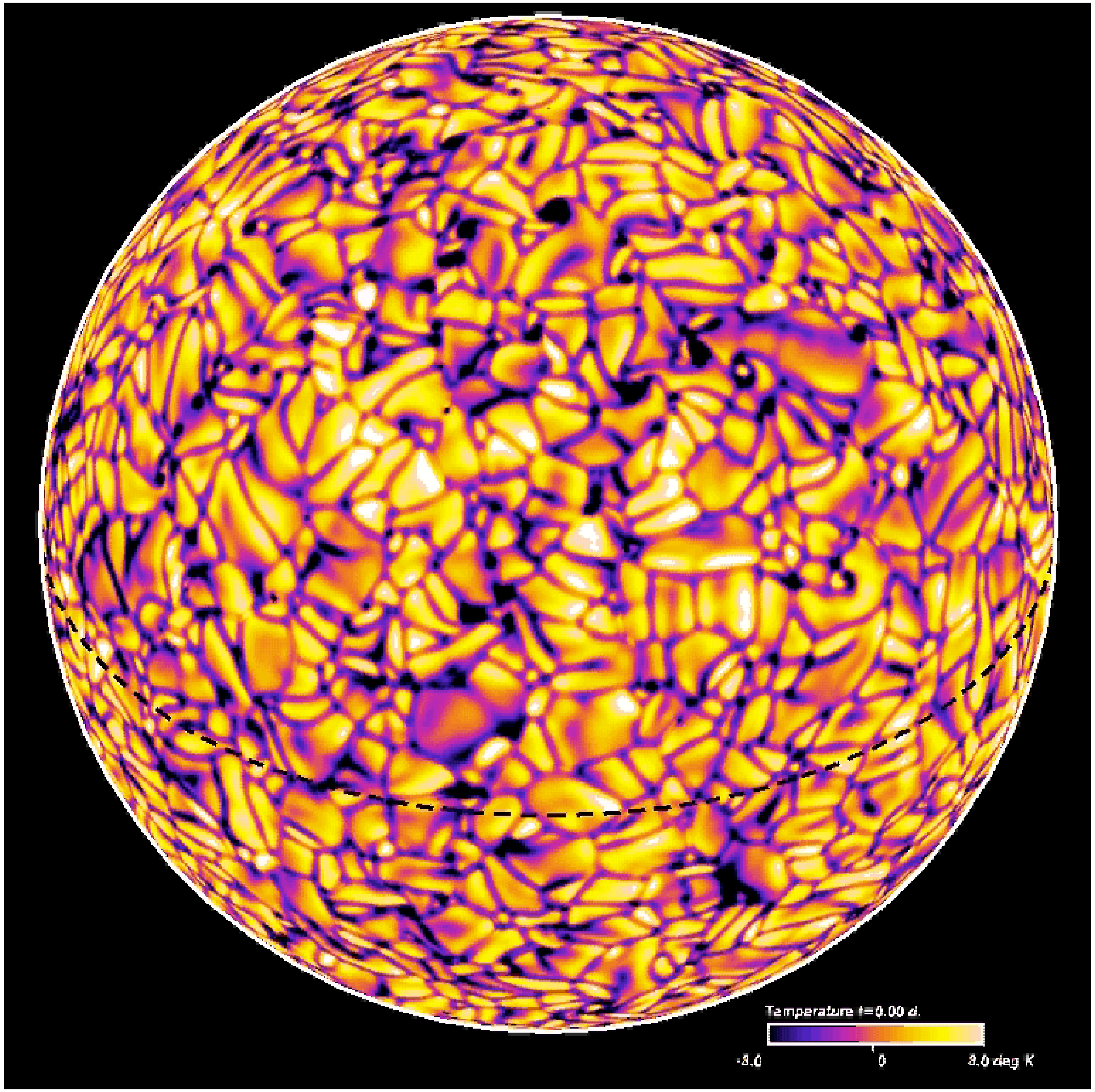
ASH code

Anelastic
Spherical
Harmonics

Clune et al. 1999

temperature at
$r = 0.98\ R_{\odot}$

Brun & Toomre 2002



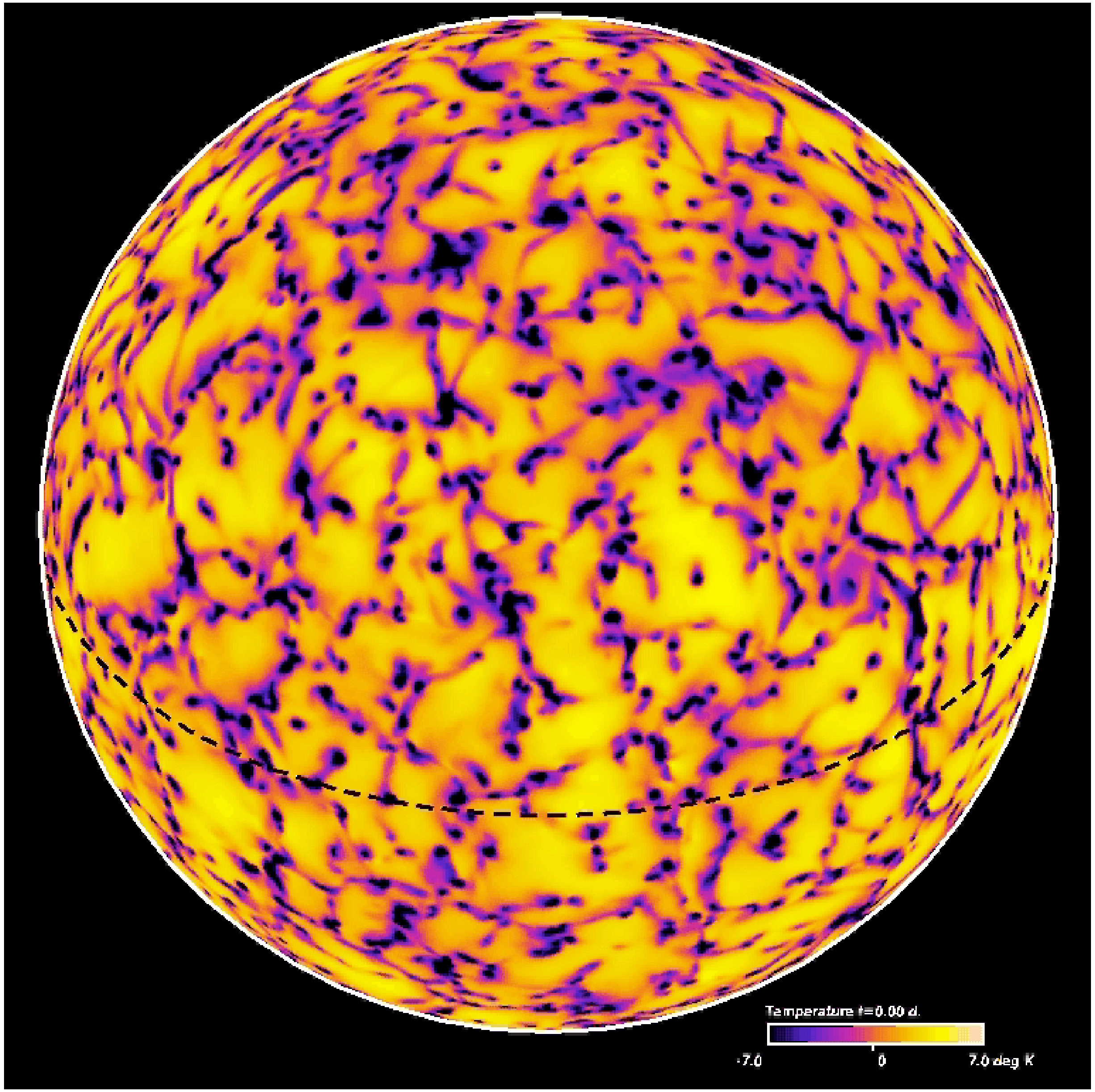
3D simulations

Global,
with rotation

ASH code

Anelastic
Spherical
Harmonics

Clune et al. 1999

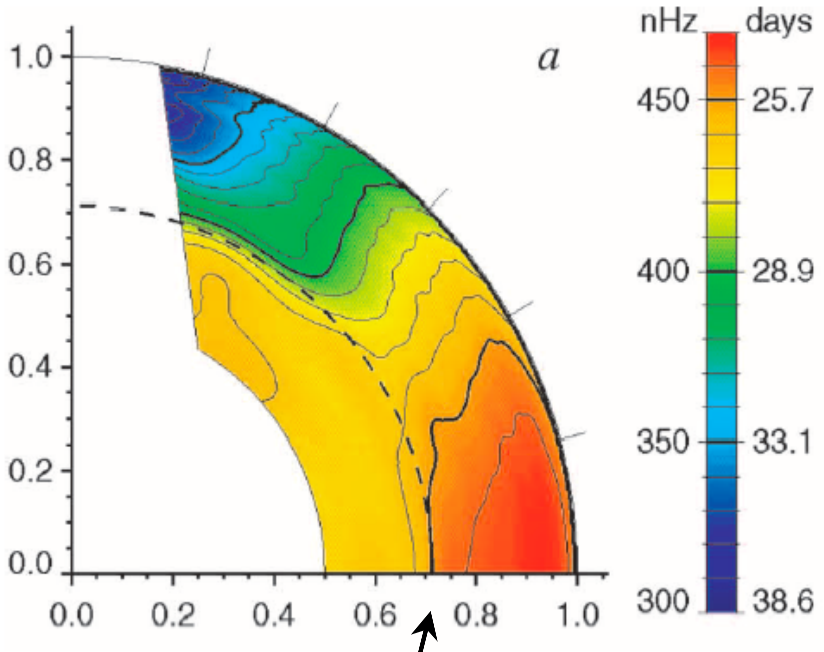


temperature at
$r = 0.95\ R_{\odot}$

Brun & Toomre 2002

Internal rotation of Sun

through helioseismology



tachocline

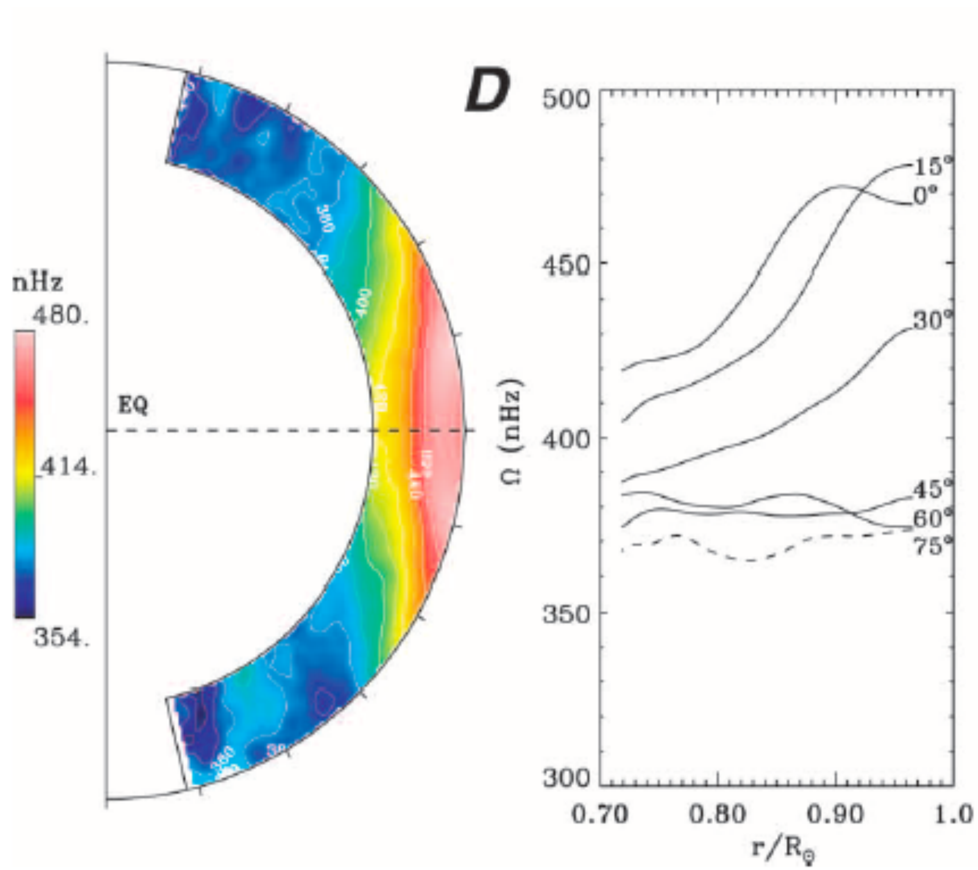
SoHO MDI

ASH code

first simulations to show equatorial acceleration

192 x 512 x 1024

Brun & Toomre 2002



ASH code

Modeling the whole Sun

0.07 - 0.97 $R_\odot$

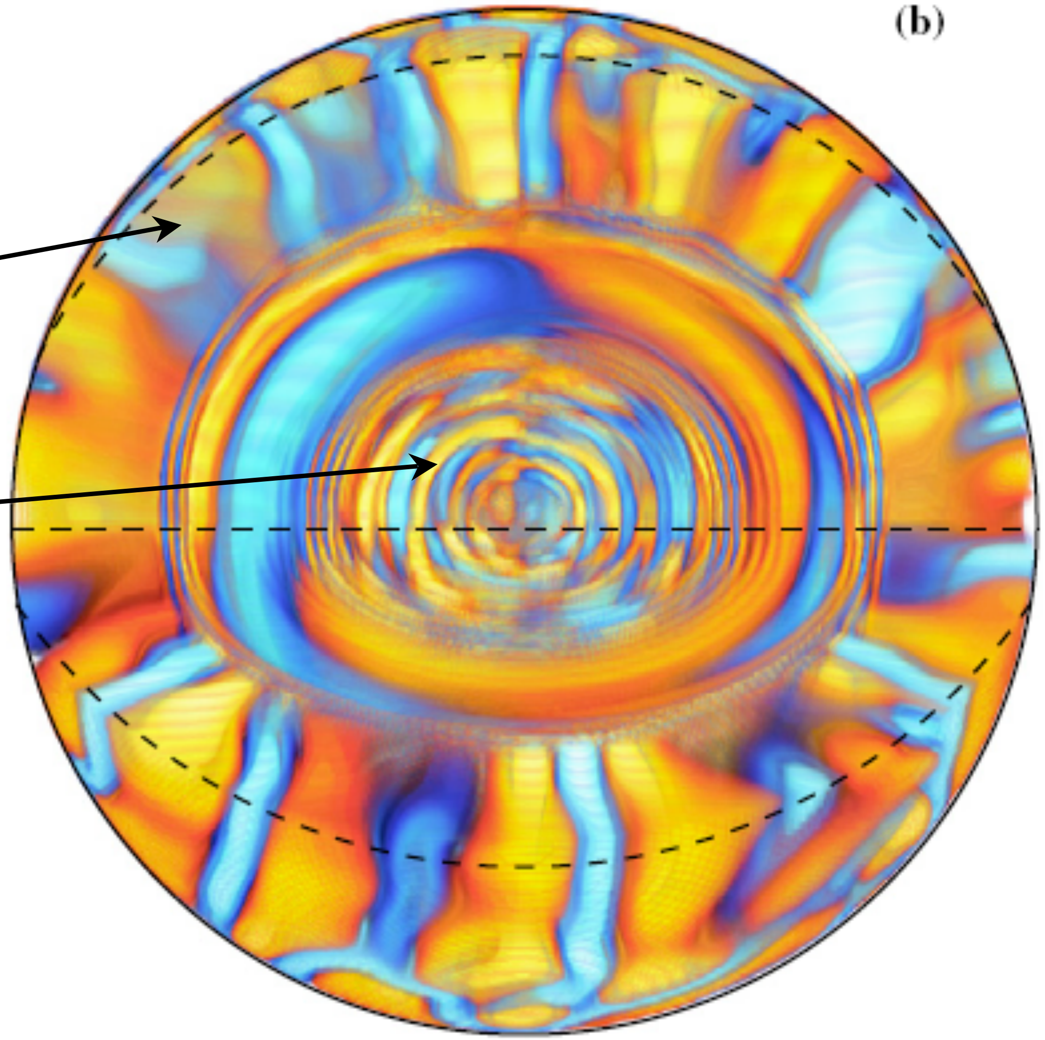
720 x 256 x 512

convection

internal gravity waves

convective penetration

0.04 $R_\odot$



Brun, Miesch & Toomre 2011

Star in a cartesian box

Red supergiant
Betelgeuse

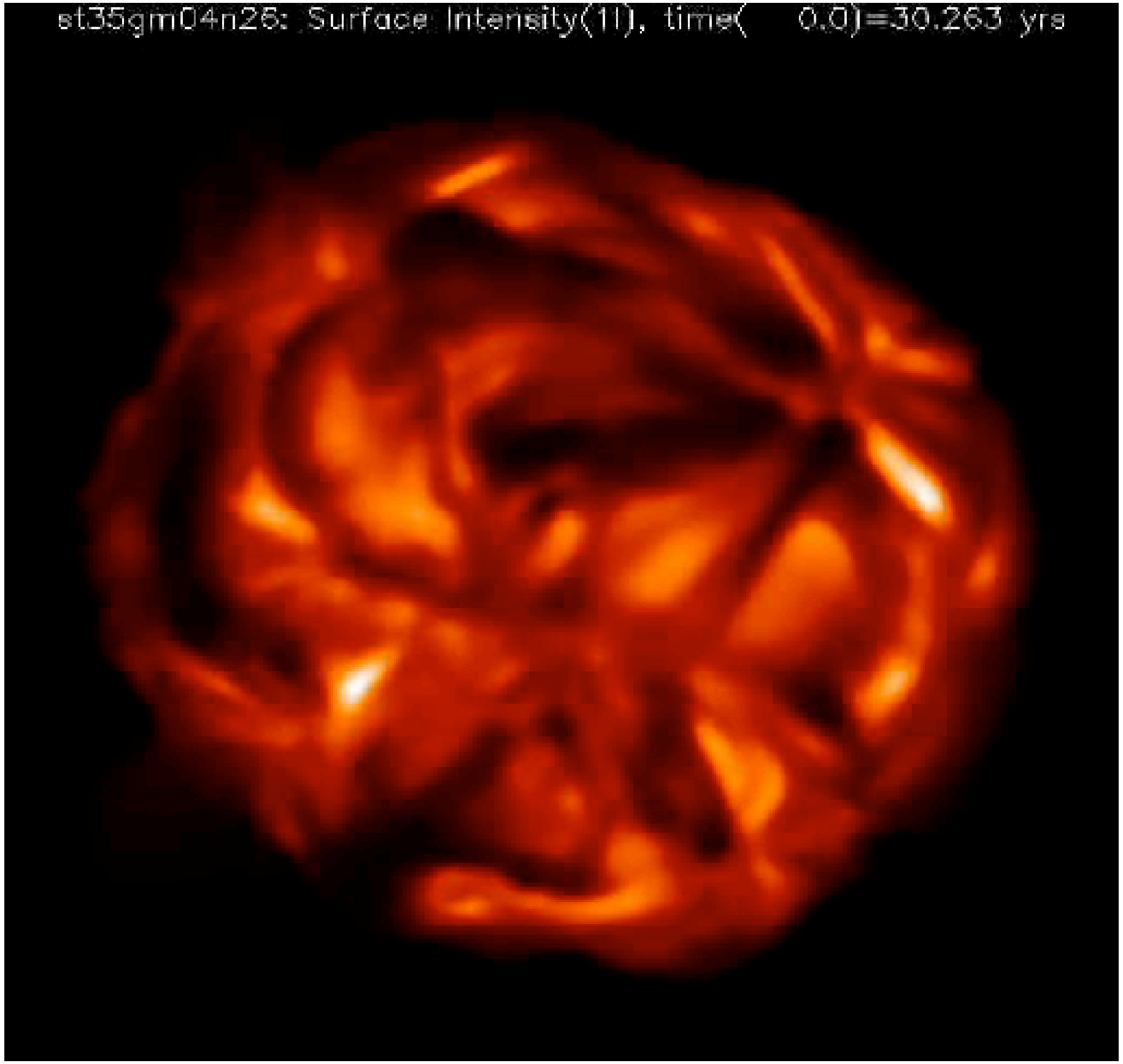
5 $M_{\odot}$

600 $R_{\odot}$

box 1674 $R_{\odot}$

171 x 171 x 171

Freytag 2000



Presently - where do we stand ?

Still two approaches, as defined by E. Spiegel in 1976 *

"Those who want to write down an algorithm for computing stellar structure that contains adjustable parameters which can be fit to well known cases"

$\Rightarrow$ Only feasible way to model secular evolution of stars

"Those who want to write down the full equations and solve them, who have virtue but no results that apply directly to stars"

$\Rightarrow$ A requirement when describing dynamical processes (effect of rotation, magnetic field, coupling with puslation, predict line profiles, etc.)

* IAU coll.39, Problems of stellar convection

What about the radiation zones ?

Are they really stable ?

Are they motionless - except for (differential) rotation ?

Signs of mixing in radiation zones

In the absence of mixing, some elements would be overabundant
at the surface of stars, others underabundant,
due to radiative levitation and gravitational settling

[Schatzman 1969, Michaud et al 1999]

Elements that are produced only in the core of stars (He, N, ^{13}C …)
appear at the surface

Consequences of such mixing

Increases life-time of stars

Modifies later stages of evolution

Determines chemical evolution of Galaxy

How to treat this extra mixing in RZ

Parametric approach

Assume all transports are achieved through turbulent diffusion

Introduce a parametrized turbulent diffusivity
for each transport process

Adjust parameters to fit observations

Physical approach

Strive to implement the physical processes
that are likely to cause mixing:

- large scale circulation induced by applied torques
  (wind, accretion, etc.)
  and structural changes

- turbulence produced by instabilities
  (shear, magnetic, thermohaline, etc.)

Mixing processes in radiation zones

Meridional circulation

Classical picture: circulation is due to thermal imbalance
caused by perturbing force (centrifugal, magnetic, etc.)

Eddington (1925), Vogt (1925), Sweet (1950), Mestel (1950) etc.

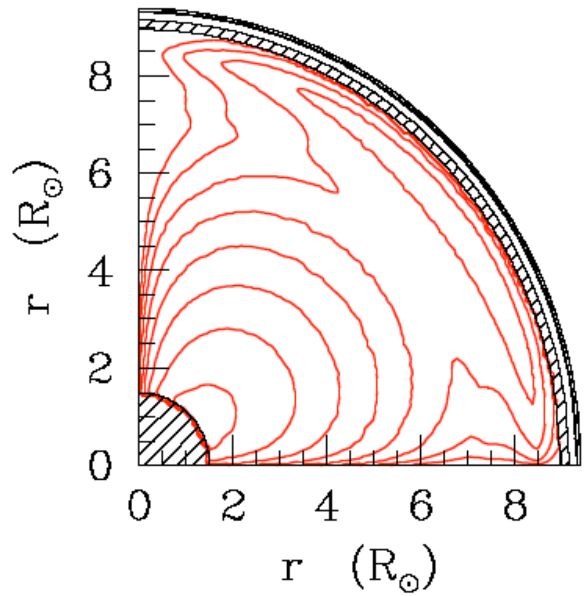
Eddington-Sweet time $$t_{ES} = t_{KH} \frac{GM}{\Omega^2 R^3}, \quad \text{with} \quad t_{KH} = \frac{GM^2}{RL}$$

Revised picture:

after a transient phase of about t_{ES},
circulation is driven by the loss (or gain) of AM
and by structural changes due to evolution

Busse (1981), JPZ (1992),
Maeder & JPZ (1998), Mathis & JPZ (2004)
Decressin et al (2009)

$\Rightarrow$ modifies the rotation profile



Rotational mixing in radiation zones

Turbulence caused by vertical shear $\Omega(\mathbf{r})$

(baroclinic instability)

- if maximum of vorticity (inflexion point) : linear instability
- if no maximum of vorticity : finite amplitude instability
- stabilizing effect of stratification

$$\frac{w\ell}{\nu} \geq R_{crit}$$

for instability

reduced by thermal diffusion

turbulence if $$Ri_c \left(\frac{dV_{\rm hor}}{dr}\right)^2 > N^2 \left(\frac{w\ell}{K}\right)$$

Richardson criterion

$$N^2 = \frac{g}{H_P}[\nabla_{ad} - \nabla]$$

Townsend 1959, Dudis 1974

from which one deduces the turbulent diffusivity

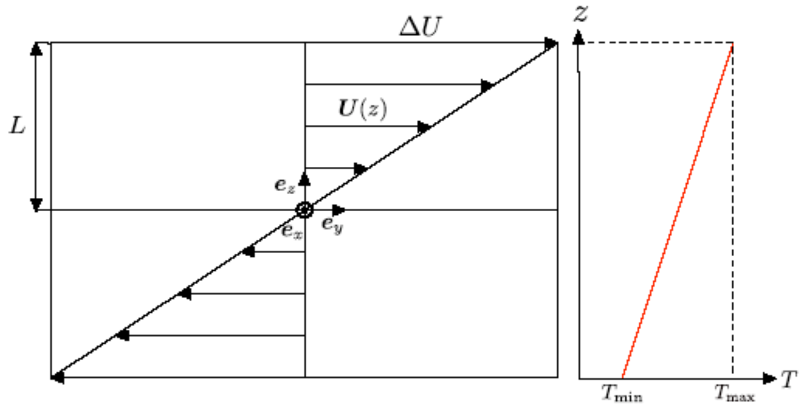
$$D_v = w\ell = Ri_c K \frac{\Omega^2}{N^2} \left(\frac{d\ln\Omega}{d\ln r}\right)^2$$

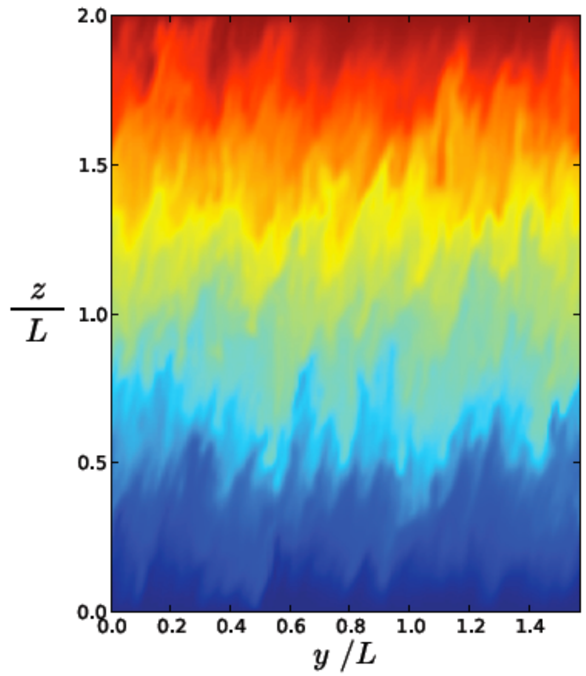
JPZ 1974
Lignières et al. 1999

K thermal diffusivity; ν viscosity; N buoyancy frequency

Turbulent transport
in stellar radiation zones

Prat & Lignières 2013





128 x 128 x 257

$Re = L\,\Delta U/\nu \qquad Pe = L\,\Delta U/\chi$

$Ri = (N/S)^2 \qquad S = \Delta U/L$

$$\nabla\cdot\tilde{\boldsymbol{u}} = 0,$$

$$\frac{\partial\tilde{\boldsymbol{u}}}{\partial\tilde{t}} + \tilde{\boldsymbol{u}}\cdot\nabla\tilde{\boldsymbol{u}} = -\nabla\tilde{p} + Ri\tilde{\theta}\boldsymbol{e}_z + \frac{1}{Re}\Delta\tilde{\boldsymbol{u}} + \tilde{\boldsymbol{f}}_{\mathrm{v}},$$

$$\cancel{\frac{\partial\tilde{\theta}}{\partial\tilde{t}}} + \cancel{\tilde{\boldsymbol{u}}\cdot\nabla\tilde{\theta}} + \tilde{w} = \frac{1}{Pe}\Delta\tilde{\theta} + \cancel{\tilde{f}_{T}},$$

In the limit of low Péclet number

Introduce a passive scalar

$$\frac{\partial c}{\partial t} + \boldsymbol{u}\cdot\nabla c = D_{\mathrm{m}}\Delta c,$$

draw the turbulent diffusivity

$$\boxed{D_{\mathrm{t}} = -\frac{\langle c' w\rangle}{\mathrm{d}C/\mathrm{d}z}.}$$

find that $D_t \propto Ri_{crit}\,\chi\,(S/N)^2$

as in Zahn 1974

Rotational mixing in radiation zones

Turbulence caused by horizontal shear $\Omega(\theta)$

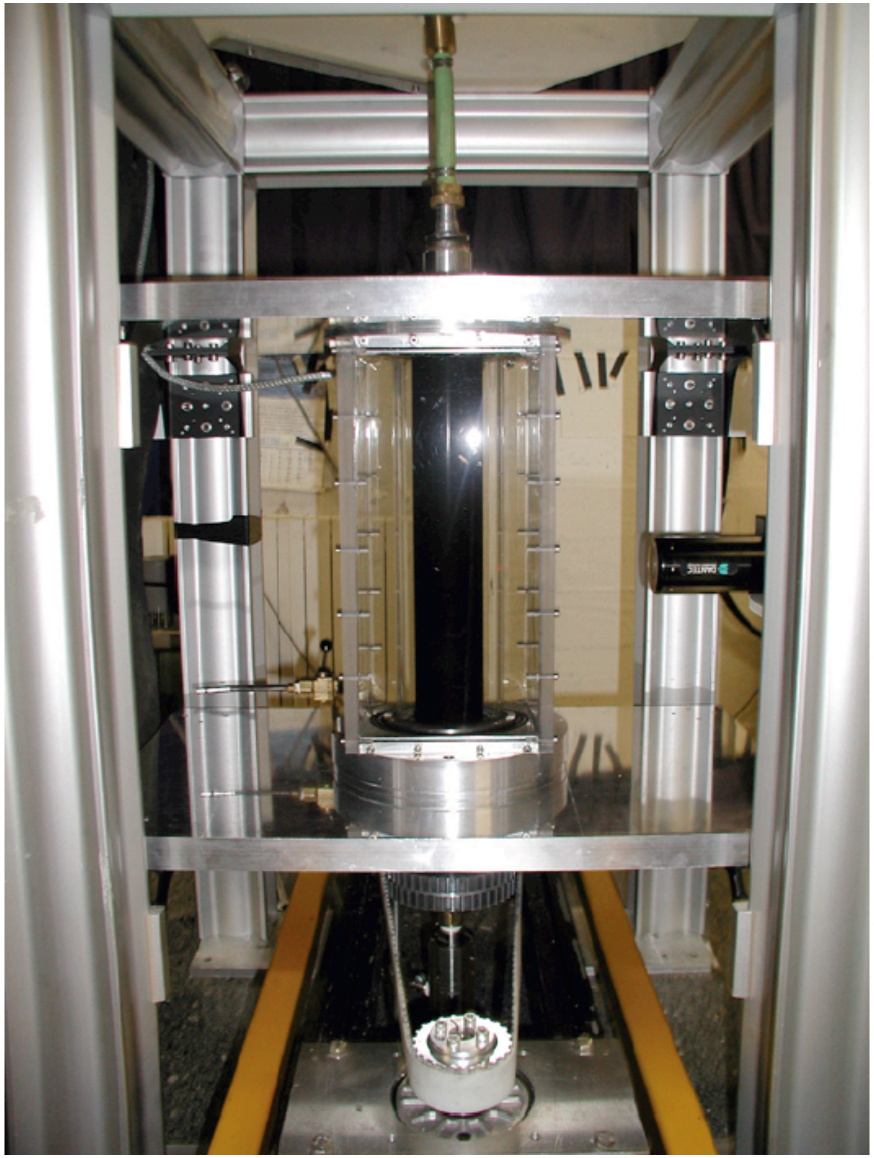
(barotropic instability)

Assumptions:

- finite amplitude instability (no linear instability expected)
- instability acts to suppress its cause, i.e. diff. rotation in latitude $\Omega(\theta)$

Experimental evidence ?

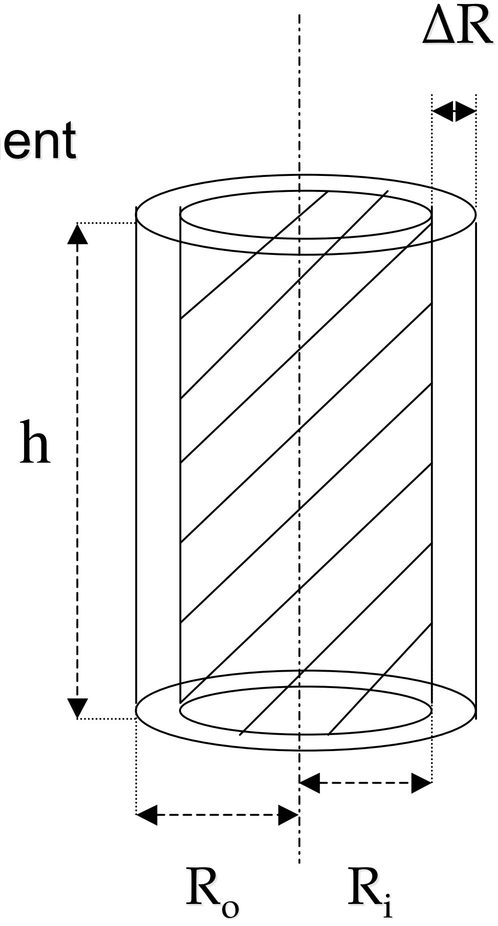
What we can learn from Couette-Taylor flow



Saclay experiment

R_i= 3.5 cm
R_o= 5 cm
h = 38 cm

Ω_{max} = 30 t/s

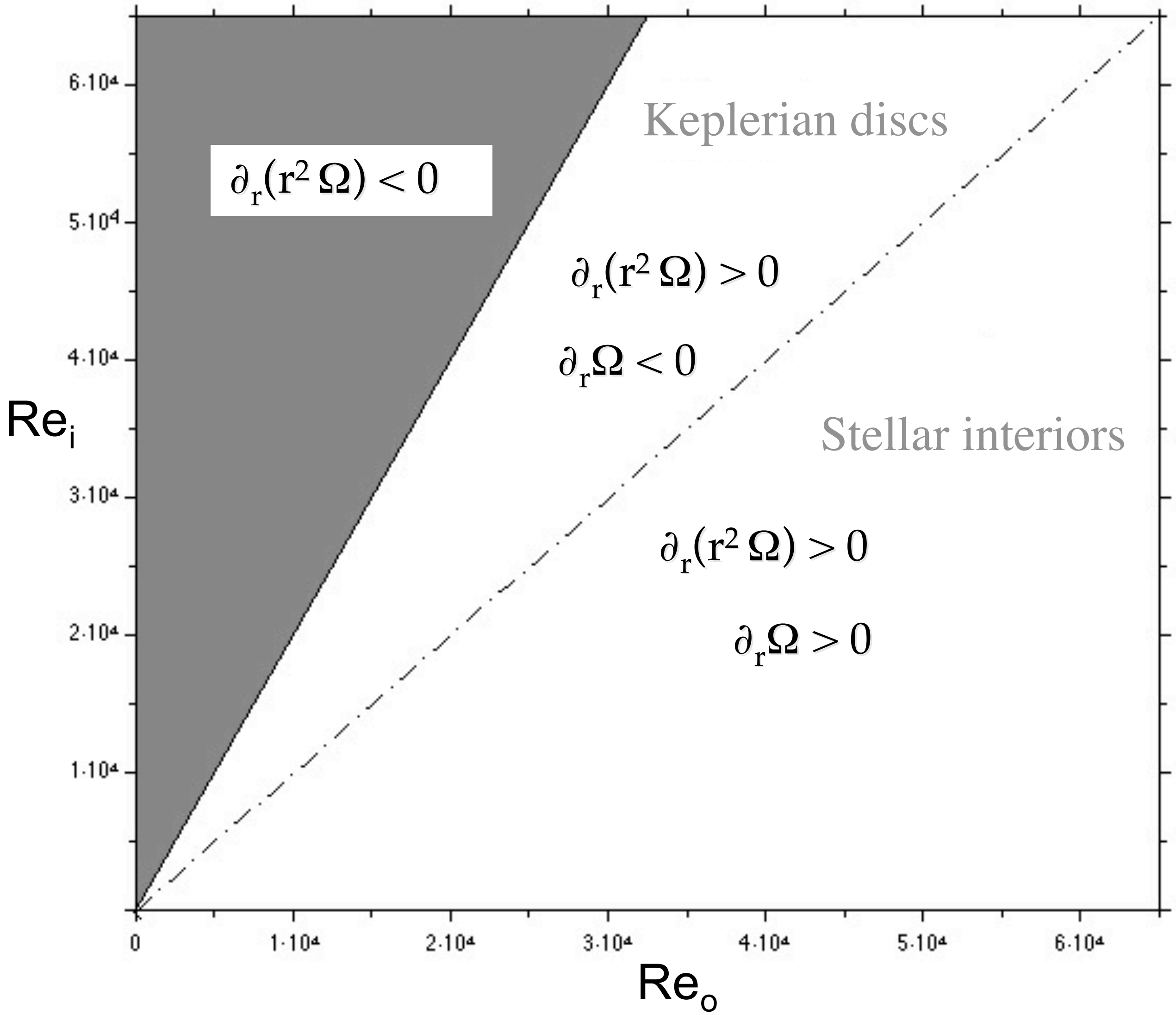


Richard, Dauchot,
JPZ 2001

$$\mathrm{Re}_i = \Omega_i \, R_0 \, \Delta R / \nu$$

$$\mathrm{Re}_0 = \Omega_0 \, R_0 \, \Delta R / \nu$$

The Saclay experiment



Main motivation:
Keplerian discs

Rayleigh criterion:
linear instability when

$$\partial_r(r^2\Omega) < 0$$

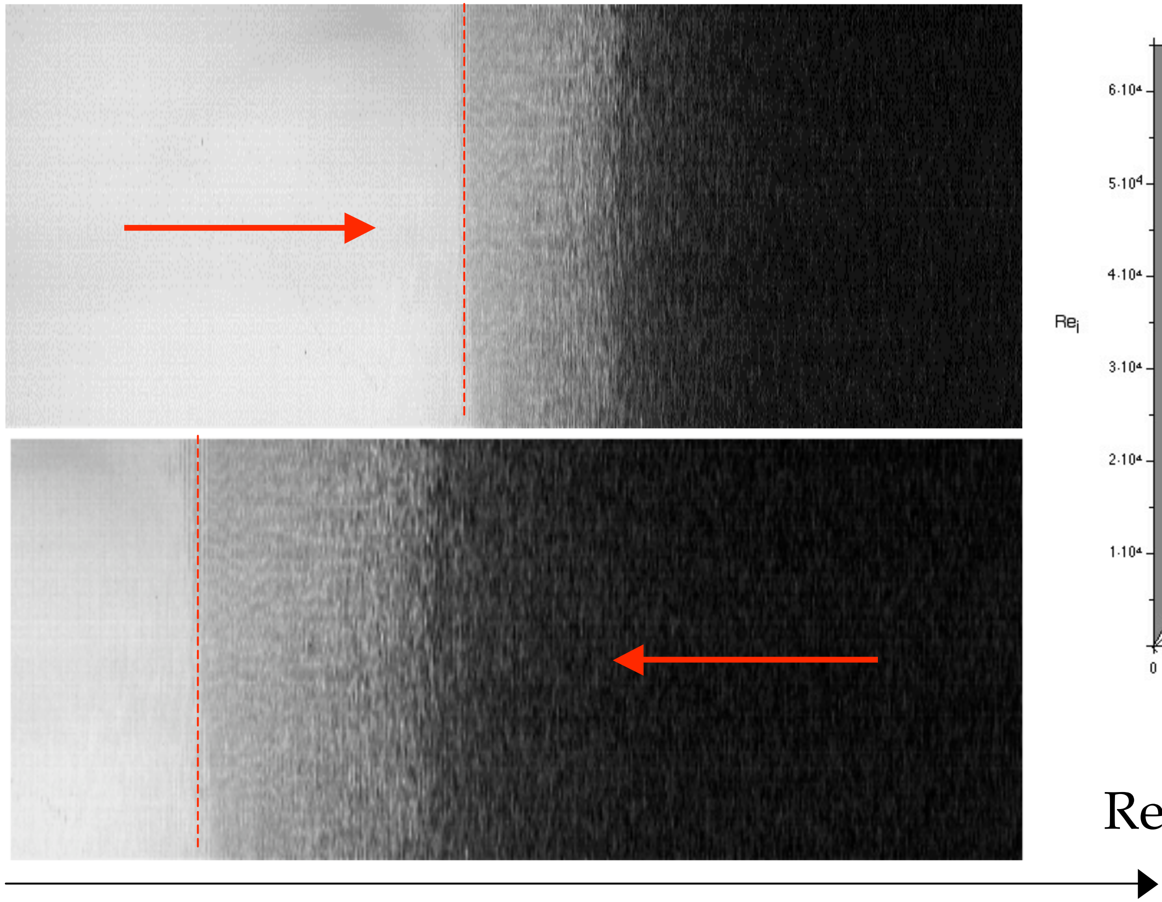
$$\mathrm{Re}_i = \Omega_i \, R_0 \, \Delta R / \nu$$

$$\mathrm{Re}_0 = \Omega_0 \, R_0 \, \Delta R / \nu$$

Finite amplitude instability

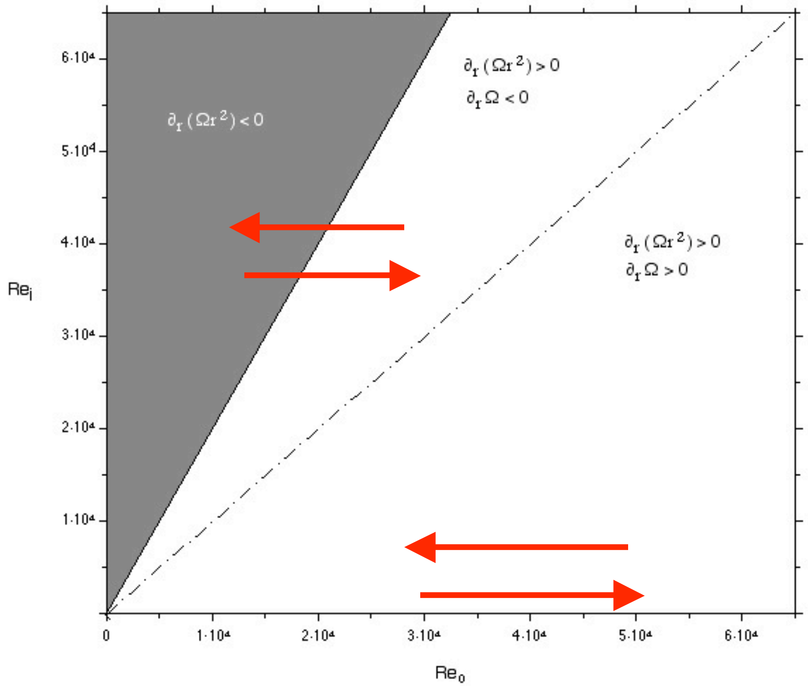
spatio-temporal diagrams

$Re_o = 3.10^{4} - 5.10^{4}$



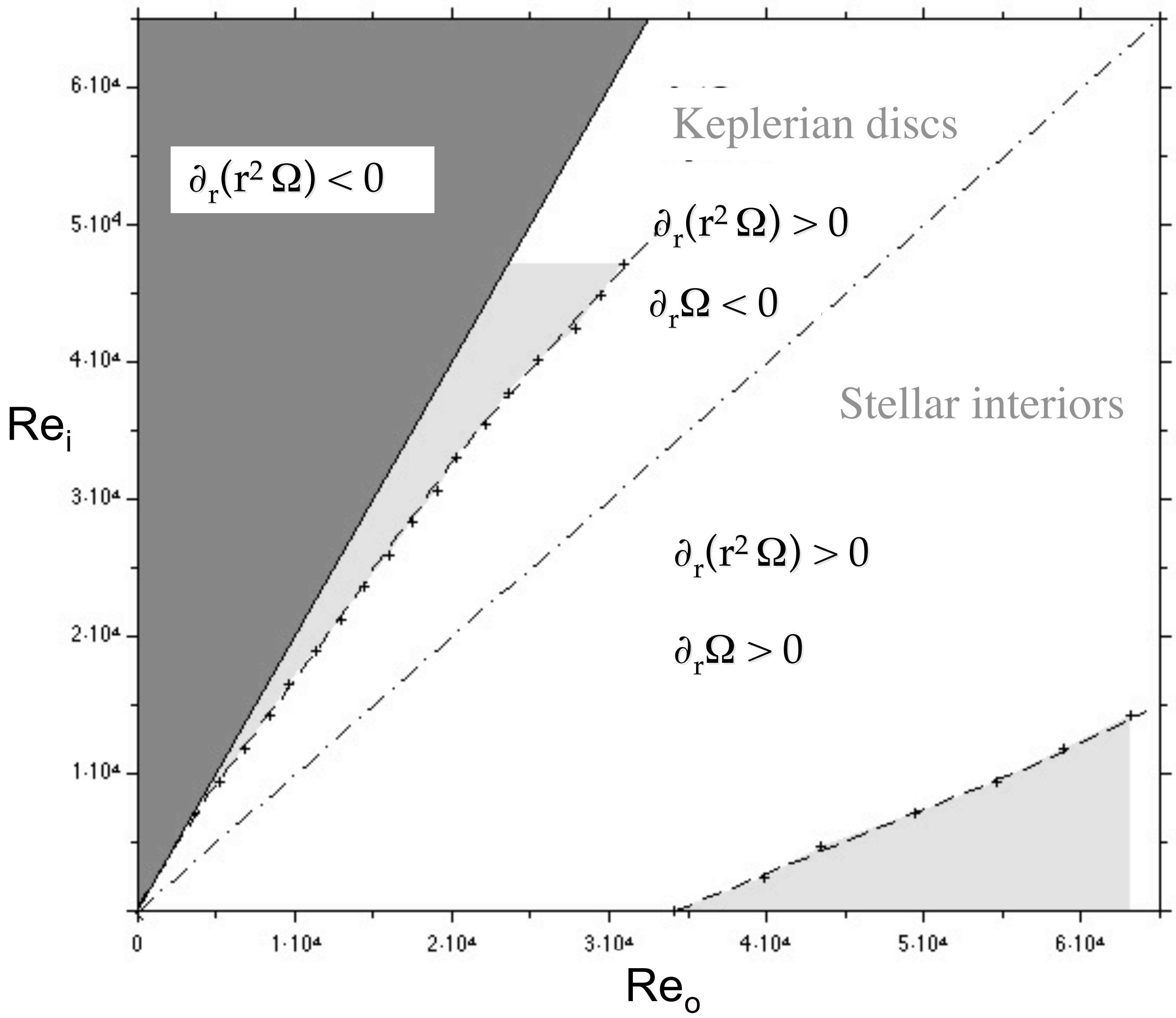
Re_o

$\Rightarrow$ Proof of hysteresis

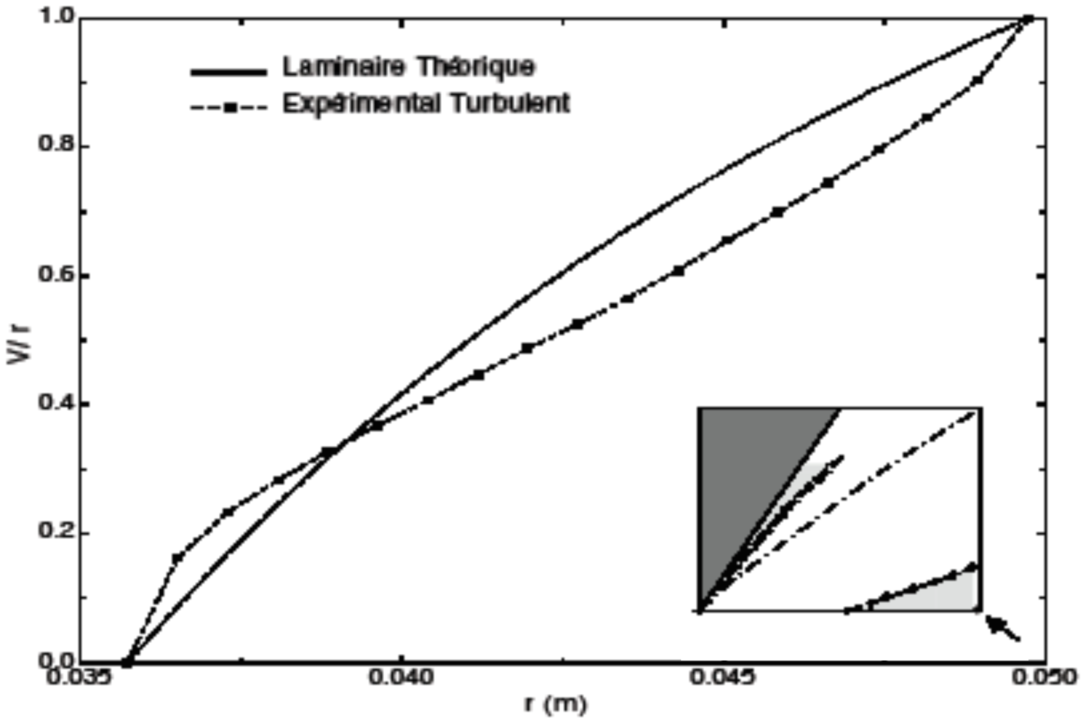


Thesis Richard 2001

Finite amplitude instability



Angular velocity profile



Shear instability

tends to flatten
the Ω profile

i.e. to suppress
the cause of instability

From exp. Ω profiles
(Wendt 33, Taylor 36)

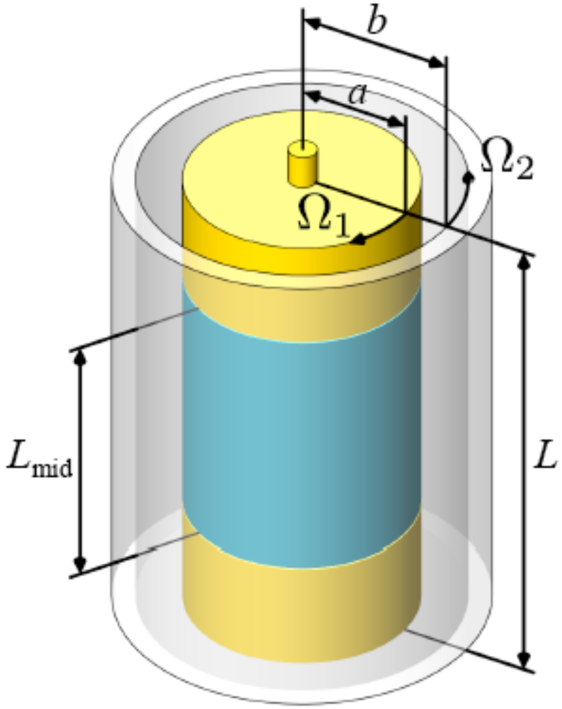
⇒ turbulent viscosity

$$\nu_{turb} = \beta \left| r^3 \partial\Omega / \partial r \right|$$

$$\beta = 1.5 \pm 0.5\, 10^{-5} \quad \text{for} \quad \Omega_i = 0$$

Richard & JPZ 99

The Maryland experiment

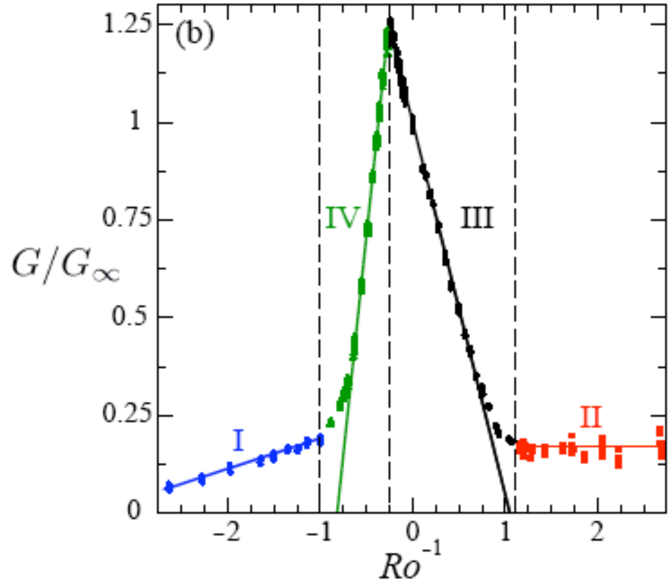


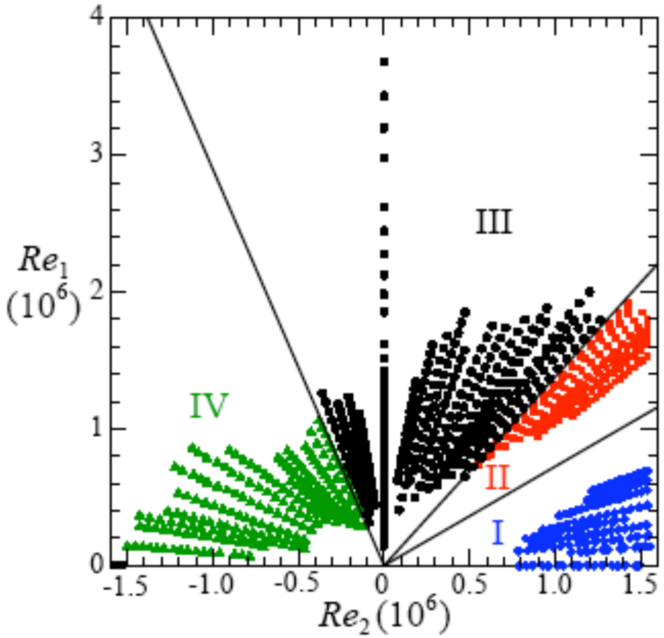
a = 16 cm
b = 22 cm

L = 69.5 cm
L_{mid} = 40.6 cm

Rossby number

$$Ro = (\Omega_1 - \Omega_2)/\Omega_2$$





Torque measurement G
$\Rightarrow$ turbulent viscosity

$$\nu_{turb} = \beta \left| r^3 \partial\Omega / \partial r \right|$$

$$\beta = 1.84\,10^{-5} \quad \text{for } Ro = -1 \quad (\Omega_1 = 0)$$

Paoletti & Lathrop 2011

Rotational mixing in radiation zones

Turbulence caused by horizontal shear $\Omega(\theta)$

(barotropic instability)

Assumptions:

- instability acts to suppress its cause, i.e. rotation in latitude $\Omega(\theta)$
- turbulent transport is anisotropic (due to stratification): $D_h >> D_v$

Main weakness: no firm prescription for D_h

Maeder 2003
Mathis, Palacios & JPZ 2004

→ anisotropic turbulence interferes with vertical transport :

- erodes stabilizing effect of stratification ; shear-unstable when

$$Ri_c \left(\frac{d\ln\Omega}{d\ln r}\right)^2 > N_t^2\left(\frac{wl}{K}\right) + N_\mu^2\left(\frac{wl}{D_h}\right) \qquad N_\mu^2 = \frac{g}{H_P}\frac{d\ln\mu}{d\ln P}$$

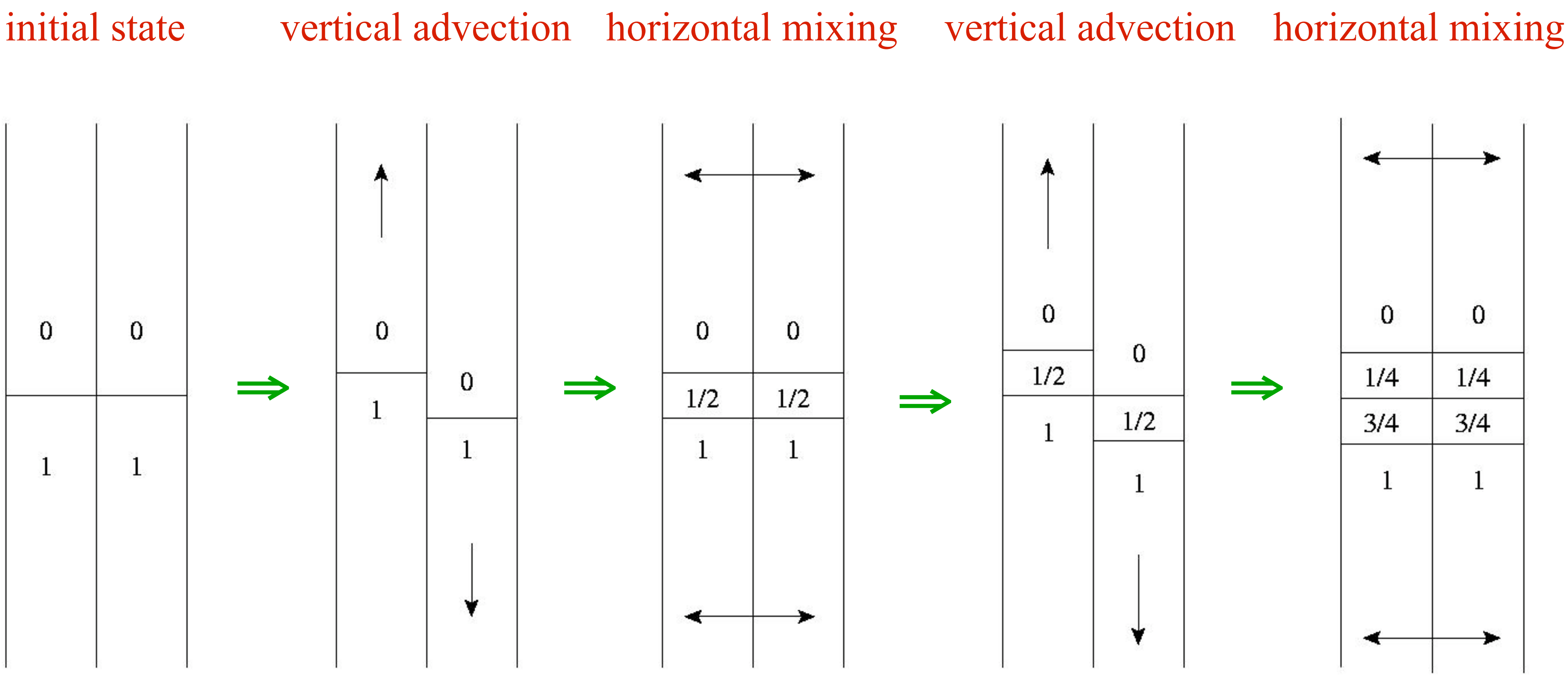
Talon & JPZ 1997

- changes advection of chemicals into vertical diffusion

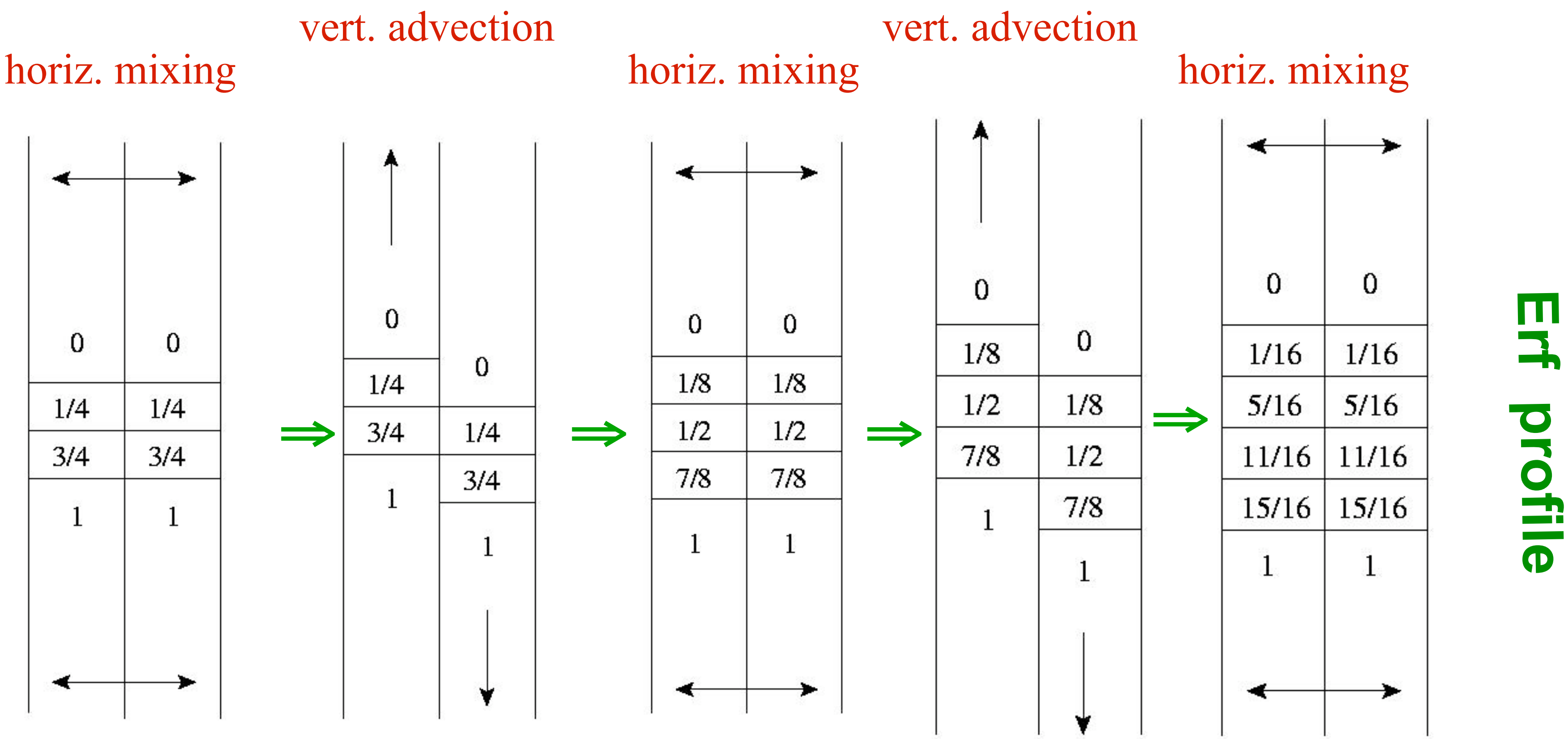
$$D_{eff} = \frac{1}{30}\frac{(rU)^2}{D_h} \qquad u_r(r,\theta) = U(r)\,P_2(\cos\theta)$$

Chaboyer & JPZ 1992

A cartoon: Turbulent erosion of advective transport



Turbulent erosion of advective transport (cont.)



transport of chemicals ⇒ vertical diffusion

transport of AM remains an advective process

Rotational mixing - the observational test

Assumption: the processes that cause the mixing of chemical elements
(i.e. circulation and turbulence)
are also responsible for the transport of angular momentum
JPZ 1992, Maeder & JPZ 1998

- quite successful with massive stars (fast rotators)
  Talon et al. 1997; Maeder & Meynet 2000; Talon & Charbonnel 1999

- for solar-like stars (which are spun down by wind) predicts
  - fast rotating core   not true: helioseismology
  - strong destruction of Be in Sun   not observed
  - mixing correlated with loss of angular momentum
    not true: Li in tidally locked binaries

⇒ Another, more powerful process is responsible
for the transport of AM in solar-like stars

· magnetic field ?   · internal gravity waves ?

Possible effects of magnetic field

Dynamo field (solar-type stars, or from convective core)

- Likely to have reversals → will not penetrate into RZ

[Garaud 1999]

Fossil field (such as in Ap/Bp stars)

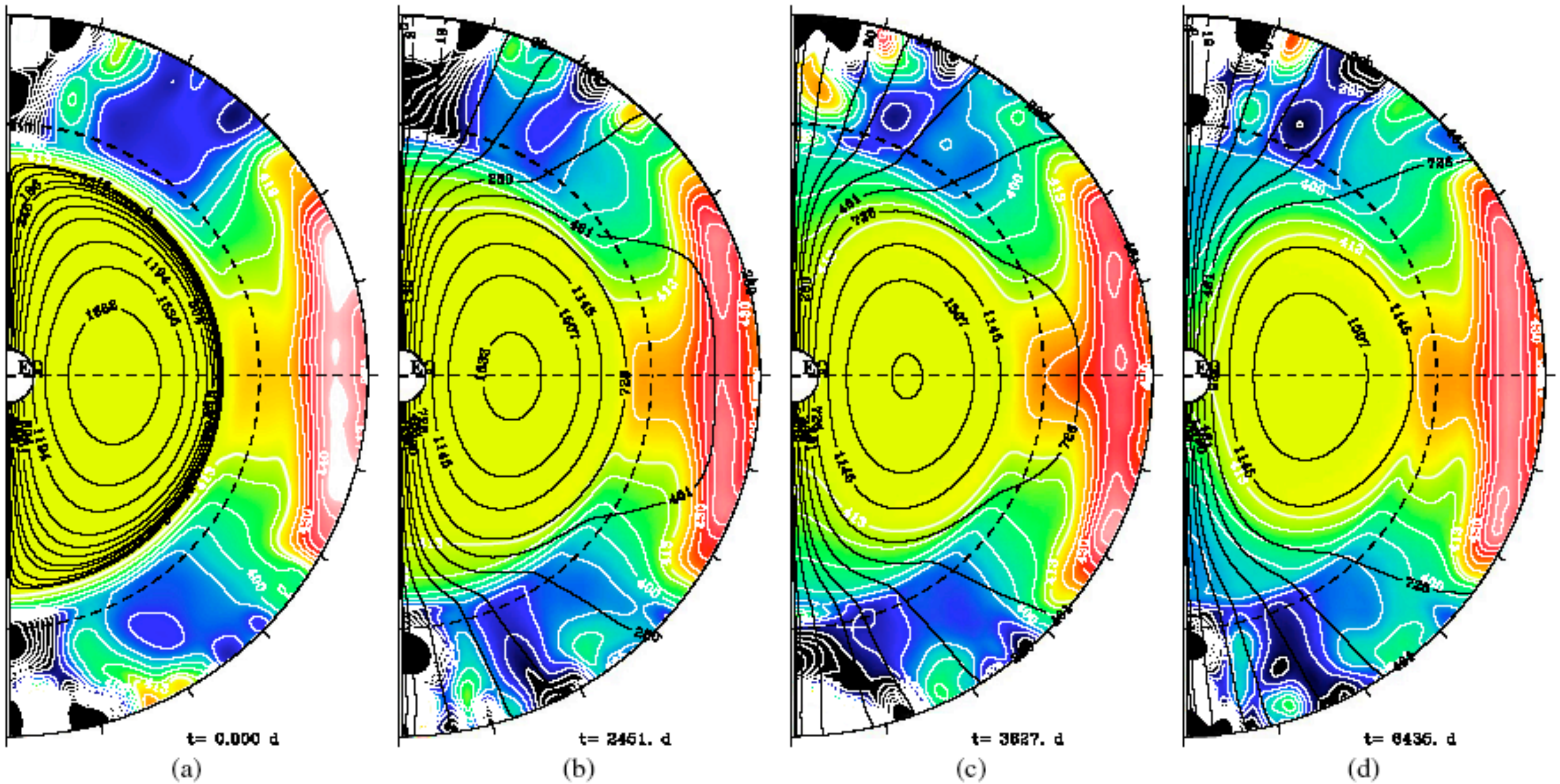
- Renders the rotation uniform [Mestel and coll.]

  along field lines if axisymmetric (Ferraro law)

- Imprints diff. rotation of CZ on RZ [Brun & JPZ 2006]

Fossil field and rotation

Fossil field expands into CZ, and prints its differential rotation on RZ



3D simulations - ASH code

Strugarek, Brun & JPZ 2011

Role of magnetic field

Dynamo field (solar-type stars, or from convective core)

- Likely to have reversals → will not penetrate into RZ

[Garaud 1999]

Fossil field (such as in Ap stars)

- Renders the rotation uniform [Mestel and coll.]

  along field lines if axisymmetric (Ferraro law)

- Imprints diff. rotation of CZ [Brun & JPZ 2006]

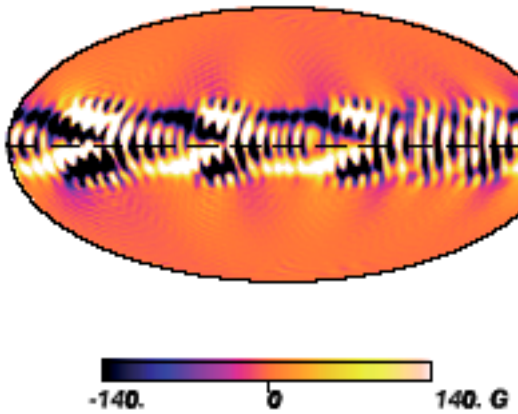
Field itself may be unstable [Tayler & coll.; Spruit 1998]

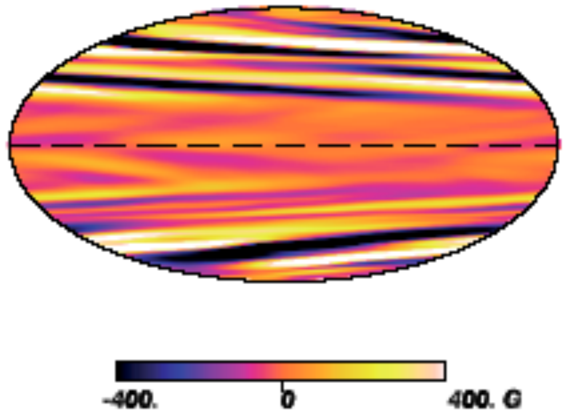
- yes - but instabilities are probably of wave type → no mixing

- may these instabilities sustain a dynamo ?

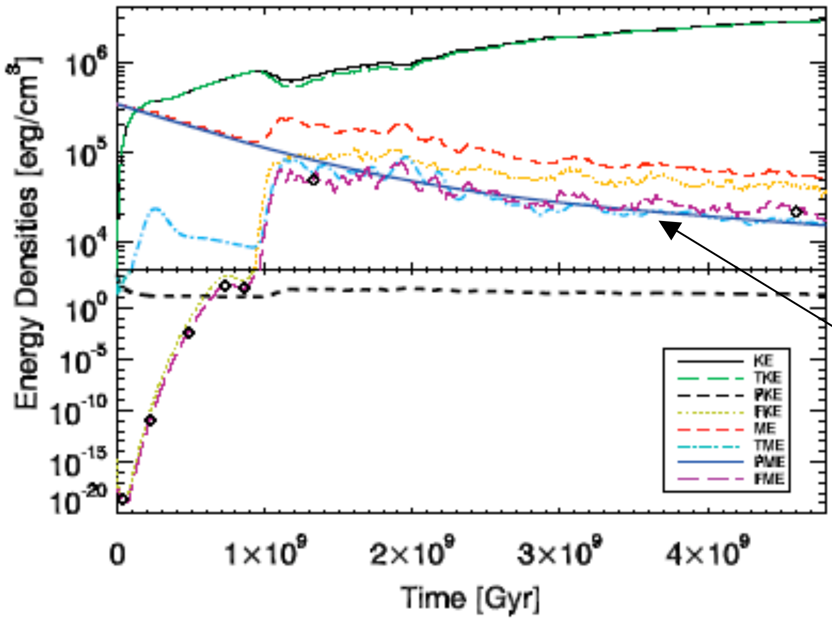
Spruit 2002; Braithwaite 2006; JPZ, Brun & Mathis 2007]

Tayler instabilities of a fossil field





Instability due
to poloidal field



Instability due
to toroidal field

JPZ, Brun, Mathis 2007

Mean poloidal field
is not destroyed

No dynamo found - contrary to the claim by Braithwaite and Spruit (2006)

Angular momentum transport by waves

Press 1981, Garcia-Lopez & Spruit 1991, Schatzman 1993, JPZ et al 1997

Internal gravity waves and gravito-inertial waves
are emitted at the edge of the convection zone

They transport angular momentum, which they deposit
where they break or are damped through thermal diffusion

damping rate $\propto \sigma^{-4}$

$$\sigma(r,m) = \sigma_c + m[\Omega(r) - \Omega_{zc}]$$

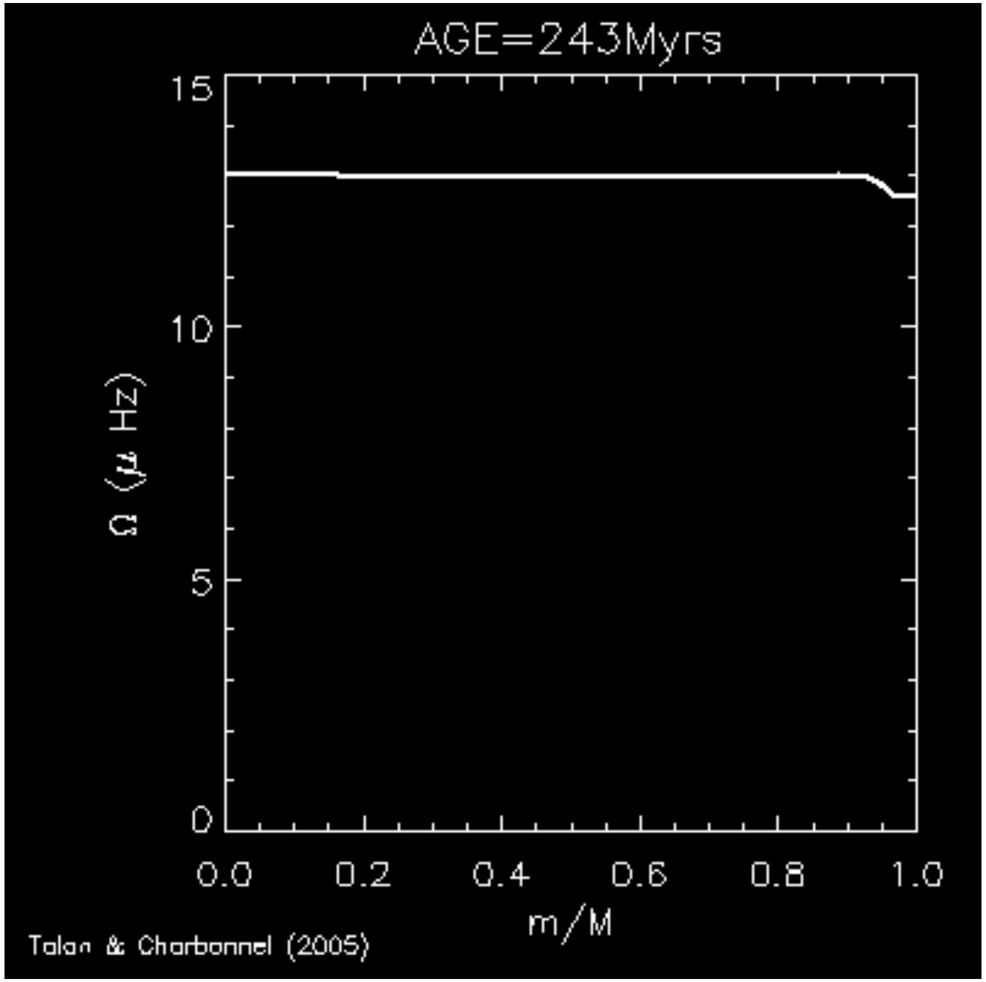
- if there is differential rotation,
  prograde and retrograde waves deposit
  their momentum (of opposite sign)
  at different depth

- waves strengthen the local differential rotation,
  until the shear becomes unstable
  $\Rightarrow$ turbulence

Talon et al 2002, Talon & Charbonnel 2005

Extraction of AM by IGW

low-degree, low-frequency waves



Angular momentum
is carried away
by solar wind

IGW are able to extract AM
from solar interior and
render the rotation uniform

Talon & Charbonnel 2005

More on IGW by S. Mathis and T. Rogers

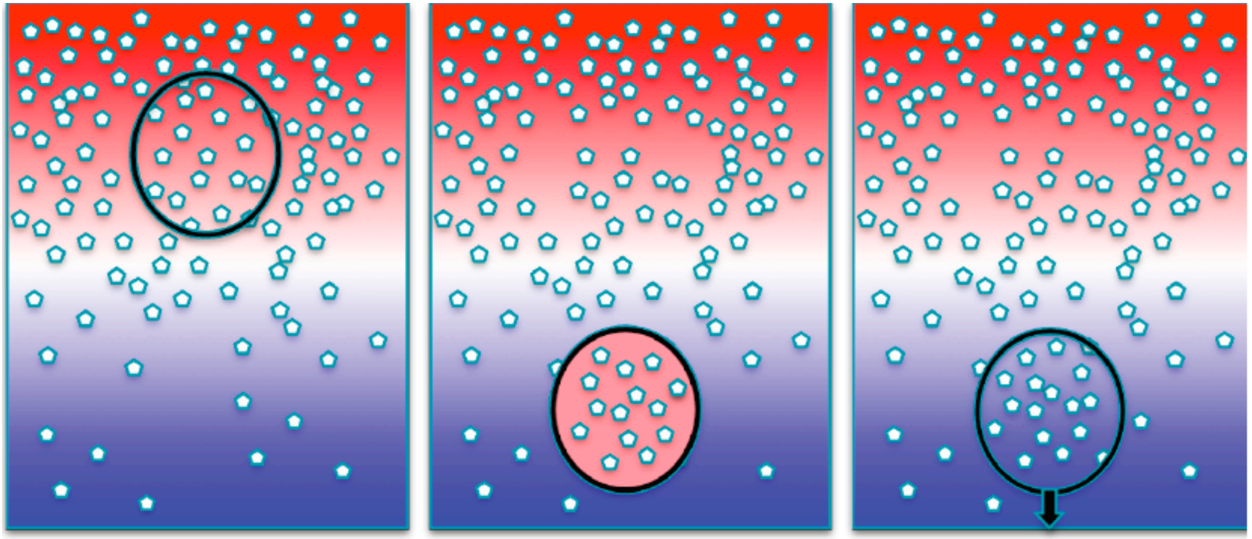
Thermohaline mixing

hot

salty

cold

fresh



after heat is lost
salt remains,
→ blob sinks

Stern 1960

courtesy P. Garaud

Thermohaline instability: a double-diffusive instability

→ it occurs in unstable salt stratification, stabilized by temperature gradient
because heat diffuses much faster than salt

In stars, such molecular weight inversions occur

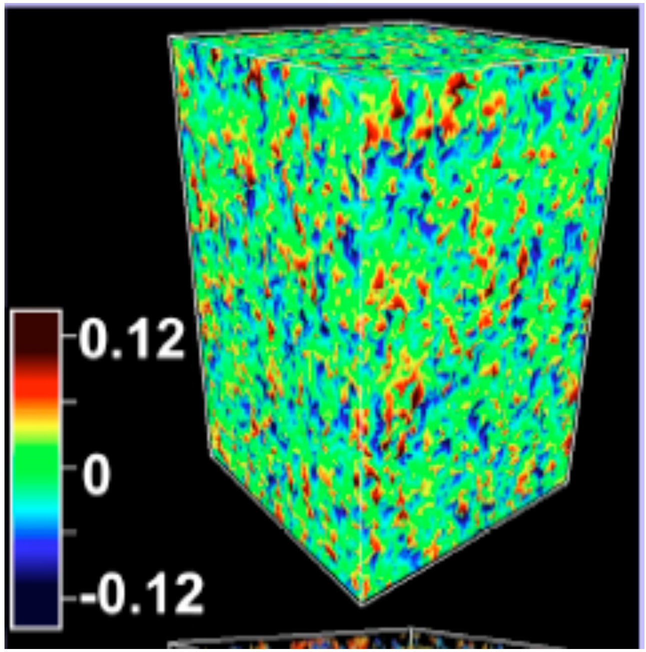
- when heavy elements are accreted (Vauclair 2004)
- in regions of hydrogen burning, due to $^3He + {}^3He \rightarrow {}^4He + 2p$
  (Ulrich 1972, Eggleton et al. 2006, Charbonnel & JPZ 2007)

→ It leads to mixing

Thermohaline mixing - fingering convection

A complex phenomenon:

- fingers
- collective instability
- staircases (ocean)
- strong dependence on BC
- strong dependence on parameters



Simplest treatment: as a diffusion

$$D_t = C_t K \frac{-N_\mu^2}{N^2} \qquad C_t = \frac{8}{3}\pi^2\alpha^2$$

Ulrich 1972; Kippenhahn et al.1980

fingers aspect ratio (from lab)

$$\alpha = 5 \quad \Rightarrow \quad C_t = 658$$

Charbonnel & JPZ 2007

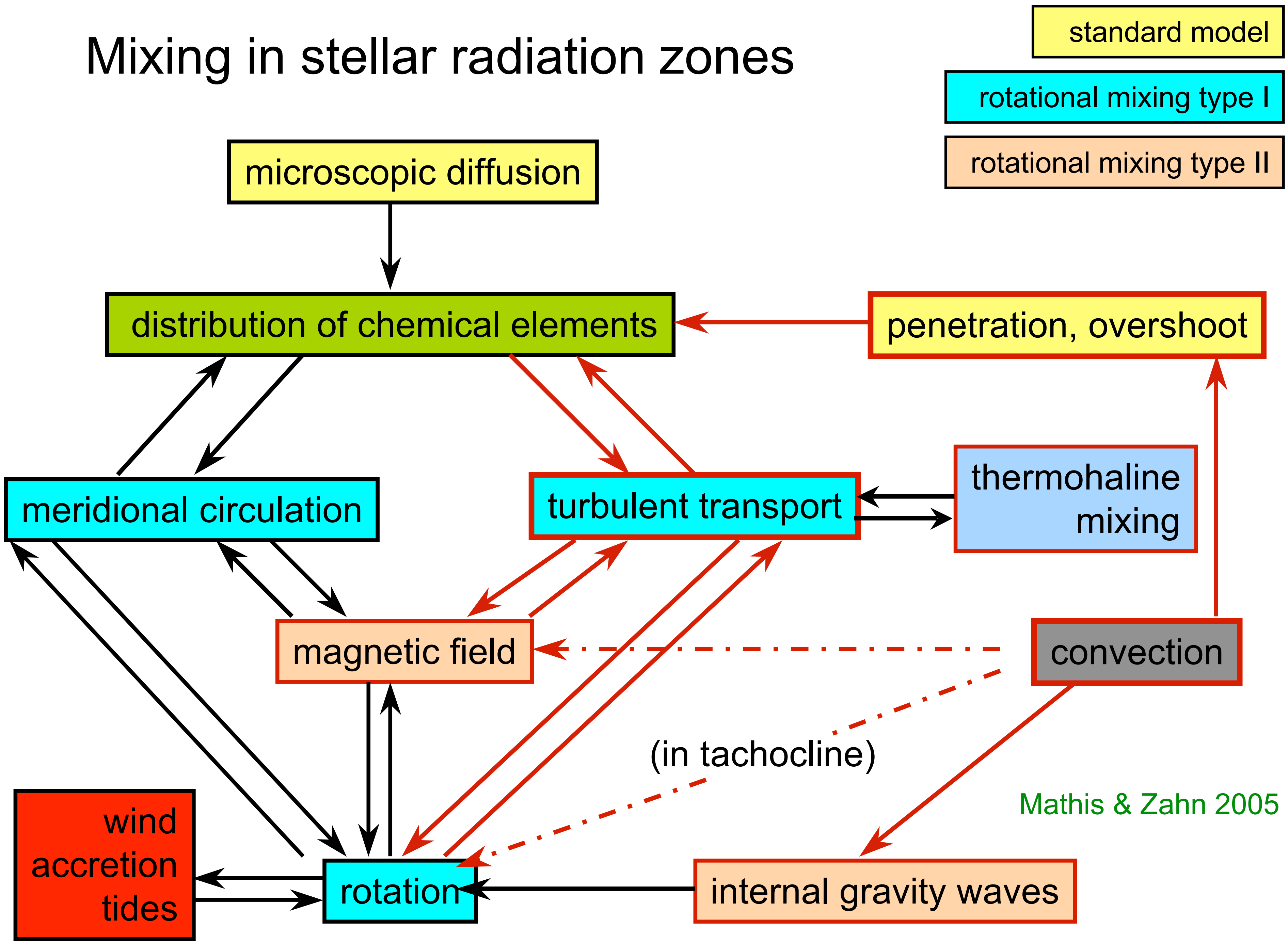
periodic BC in z   Stellmach et al 2010

Numerical simulations yield smaller aspect ratio

Traxler et al 2011

but they don't reach yet realistic Pe

Mixing in stellar radiation zones



Weakest points of present models with mixing in radiation zones

- Parametrization of the turbulence caused by differential rotation
- Power spectrum for IGW emitted at base of convection zone
- Particle transport by IGW ?
- Role of instabilities due to magnetic field ?
- Prescription for thermohaline mixing

Fortunately, the art of modeling stellar interiors progresses rapidly, thanks mainly to numerical simulations and to asteroseismology